Chapter 4

Radial oscillations and stability of quark stars with strongly coupled QGP in their interior

4.1 Introduction

The pioneering work of Chandrasekhar[1, 2] in the framework of general relativity revealed the existence of a dynamical instability, wherein gaseous masses become unstable with respect to radial pulsations well before the Schwarzschild limit is reached. His conclusion defines the sufficiency condition for the stability of a compact star - that it should be able to withstand small radial perturbations (The necessary condition being $\frac{\partial M(\epsilon_c)}{\partial \epsilon_c} > 0$, $M$ being the equilibrium stellar mass and $\epsilon_c$ the central energy density). In the previous chapter we have developed the SCQGP equation of state describing matter within cold, pure quark stars composed of massive and strongly
coupled quarks. From the mass sequences, obtained employing the equation of state, the necessary condition for stability is found to be satisfied (see Fig(3.3)). To check the sufficiency condition, the normal mode analysis of radial oscillations has to be carried out. The radial pulsations that preserve the spherical symmetry of a star do not result in the emission of gravitational radiation. Hence the normal mode analysis of such oscillations is less complicated and straightforward. The eigenequation of Chandrasekhar which governs the normal modes has the Sturm-Liouville form. The eigenmodes hence constitute a complete set and any arbitrary periodic radial motion can be expressed as their superposition.

Studies on radial pulsations of quark stars – hypothetical stars with quark matter in the interior – have been carried out earlier by a number of authors for different proposed equations of state for dense quark matter (For e.g. [3], [4], [5], [6], [7]). Here we start by analysing the radial oscillations of quark stars described by the SCQGP equation of state. We calculate the oscillation periods of the fundamental and first overtone for different values of the confining bag parameter ($B$). The eigen functions of the lowest three normal radial modes are then analysed, for the particular example of $B^{1/4} = 210\text{MeV}$. We then go on and study the damping of pulsations due to non-equilibrium processes. The corresponding neutrino emissivities are derived and the temporal evolution of pulsation energies are analysed. For illustration, calculations are performed for SCQGP stars with bag value, $B^{1/4} = 210\text{MeV}$ and the results are plotted for stellar masses 1.518, 1.845, $2\,M_\odot$. 
4.2 Normal radial modes of quark stars in the SCQGP model

When a compact star is in a state of hydrostatic equilibrium, the Einstein field equations, given the space-time metric (3.14) and the energy momentum tensor (3.15), yield the equations of structure known as the Tolman-Oppenheimer-Volkoff equations [8, 9]

\[
\frac{dm}{dr} = 4\pi r^2 \epsilon, \quad (4.1)
\]

\[
\frac{dp}{dr} = -\frac{(p + \epsilon)(m + 4\pi r^3 p)}{r(r - 2m)}, \quad (4.2)
\]

\[
\frac{d\nu}{dr} = -\frac{1}{p + \epsilon} \frac{dp}{dr}, \quad (4.3)
\]

Here \(m\) is the included mass within the coordinate \(r\). The metric function \(\lambda\) is given by

\[e^{2\lambda} = (1 - \frac{2m}{r})^{-1} \quad (4.4)\]

\(\lambda\) has the same form both inside and outside the star although it is the included mass \(m\) and not the total mass that appears in the interior solution. In order to match the exterior Schwarzschild solution the metric function \(\nu\) should obey the boundary condition

\[\nu(r = R) = \frac{1}{2} \ln(1 - \frac{2M}{r}), \text{ where } M \text{ is the mass of the star and } R \text{ its radius.}\]

The equations governing radial oscillations were originally obtained by Chandrasekhar[1, 2] on perturbing the equilibrium con-
configurations governed by equations (4.1)-(4.3) in a manner such that
the spherical symmetry is not violated and only the terms linear in
order are retained. In our discussion we denote the normal mode
motions of the star by $\delta r(r,t) = \xi_n(r) e^{i\omega_n t}$. Here $\xi_n(r)$ are the
normal mode amplitudes (or ‘eigenfunctions’) of the nth normal mode,
n=0 being the fundamental or nodeless mode. The quantities $\omega_n$
are the radial eigenfrequencies of the perturbed star.

Employing a new variable $u_n = r^2 e^{-\nu} \xi_n$, the Chandrashekar
eigenequation governing the radial modes appears in the the Sturm-
Liouville form

$$\frac{d}{dr} \left( P \frac{du_n}{dr} \right) + (Q + \omega_n^2 W) u_n = 0 \tag{4.5}$$

The functions $P(r)$, $Q(r)$ and $W(r)$ expressed in terms of equilibrium
configurations of the star are given by

$$P = e^{(\lambda+3\nu)r^{-2}\gamma p},$$

$$Q = -4e^{(\lambda+3\nu)r^{-3}} \frac{dp}{dr} - 8\pi e^{3(\lambda+\nu)r^{-2}} p(\epsilon + p)$$

$$+ e^{(\lambda+3\nu)r^{-2}} (\epsilon + p)^{-1} \left( \frac{dp}{dr} \right)^2,$$

$$W = e^{(3\lambda+\nu)r^{-2}} (\epsilon + p),$$

Here $\gamma = \frac{(\epsilon + p)}{p} \frac{dp}{d\epsilon}$ denotes the adiabatic index. Solutions to the
eigenequation are physically acceptable only if they satisfy certain
boundary conditions. At the center of the star the requirement that
$\delta r$ and $\frac{d\delta r}{dr}$ are finite there leads to the condition

$$\frac{u_n}{r^3} \text{ should be finite or zero as } r \to 0 \tag{4.7}$$
At the surface of the star the Lagrangian change in pressure, $\Delta p$ should vanish which leads to the condition

$$\Delta p = -\frac{\gamma p e^{\nu} du_n}{r^2} = 0 \text{ at } r = R.$$ (4.8)

The eigenvalue can now be solved using standard procedure to obtain the frequency spectrum $\omega_n^2 (n = 0, 1, 2, \ldots)$ of the normal modes. The squared normal mode frequencies being eigenvalues of the Sturm-Liouville equation are real and form an infinite discrete sequence, $\omega_0^2 < \omega_1^2 < \omega_2^2 < \ldots$. For a star to be stable against radial perturbations $\omega^2$ should be positive since then $\omega$ itself is real. If any of the eigenvalues, $\omega^2$, is negative then $\omega$ would be purely imaginary leading to a solution that grows exponentially as $e^{i\omega t}$. Thus a negative value of $\omega^2$ indicates instability. Since the frequencies increase sequentially with n, $\omega_0^2 > 0$ is the sufficient condition for stability.

The eigenmode analysis performed employing the equation of state for SCQGP with different values for the confinement parameter $B$, yields the spectrum of eigenfrequencies, $\omega_n^2$. The squared frequencies $\omega_n^2$ go to zero as the maximum mass star is reached, as expected. We have plotted the results obtained by the normal mode analysis in Fig.(4.1) and Fig.(4.2).

Solid lines indicate the period $(\tau_n = 2\pi/\omega_n)$ calculated for the SCQGP equation of state for different $B$ values ($B^{1/4} = 190, 200, 210MeV$). Dotted lines indicate the period calculated for strange stars composed of non-interacting quarks with $m_u = m_d = 0, m_s = 150MeV$.
Figure 4.1: (a),(b)-Oscillation periods ($\tau$), calculated for the SCQGP EOS, as a function of central energy density ($\epsilon_c$) for the fundamental and first excited modes respectively. The curves are labelled I,II & III corresponding to values of confining bag parameter $B^{1/4} = 190, 200 & 210 MeV$ in order.

within the MIT Bag Model for $B^{1/4} = 145 MeV$, which we have plotted for comparison. Figures (4.1a), (4.1b) show the variation in period with central energy density ($\epsilon_c$) for the fundamental and first excited modes respectively. The resulting pattern of curves indicates that for a particular value of $\epsilon_c$, of the various mass sequences obtained for different values of the bag constant, if we pick the more massive star - it has a higher value of pulsation period . It is found that pulsation periods tend to zero as the central density approaches its minimum possible value, a property characteristic to quark stars as opposed to hadronic stars [4]. The Fig(4.2a) shows the period as a function of the stellar mass $M$ (in units of solar mass) for the fundamental radial mode ($n=0$). It is found that for lower mass stars the pulsation periods of the fundamental mode are typically of the order of one tenth of a millisecond and have negligible dependence on the bag parameter. For medium and higher mass stars a variation
Figure 4.2: (a),(b)-Oscillation periods as a function of mass M for the fundamental and first excited modes respectively. Solid lines represent the period calculated for the SCQGP EOS and are labelled I,II & III corresponding to values of confinement parameter $B^{1/4} = 190, 200 & 210 MeV$ respectively. Dotted lines indicate the period obtained for strange stars composed of non-interacting quarks with $m_u = m_d = 0, m_s = 150 MeV$ within the MIT Bag Model for $B^{1/4} = 145 MeV$.

of pulsation periods with change in the confining bag parameter(B) is seen - the periods show a decrease with decrease in bag constant. The behaviour may be explained by noting that the stiffness of the equation of state tends to increase with decrease in the confining bag constant. Thus for quark stars of the same mass decreasing B value indicates a stronger coupling between the quark constituents which increases the normal mode frequencies/ lowers the pulsation periods.

The behaviour is more pronounced in the case of intermediate to higher mass stars. Comparing with strange stars composed of non-interacting quarks with $m_u = m_d = 0, m_s = 150 MeV$ treated within the MIT Bag model with bag constant $B^{1/4} = 145 MeV$ we see that the oscillation periods show considerable difference throughout the entire range of stellar masses with the difference increasing
with decrease in bag parameter value (increasing stiffness) for the SCQGP equation of state. For SCQGP stars the oscillation periods are 2 to 3 times lower than that for strange stars with non-interacting quarks for the chosen bag constants (In earlier work by [10], [11] a similar difference was seen in the case of hadronic stars with and without interaction). In Fig(4.2b) we have plotted the pulsation periods for the first excited mode (n=1) as a function of stellar mass M again for SCQGP stars as well as for strange stars within the non-interacting bag model. For the first excited mode the pulsation periods have typical values in the range, approx. $1/2 - 1/3$ that of the fundamental mode. The distribution of curves follow the pattern akin to that of the fundamental mode discussed above, with the compared difference with the non-interacting model now relatively less.

### 4.3 Normal mode eigenfunctions of the radial modes and the energy stored in the pulsations

A careful study of the behaviour of the normal mode amplitudes with radial distance $r$ provides insight into how matter described by a particular equation of state responds to radial perturbations. The normal mode amplitudes of radial oscillations are given by the eigenfunctions, $\xi_n(r)$, of the Sturm-Liouville equation (eqn.(4.5)). We now examine the variation of the normal mode amplitudes with radius $r$, by plotting and analysing the eigenfunctions, $\xi_n(r)$ for strongly coupled quark matter. The eigenfunctions are normalized
Figure 4.3: The 'relative eigenfunctions' $\xi/r$, for the SCQGP equation of state, plotted against the 'relative radius' $r/R$, for central energy densities $\epsilon_c = 0.6, 0.84, 1.3$ and $1.55 \text{ GeV/fm}^3$ and bag parameter $B^{1/4} = 210 \text{ MeV}$.

Fig(4.3a) represents the fundamental mode ($n=0$). Fig(4.3b) and Fig(4.3c) represent the first ($n=1$) and second ($n=2$) excited modes respectively.

using the condition

$$\lim_{r \to 0} \frac{\xi_n}{r} = \Delta$$

$\Delta$ is a small normalization parameter. For illustration we choose the bag parameter as $B^{1/4} = 210 \text{ MeV}$. Since it allows for a better comparison the 'relative eigenfunctions' $\xi_n/r$ are plotted against 'relative radius' $r/R$, $R$ being the radius of the star. The typical
eigenfunctions for the fundamental \((n = 0)\), first excited \((n = 1)\) and second excited \((n = 2)\) modes, with normalization parameter \(\Delta = 1\), are shown in Fig(4.3), each for central energy densities \(\epsilon_c = 0.6, 0.84, 1.3\) and 1.55 \(GeV/fm^3\). The corresponding stars have masses 0.56, 1.65, 1.98, 2.01\(M_\odot\) respectively. For the fundamental mode the relative amplitude deviates from the homologous behaviour \((\xi/r = 1)\) starting from the core and continues to decrease with an increasing slope towards the outer layers of the star.

The shapes of the normal mode amplitudes tend to be determined by the degree of homogeneity of the stellar model. The fundamental mode is approximately homologous only if the logarithm of energy density \((\log_{10}\epsilon)\) and the adiabatic index \((\gamma)\) are roughly constant throughout the configuration barring the outermost layers [11]. In the current model both \(\log_{10}\epsilon\) and \(\gamma\) are found to vary throughout the star. The variation in \(\gamma\) is more striking. In Fig(4.4) a plot of the adiabatic index as a function of energy density is shown.

![Figure 4.4: The variation in the adiabatic index \(\gamma\) with energy density \(\epsilon\) for the SCQGP equation of state with bag parameter \(B^{1/4} = 210 MeV\)](image)

The adiabatic index is found to increase first slowly and then
steeply with decreasing energy density. The behaviour of the adiabatic index can be attributed to the stiffness of quark matter due to strong coupling between quarks as well as the effect of the bag. With increasing adiabatic index the compressibility of matter decreases thereby accounting for the steady decrease in $\xi/r$. The fundamental mode is hence found to be much sensitive to changing adiabatic index. Though not so evident, on careful examination it is seen that there is a slight increase in the relative amplitude of the fundamental mode with increasing central density as we move towards the maximum mass star. The relative eigenfunctions of the first and second excited modes show sinusoidal behaviour but with increasingly smaller values in the outer layers of the star.

Once the spatial distribution of normal mode amplitudes $\xi_n(r)$ are given, the pulsation energy stored in the radial oscillations can be computed. Just like an arbitrary pulsation can be expressed as a superposition of the normal modes, so the pulsation energy can be written in terms of the normal mode components [11]

$$E_{\text{puls}} = \sum_n A_n^2 E_{\text{puls}}^{(n)}$$  \hspace{1cm} (4.10)$$

with

$$E_{\text{puls}}^{(n)} = 2\pi \omega_n^2 \int_0^R W(r^2 e^{-\nu} \xi_n)^2 dr$$  \hspace{1cm} (4.11)$$

The function $W(r)$ is given by eqn(4.6). The dimensionless amplitudes $A_n$ can be determined from pulsation damping mechanisms which operate to dissipate the energy stored in the pulsations. We consider the damping of radial pulsations and the resultant temporal evolution of pulsation energy in the next section. We restrict
our calculations to small amplitude pulsations ($\Delta \ll 1$) and for illustration choose the bag parameter value $B^{1/4} = 210 \text{MeV}$.

### 4.4 Damping of pulsations by non-equilibrium processes

For a non-vibrating quark star with matter described by the zero temperature SCQGP equation of state, the condition for $\beta$ equilibrium is given by the relation\(^1\), $\delta \mu = \mu_d - \mu_u - \mu_e = 0$. Here $\mu_i$ are the chemical potentials of the particle species $u$, $d$ and $e^-$. Radial pulsations drive the stellar matter out of chemical equilibrium in which case $\delta \mu(r, t) \neq 0$. The processes tending to restore the matter back to equilibrium lead to the damping of pulsations. The most efficient of the non-equilibrium processes is the direct-Urca process.

The direct-Urca processes are simple $\beta$-decay processes which in the case of ordinary neutron star matter are the reactions $n \rightarrow p + e^- + \nu_e$ and $p + e^- \rightarrow n + \nu_e$. In ordinary neutron stars the direct-Urca process is forbidden since the laws of conservation of momentum and energy cannot be satisfied for the expected composition of neutron star matter. On the other hand the analogous processes can occur for quark matter - the energy and momentum conservation laws are satisfied once we take into consideration the the finite quark masses and/or the interaction between quarks [12]. In the SCQGP phase we have assumed the quarks to be massive and interacting. Hence we

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\(^1\)Here we consider quark matter to be composed of u,d quarks and electrons such that conditions of $\beta$-equilibrium and charge neutrality are satisfied. In the original SCQGP EOS, the contribution due to electrons was ignored since the corresponding modification to the EOS is negligible at quark matter densities.
expect direct-Urca process to be the primary and dominant source of pulsation damping in such quark stars.

In what follows we analyse the quark direct-Urca process in the domain of SCQGP model and calculate the associated neutrino luminosities. Initially we consider the general case of a vibrating quark star at finite temperature $T$. In this connection we closely follow Iwamoto[12, 13] while making appropriate modifications pertinent to the context. Iwamoto has derived the neutrino luminosities for equilibrium quark matter at finite $T$ which we adapt to the non-equilibrium case.

For quark matter devoid of strange quarks the direct-Urca process is given by the reactions

$$d \rightarrow u + e^- + \bar{\nu}_e \quad (4.12)$$

and

$$u + e^- \rightarrow d + \nu_e \quad (4.13)$$

The rate at which energy is lost due to neutrino emission process (4.12) in a unit volume, the neutrino emissivity, is given by

$$\varepsilon_{\nu_e}(T, \delta \mu) = 6V^{-1} \left( \prod_{i=1}^{4} V \int \frac{d^3 p_i}{(2\pi)^3} \right) E_2 \times W_{fi} n(p_1) [1 - n(p_3)] [1 - n(p_4)] \quad (4.14)$$

$W_{fi}$ is the transition rate for $\beta$-decay given by

$$W_{fi} = V (2\pi)^4 \delta^4(p_1 - p_2 - p_3 - p_4) \mid M \mid^2 / \prod_{i=1}^{4} 2E_i V \quad (4.15)$$

where the four-vectors, $p_i = (E_i, \vec{p}_i)$, numbered from $i = 1$ to 4, de-
note the particles $d, \bar{\nu}_e, u, e^-$ in order. The factor 6 stands for the three color and two spin degrees of freedom of the initial $d$ quark, $V$ represents the normalization volume and $n(p_i) = (1 + \exp[(E_i - \mu_i)/kT])^{-1}$ is the Fermi-distribution function. The term $(1-n(p))$ ensures that the exclusion principle is obeyed. $|M|^2$ is the squared invariant amplitude averaged over initial $d$ quark spin ($\sigma_1$) and summed over the final spins of $u$ quark ($\sigma_3$) and electron ($\sigma_4$),

$$|M|^2 = \frac{1}{2} \sum_{\sigma_1,\sigma_3,\sigma_4} |M_{fi}|^2 = 64 G^2 \cos^2 \theta_c (p_1.p_2)(p_3.p_4)$$ \hspace{1cm} (4.16)$$

where the weak-coupling constant, $G \approx 1.435 \times 10^{-49} \text{erg cm}^3$ and $\theta_c$ is the Cabibbo angle ($\cos^2 \theta_c \approx 0.948$). In the SCQGP model $u$ and $d$ quarks are massive and non-relativistic while electrons are ultra-relativistic. We therefore have

$$(p_1.p_2)(p_3.p_4) \simeq E_1 E_2 E_3 E_4 \left(1 - \frac{|\vec{p}_3|}{E_3} \cos \theta_{34}\right)$$

$$\times \left(1 - \frac{|\vec{p}_i|}{E_1} \cos \theta_{12}\right)$$ \hspace{1cm} (4.17)$$

with $\cos \theta_{ij} = \vec{p_i}.\vec{p_j}/|\vec{p_i}||\vec{p_j}|$. In degenerate Fermi systems only those particles with momenta that lie close to the respective Fermi surfaces can participate in a reaction. Hence we can replace the magnitude of quark and electron momenta by the respective Fermi momenta. Accordingly $\cos \theta_{34}$ can be expressed as a function of Fermi-momenta ($p_F(i)$) of the involved fermions. We neglect the neutrino momentum in our calculations. The integrals in the ex-
pression for emissivity can now be decoupled and one can write

$$\varepsilon_{\nu}(T, \delta \mu) = \frac{24G^2 \cos^2 \theta_c}{(2\pi)^8} \left( 1 - \frac{p_F(u)}{m_u^*} \cos \theta_{34} \right) A B$$  \hspace{1cm} (4.18)


$m_i^*$ denotes the quark effective mass, $i = d, u$. Here

$$A = \left( \prod_{i=1}^{4} \int \, d\Omega_i \right) \delta^3(p_1^\nu - p_3^\nu - p_4^\nu) \left( 1 - \frac{|p_1^\nu|}{E_1} \cos \theta_{12} \right)$$  \hspace{1cm} (4.19)

is an angular integral and

$$B = p_F(d) p_F(u) p_F^2(e) m_d^* m_u^* \int_0^\infty dE_1 \int_0^\infty E_2^2 dE_2 \times \int_0^\infty dE_3 \int_0^\infty dE_4 E_2 S \delta(E_1 - E_2 - E_3 - E_4)$$  \hspace{1cm} (4.20)

is the energy integral with $S = n(p_1^\nu)(1 - n(p_3^\nu))(1 - n(p_4^\nu))$. The angular integral can be done analytically to give $A = 32 \pi^3 / p_F(d) p_F(u) p_F(e)$.

The energy integral can be evaluated using standard procedure and can be written, in terms of dimensionless variables $y = E_2/kT$ and $\delta \tilde{\mu} = \delta \mu/kT$ as

$$B = p_F(d) p_F(u) p_F^2(e) m_d^* m_u^* \frac{(kT)^6}{2} F(\delta \tilde{\mu})$$  \hspace{1cm} (4.21)

The dimensionless function $F$ is defined as

$$F(x) = \int_0^\infty y^3 dy \left( \frac{\pi^2 + (y - x)^2}{1 + \exp(y - x)} \right)$$  \hspace{1cm} (4.22)

Next we consider the limiting case in which $\delta \tilde{\mu} \gg 1$. Then the function $F$ can be represented by the asymptotic formula $F \approx (1/60)(\delta \tilde{\mu})^6$ [14]. If we finally write down the neutrino emissivity
in this particular limiting case \( \delta \mu \gg 1 \), we obtain the temperature independent form

\[
\varepsilon_{\nu_e}(\delta \mu) = \frac{G^2 \cos^2 \theta_c}{40 \pi^5 \hbar^4 c^4} p_F(e) m_u^* m_u^* \\
\times \left( 1 - \frac{p_F(u)}{m_u^* c \cos \theta_{34}} \right) (\delta \mu)^6
\]

(4.23)

This final equation for neutrino emissivity is what we need in our specific case of a zero temperature pulsating SCQGP quark star. Following the same method we can calculate the \( \nu_e \) emission rate from the inverse process (4.13) which yields the same expression as the above in the limiting case \( -\delta \mu \gg 1 \). In this latter limiting condition the inverse process of \( \nu_e \) emission (4.13) dominates over the \( \bar{\nu}_e \) emission process (4.12) (which is then negligible in comparison). In the former limit the converse is true. It has to be pointed out that during our calculation of neutrino emissivities we naively replace the quark effective masses by their constituent masses. To obtain an expression for the Fermi momenta \( (p_F(q)) \) of strongly coupled degenerate quarks we have utilised the formula for Fermi momentum derived in the case of an electron fluid with Coulomb interactions by Isihara & Kojima [15]. Appropriate modifications are done to include the quark color degrees of freedom primarily by replacing \( r_s \) for degenerate electron system by \( r_s \) for degenerate, massive quarks given by eqn(3.8). In natural units we can write

\[
p_F(q) = \frac{8}{3} M_q \alpha_s \left( \frac{0.95957}{r_s} A(r_s) \right) MeV
\]

(4.24)

with \( A(r_s) = 1 - 0.16586 r_s + r_s^2 (0.0084411 ln r_s - 0.027620) \)
To proceed with our analysis of pulsation damping we next write down the relationship between the chemical potential difference $\delta \mu(r, t)$ and an arbitrary pulsation $\xi(r, t)$. The relation has been derived in the context of neutron $\beta$-decay [16], but can be readily applied to quarks which possess a baryon number. The relation is as follows

$$\delta \mu(r, t) = -\frac{\partial \delta \mu(n_b, x_e)}{\partial n_b} n_b \frac{e^\nu}{r^2} \frac{\partial}{\partial r} \left( r^2 e^{-\nu} \xi(r, t) \right)$$  \hspace{1cm} (4.25)

Here the partial derivative with respect to $n_b$ is taken at constant $x_e = n_e/n_b$. The variables $n_b, n_e$ are the equilibrium baryon number density, electron number density respectively. Using the above equation we can express the neutrino emissivities in terms of $\xi(r, t)$. If we denote the total neutrino emissivity by $\varepsilon = \varepsilon_\nu + \varepsilon_\nu$, then the total redshifted neutrino luminosity $L$ of the star is given by

$$L = \int_0^R \overline{\varepsilon} e^{2\nu} dV$$  \hspace{1cm} (4.26)

where $\overline{\varepsilon}$ is the total neutrino luminosity averaged over a pulsation period and $dV$ is the proper volume, $dV = 4\pi r^2 e^\lambda dr$.

We now go forth and calculate the pulsation damping in the fundamental mode of a zero temperature SCQGP quark star. In the presence of pulsation damping the normal mode motions can be represented by, $\xi_0(r, t) = A_0(t) \xi_0(r) \cos \omega t$. The dimensionless amplitude $A_0(t)$ is a slowly decreasing function of time with $A_0(t = 0) = 1$. The pulsation energy in the fundamental mode with damping is $E_{puls} = E_{puls}^{(0)} A_0^2(t)$. Now assuming the direct-Urca reactions to be the only major damping mechanism present,
the rate of energy loss from pulsations is equal to the total neutrino luminosity:

$$- \frac{dE_{\text{puls}}}{dt} = L$$

(4.27)

On solving the above equation with the initial condition $A_0(0) = 1$, we obtain the variation in pulsation energy with time. We have calculated the neutrino luminosities and pulsation energies in the fundamental mode for SCQGP quark stars for the bag parameter $B^{1/4} = 210$ MeV and normalization parameter $\Delta = 0.01$. The chosen central densities are $\rho_c = 1.392 \times 10^{15}, 1.783 \times 10^{15}, 2.54 \times 10^{15}$ gm cm$^{-3}$ which yield stars of masses 1.518, 1.845, 2$M_\odot$ respectively. The corresponding neutrino luminosities are plotted in Fig(4.5a) as a function of time, $t$. The variation in pulsation energies with time is shown in Fig(4.5b). The figure demonstrates that the capacity of higher mass quark stars, to store the pulsation energy is much less than that of lower mass stars. Lower mass stars can store a relatively increased pulsation energy for longer intervals of time as compared to higher mass stars. This behaviour can be explained by noting that the pulsation energy is dependent on the square of the normal mode frequency. From earlier calculations (see figures Fig(4.1) and Fig(4.2)) it is evident that the normal mode frequencies of quark stars decrease with increase in stellar mass $M$. Now the rate of energy loss due to pulsation damping tapers off with time. Therefore it comes out that lower mass stars can retain higher pulsation energies for longer durations when compared to higher mass stars. The neutrino luminosities plotted in Fig(4.5a)
Figure 4.5: Fig. (4.5a) shows the neutrino luminosities (L), in the fundamental mode for SCQGP stars with bag parameter $B^{1/4} = 210\,\text{MeV}$, as a function of time. The chosen central densities are $\rho_c = 1.392 \times 10^{15}, 1.783 \times 10^{15}, 2.54 \times 10^{15} \,\text{gm cm}^{-3}$ with stellar masses $1.518, 1.845, 2M_{\odot}$ respectively. The normalization parameter $\Delta = 0.01$. Fig. (4.5b) shows the temporal evolution of pulsation energy in the fundamental mode for SCQGP stars with chosen bag parameter, central densities and corresponding stellar masses as in Fig. (4.5a).
show a more dramatic behaviour. The neutrino luminosity initially follows the pattern \( L_{2\odot} > L_{1.845\odot} > L_{1.518\odot} \). In a matter of hours this pattern is found to be reversed. The baryon number density \( n_b(r) \) of higher mass quark stars have values larger compared to their lower mass counterparts. Hence the moment the damping is ‘switched on’, the number of triggered \( \beta \)-reactions and the corresponding number of emitted neutrinos are far greater than that for lower mass stars - the initial pattern for luminosity results. Now the oscillation frequencies of lower mass quark stars are larger compared to higher mass quark stars. Therefore with passage of time the \( \beta \)-reaction rates of lower mass stars catch up with the higher mass stars which oscillate with a lower frequency. Hence a reversal of pattern results.
References


