CHAPTER 5

CONCLUSION
The study of oscillating systems (oscillators) has been of prime interest in engineering and physics for long time. An oscillator is a device or system which is slightly disturbed from its stable equilibrium position and then the change in distance from the equilibrium point is studied over the time. In our daily life the oscillations are commonly seen in many mechanical systems for example motion of a clock pendulum, vibrations of strings on musical instruments, playground swing and tuning fork etc. Electric oscillations are seen in alternating current, electronic oscillator, Hartley oscillator, RLC circuit etc. There are some other applications in various fields of science and technology for example in electro mechanical systems such as loudspeaker and microphone, in biological systems such as neural oscillations and circadian rhythm, in astrophysics such as neutron stars and cyclic model etc. All physical oscillators can be broadly classified into two categories: linear and nonlinear oscillators. In linear oscillatory systems the forces acting are completely linear in nature however for the nonlinear systems the forces or some of the forces are nonlinear. The modeling of these systems respectively can be done through linear and nonlinear differential equations hence the principle of superposition does not hold for the nonlinear systems. Two of the very interesting features of nonlinear systems are: (i) the frequency depends on the amplitude of oscillations and (ii) the dynamics of these systems depends heavily on the initial conditions. The study of nonlinear systems i.e. nonlinear dynamics has become an important subject with the works of Van der Pol & Van der Mark [Van der Pol and Van der Mark (1927)] studying a simple nonlinear electronic circuit consisting of a neon tube as a nonlinear element and Henry Poincare [Poinacare (1885)] solving the three body problem. Later approaches from this field had been used in many other fields. With the discovery of modern computers, scientists solved many nonlinear problems and Lorentz [Lorentz (1963)] numerically first coined the term chaos while studying the weather prediction through fluid convection model. Since then chaos theory flourished very rapidly in different branches of natural sciences. Nonlinear problems acquired a gradually increasing importance in various branches of applied science. The theory of oscillations is the most explored branch, where the
nonlinear theory has been mostly worked out and applied to numerous phenomena. There are other branches into which nonlinear problems began to penetrate in recent years, such as theory of automatic controls systems, econometrics, biology, astronomy, atomic theory etc., but in all these new developments the fundamentals remain same.

Nonlinearity is ubiquitous in physical phenomena. Fluid and plasma mechanics, gas dynamics, elasticity, relativity, chemical reactions, combustion, ecology, biomechanics, and many other phenomena are all governed by inherently nonlinear processes and governed/modelled by nonlinear differential or difference equations. The nonlinear systems are generally difficult to analyze due to the facts- existence and uniqueness of solutions are not guaranteed; explicit formulae are difficult to come by; linear superposition is no longer available; numerical approximations are not always sufficiently accurate etc. A thorough understanding of linear phenomena and linear mathematics is an essential prerequisite for progress in the nonlinear arena. Moreover, many important physical systems are “weakly nonlinear”, in the sense that, while nonlinear effects do play an essential role, the linear terms tend to dominate the physics, and so, to a first approximation, the system is essentially linear. As a result, such nonlinear phenomena are best understood as some form of perturbation of their linear approximations.

Nonlinearity in the oscillating system may exist in various forms e.g. in a mechanical system the nonlinearity may be due to the presence of nonlinear elastic / spring elements, nonlinear damping, system with fluid, nonlinear boundary conditions etc., in an electromagnetic system the nonlinear resistive, inductive, capacitive elements, hysteresis of ferromagnetic materials, nonlinear active elements like vacuum tube, transistor etc. may be responsible for nonlinear effects in the systems. The mechanism of supply (source) and dissipation of energy in the oscillator combinedly plays a very important role in deciding the dynamical behaviour of the system. One of the very common and ubiquitous forms of dissipation in oscillating systems is damping. Damping mainly reduces the amplitude of oscillations. The linear (viscous) damping is one of the most
common forms of damping present in many physical systems. However, the consideration of \textit{nonlinear damping} is also necessary in many of the engineering/physical problems such as rolling in the ship dynamics [Thompson et al. (1990)], vibration isolators [Mallik (1990)], drag forces in flow induced vibrations [Pippard (1989)] etc. Hence, the study of oscillating physical systems under the presence of \textit{nonlinear damping} is also an important area of active research and specifically the study on \textit{nonlinear oscillating system} under the \textit{nonlinear damping} is relatively new and unexplored area of research. Hence, the subject of the present thesis has been chosen centered around the study of dynamical behaviour of some of the very common nonlinear oscillators under the presence of nonlinear damping.

The main objective of this thesis work was to see the effect of nonlinear damping on the dynamical behaviour of some of the ubiquitous classical nonlinear oscillators. We have mainly concentrated on the theoretical and computational studies on the driven pendulum, forced Duffing, forced Helmholtz-Duffing, parametrically driven Duffing oscillators using the nonlinear damping term proportional to the $\nu |\nu|^{p-2}$, where, ‘$p$’ is known as damping exponent and ‘$\nu$’ is the velocity. We have particularly focused our attention on how the damping exponent affects the global dynamical behaviour of the oscillator. In particular, we have obtained analytically the threshold condition for the occurrence of homoclinic bifurcation using Melnikov technique and compare the results with the computational results. We have also identified the major route to chaos and the regions of the 2D parameter space (consists of external forcing amplitude and damping coefficient) corresponding to the various types of asymptotic dynamics under linear and nonlinear damping. We have also attempted to analyze how the basin of attraction patterns change with the introduction of nonlinear damping.

One of the prime objectives of the present study has been to identify some general dynamical features of nonlinear oscillators under nonlinear damping. To achieve these objectives, we have done the extensive computational and analytical study on above mentioned four important physical oscillators under the effect of
nonlinear damping term proportional to the power of velocity and the main important conclusions are as follows:

- For the forced nonlinearly damped Duffing oscillator, we have observed that the nonlinear damping:
  - Lowers the critical value of the forcing amplitude corresponding to first transition to chaos.
  - Increases the chaos in parameter space.
  - Increases the fractalness of the phase space attractor.
  - Increases the fragileness of the chaotic regions in the parameter space.
  - Affects the route to chaos in the system.

- For forced nonlinearly damped simple pendulum, we have observed that the nonlinear damping:
  - Decreases the number of periodic windows i.e. chaos becomes less fragile.
  - Decreases the range of external forcing amplitude (F) for which chaos exists.
  - Decreases the complexity of the phase space attractors.
  - Chaos is less global in parameter space.
  - Does not increase the fractalness of the basin boundaries.

- For parametrically driven Duffing oscillator, we have observed that the nonlinear damping:
  - Lowers the critical value of the forcing amplitude corresponding to first transition to chaos.
  - Periodic windows are wider.
  - Decreases the chaos in parameter space.

- For Helmholtz-Duffing oscillator, we have observed that the nonlinear damping:
• Lowers the critical values of the forcing amplitude corresponding to first transition to chaos.
• Increases the chaos in parameter space.
• Chaos is more global in symmetric case.

Our extensive study on four oscillators under the effect of nonlinear damping reveals that presence of nonlinear damping has similar effects on the three of the oscillators namely: forced Duffing, forced Helmholtz-Duffing and parametrically forced Duffing oscillator. However the effect of nonlinear damping on the forced simple pendulum is opposite in nature than what we observe for the other three oscillators. The possible reason may be that the three oscillators sharing the common features/effect have a double well potential however the simple pendulum is a multi-well potential system. Hence more such studies on various nonlinearly damped nonlinear oscillators are required to find the universalities in the patterns of dynamical behaviour of these systems. The studies with other models of nonlinear damping will also be helpful in understanding the hidden features of nonlinear oscillators.