CHAPTER 4
COMPLEMENT AND OPERATIONS ON
INTUITIONISTIC FUZZY GRAPHS

4.1 INTRODUCTION


In this chapter, the definition of complement of intuitionistic fuzzy graphs and some of its properties are analyzed. The operations “∪”, “+” and cartesian product “×”, have been modified and redefined for the sets with one or more common vertices. The operations “∪” and “+” which are found to be complementary to each other have been analyzed with relevant examples. Application of complement intuitionistic fuzzy graphs in knowledge management system has been analyzed.

4.2 COMPLEMENT OF INTUITIONISTIC FUZZY GRAPHS

Definition : 4.2.1

The complement of an intuitionistic fuzzy graph \( G : (V, E) \) denoted by \( \overline{G} : (\overline{V}, \overline{E}) \) is defined by

(i) \(-\mu_A(x) = \mu_A(x); -\nu_A(x) = \nu_A(x) \quad \forall x \in V\)

(ii) \(-\mu_B(xy) = \begin{cases} \frac{[\mu_A(x) \cap \mu_A(y)] - \mu_B(xy)}{[\mu_A(x) \cap \mu_A(y)]} & \forall xy \in E \\
\frac{[\nu_A(x) \cap \nu_A(y)] - \nu_B(xy)}{[\nu_A(x) \cap \nu_A(y)]} & \forall xy \notin E \end{cases}\)

\(-\nu_B(xy) = \begin{cases} \frac{[\nu_A(x) \cup \nu_A(y)] - \nu_B(xy)}{[\nu_A(x) \cup \nu_A(y)]} & \forall xy \in E \\
\frac{[\nu_A(x) \cup \nu_A(y)] - \nu_B(xy)}{[\nu_A(x) \cup \nu_A(y)]} & \forall xy \notin E \end{cases}\)
Example: 4.2.2

Consider the graph G with V = {1, 2, 3, 4, 5} and E = {12, 13, 23, 24, 35, 45}.

<table>
<thead>
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<table>
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<td>0.3</td>
<td>0.2</td>
<td>0.15</td>
<td>0.1</td>
</tr>
<tr>
<td>( \nu_B )</td>
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<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
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</tbody>
</table>

Table 4.1

![Graph G: (V, E)](image)

Fig. 4.1 G : (V, E)

The corresponding adjacency Matrix is

\[
A = \begin{bmatrix}
(0.4, 0.3) & (0.2, 0.1) & (0.4, 0.3) & (0.0) & (0.0) \\
(0.2, 0.1) & (0.3, 0.3) & (0.3, 0.3) & (0.2, 0.1) & (0.0) \\
(0.4, 0.3) & (0.3, 0.3) & (0.6, 0.2) & (0.0) & (0.15, 0.4) \\
(0.0) & (0.2, 0.1) & (0.0) & (0.5, 0.2) & (0.1, 0.3) \\
(0.0) & (0.0) & (0.15, 0.4) & (0.1, 0.3) & (0.4, 0.6)
\end{bmatrix}
\]

We compute the complement of the above G : (V, E) as given in the definition 4.2.1
Theorem : 4.2.3

For any intuitionistic fuzzy graph $G : (V, E)$, $G : (V, E) \cong \overline{G} : (\overline{V}, \overline{E})$.

Proof :

(i) $\overline{\mu_A(x)} = \overline{\mu_A(x)} = \mu_A(x) ; \overline{\nu_A(x)} = \overline{\nu_A(x)} = \nu_A(x) \forall x \in \overline{V}$

(ii) $\overline{\mu_B(xy)} = \begin{cases} \begin{align*} &[[\mu_A(x) \wedge \mu_A(y)] - \overline{\mu_B(xy)}] \quad \forall xy \in \overline{E} \\ &[[\mu_A(x) \wedge \mu_A(y)] \quad \forall xy \not\in \overline{E} \end{align*} \end{cases}$

$= [[\mu_A(x) \wedge \mu_A(y)] - (\mu_A(x) \wedge \mu_A(y)) - \mu_B(xy)] \forall xy \in \overline{E}$

$= \mu_A(x) \wedge \mu_A(y)] - (\mu_A(x) \wedge \mu_A(y))] + \mu_B(xy) \forall xy \in \overline{E}$

$= \mu_B(xy) \forall xy \in \overline{E}$

Similarly it can be shown that $\overline{\nu_B(xy)} = \nu_B(xy) \forall xy \in \overline{E}$

Hence $G : (V, E) \cong \overline{G} : (\overline{V}, \overline{E})$.

Theorem : 4.2.4

If $G$ is a strong intuitionistic fuzzy graph, then $\overline{G}$ is also strong intuitionistic fuzzy graph.
Proof:

Let \( xy \in A \), 
\[
\mu_B(xy) = [\mu_A(x) \land \mu_A(y)] - \mu_B(xy)
\]
\[
= [\mu_A(x) \land \mu_A(y)] - [\mu_A(x) \land \mu_A(y)] \quad [\text{since } G \text{ is strong}]
\]
\[
= 0
\]
\[
\nu_B(xy) = [\nu_A(x) \lor \nu_A(y)] - \nu_B(xy)
\]
\[
= [\nu_A(x) \lor \nu_A(y)] - [\nu_A(x) \lor \nu_A(y)] \quad [\text{since } G \text{ is strong}]
\]
\[
= 0
\]

If \( xy \notin A \), then 
\[
\mu_B(xy) = [\mu_A(x) \land \mu_A(y)] - \mu_B(xy)
\]
\[
= \mu_A(x) \land \mu_A(y) \quad [\text{since } \mu_B(xy) = 0]
\]
\[
\nu_B(xy) = [\nu_A(x) \lor \nu_A(y)] - \nu_B(xy)
\]
\[
= \nu_A(x) \lor \nu_A(y) \quad [\text{since } \nu_B(xy) = 0]
\]

Hence \( \bar{G} \) is strong intuitionistic fuzzy graph.

4.3 OPERATIONS ON INTUITIONISTIC FUZZY GRAPHS

In this section, we define the operations "\( \bigcup \)" and "\( \bigoplus \)" between two intuitionistic fuzzy graphs \( G_1 : (V_1, E_1) \) and \( G_2 : (V_2, E_2) \) which has one or more vertices in common \( (V_1 \cap V_2 \neq \emptyset) \).

Definition: 4.3.1

Let \( G_1 : (V_1, E_1) \) and \( G_2 : (V_2, E_2) \) be two intuitionistic fuzzy graphs with one or more vertices in common \( (V_1 \cap V_2 \neq \emptyset) \). Then the union of \( G_1 \cup G_2 \) is another intuitionistic fuzzy graph \( G : (V, E) \) defined by,

(i) 
\[
\mu_A(x) = \begin{cases} 
\mu_{1A}(x) & \forall x \in V_1 \\
\mu_{2A}(x) & \forall x \in V_2
\end{cases}
\]
\[
\nu_A(x) = \begin{cases} 
\nu_{1A}(x) & \forall x \in V_1 \\
\nu_{2A}(x) & \forall x \in V_2
\end{cases}
\]

(ii) 
\[
\mu_B(xy) = \begin{cases} 
\mu_{1B}(xy) & \forall xy \in E_1 \\
\mu_{2B}(xy) & \forall xy \in E_2
\end{cases}
\]
\[
\nu_B(xy) = \begin{cases} 
\nu_{1B}(xy) & \forall xy \in E_1 \\
\nu_{2B}(xy) & \forall xy \in E_2
\end{cases}
\]
Definition: 4.3.2

Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be two intuitionistic fuzzy graphs with one or more vertices in common ($V_1 \cap V_2 \neq \emptyset$). Then their sum $G_1 + G_2$ is another intuitionistic fuzzy graph $G : (V, E)$ defined by,

(i) $\mu_A(x) = \begin{cases} \mu_{1A}(x) & \forall x \in V_1 \\ \mu_{2A}(x) & \forall x \in V_2 \end{cases}$ and $\nu_A(x) = \begin{cases} \nu_{1A}(x) & \forall x \in V_1 \\ \nu_{2A}(x) & \forall x \in V_2 \end{cases}$

(ii) $\mu_B(xy) = \begin{cases} \mu_{1B}(xy) & \forall xy \in E_1 \\ \mu_{2B}(xy) & \forall xy \in E_2 \end{cases}$ and $\nu_B(xy) = \begin{cases} \nu_{1B}(xy) & \forall xy \in E_1 \\ \nu_{2B}(xy) & \forall xy \in E_2 \end{cases}$

(iii) There exists a strong edge between every pair of non-common vertices in $G_1$ and $G_2$.

Remark: 4.3.3

The condition (iii) in above definition can be explained as follows:

If $G = \{v_1, v_2, v_3, v_4\}$ and $G_2 = \{v_3, v_4, v_5, v_6\}$, the vertices $v_3$ and $v_4$ are common in $G_1$ and $G_2$. Hence there exist edges between vertex pairs $(v_1, v_5)$, $(v_1, v_6)$, $(v_2, v_5)$ and $(v_2, v_6)$. These edges are strong implies their membership is the minimum of memberships of their adjacent vertices and non-membership is the maximum of non-membership of their adjacent vertices.

Example: 4.3.4

Consider two intuitionistic fuzzy graphs $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ with corresponding vertex and edge sets $V_1 = \{1, 2, 3\}$, $V_2 = \{1, 2, 4, 5\}$, $E_1 = \{12, 13, 23\}$ and $E_2 = \{12, 14, 25, 45\}$ then $G_1 \cup G_2$ and $G_1 + G_2$ are as follows.
Theorem: 4.3.5

If $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ are two intuitionistic fuzzy graphs, then $G_1 \cup G_2 \simeq G_1 + G_2$ if and only if $V_1 \subseteq V_2$ or $V_2 \subseteq V_1$.

Proof:

Let $G_1 \cup G_2 \simeq G_1 + G_2$. Then condition (iii) of definition 4.3.2 is zero.

$\Rightarrow$ There doesn't exist any non-common vertex between $G_1$ and $G_2$.

Hence either of the vertex set should be the subset of the other.

$\therefore V_1 \subseteq V_2$ or $V_2 \subseteq V_1$.

Conversely, if $V_1 \subseteq V_2$ or $V_2 \subseteq V_1$, then from definitions 4.3.1 and 4.3.2 we get $G_1 \cup G_2 \simeq G_1 + G_2$.  

Fig. 4.3 $G_1 : (V_1, E_1)$  

Fig. 4.4 $G_2 : (V_2, E_2)$

Fig. 4.5 $G_1 \cup G_2$  

Fig. 4.6 $G_1 + G_2$
**Theorem : 4.3.6**

Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be two intuitionistic fuzzy graphs with one or more common vertices. Then (a) $G_1 + G_2 \simeq \overline{G_1} \cup \overline{G_2}$.

(b) $G_1 \cup G_2 \simeq \overline{G_1} + \overline{G_2}$.

**Proof :**

Consider the identity map $I : V_1 \cup V_2 \to V_1 \cup V_2$.

To prove (a) we have to show that,

(1) (i) $$(\mu_{1A} + \mu_{2A})(x) = (\overline{\mu_{1A}} \cup \overline{\mu_{2A}})(x)$$

(ii) $$(v_{1A} + v_{2A})(x) = (v_{1A} \cup v_{2A})(x)$$

(2) (i) $$(\mu_{1B} + \mu_{2B})(xy) = (\overline{\mu_{1B}} \cup \overline{\mu_{2B}})(xy)$$

(ii) $$(v_{1B} + v_{2B})(xy) = (v_{1B} \cup v_{2B})(xy)$$

(1) (i) Consider $$(\mu_{1A} + \mu_{2A})(x) = (\mu_{1A} + \mu_{2A})(x) \text{ [by definition]}$$

$$= \begin{cases} 
\mu_{1A}(x) & \forall x \in V_1 \\
\mu_{2A}(x) & \forall x \in V_2 
\end{cases}$$

$$= \mu_{1A}(x) \cup \mu_{2A}(x)$$

$$= (\overline{\mu_{1A}} \cup \overline{\mu_{2A}})(x)$$

(ii) Similar to above proof it can be shown that $$(v_{1A} + v_{2A})(x) = (v_{1A} \cup v_{2A})(x)$$

(2) (i) Consider $$(\mu_{1B} + \mu_{2B})(xy)$$

$$= \begin{cases} 
(\mu_{1A} + \mu_{2A})(x) \land (\mu_{1B} + \mu_{2B})(y) - (\mu_{1B} + \mu_{2B})(xy) & xy \in E \\
(\mu_{1A} + \mu_{2A})(x) \land (\mu_{1A} + \mu_{2A})(y) & xy \notin E 
\end{cases}$$

$$= \begin{cases} 
[\mu_{1A}(x) \cup \mu_{2A}(x)] \land [\mu_{1A}(y) \cup \mu_{2A}(y)] - (\mu_{1B} + \mu_{2B})(xy) & xy \in E \\
[\mu_{1A}(x) \cup \mu_{2A}(x)] \land [\mu_{1A}(y) \cup \mu_{2A}(y)] & xy \notin E 
\end{cases}$$
\[
\begin{align*}
&= \left[ \mu_{1A}(x) \cup \mu_{2A}(x) \right] \land \left[ \mu_{1A}(y) \cup \mu_{2A}(y) \right] - \mu_{1B}(xy) & \quad xy \in E_1 \\
&= \left[ \mu_{1A}(x) \cup \mu_{2A}(x) \right] \land \left[ \mu_{1A}(y) \cup \mu_{2A}(y) \right] - \mu_{2B}(xy) & \quad xy \in E_2 \\
&= \left[ \mu_{1A}(x) \cup \mu_{2A}(x) \right] \land \left[ \mu_{1A}(y) \cup \mu_{2A}(y) \right] - [\mu_{1A}(x) \land \mu_{2A}(y)] & \quad x \in V_1 \text{ and } y \in V_2 \\
&= \left[ \mu_{1A}(x) \cup \mu_{2A}(x) \right] \land \left[ \mu_{1A}(y) \cup \mu_{2A}(y) \right] & \quad xy \notin E
\end{align*}
\]

\[
\begin{align*}
&= \left[ \mu_{1A}(x) \land \mu_{1A}(y) \right] - \mu_{1B}(xy) & \quad xy \in E_1 \\
&= \left[ \mu_{1A}(x) \land \mu_{1A}(y) \right] & \quad xy \in E_1 \\
&= \left[ \mu_{2A}(x) \land \mu_{2A}(y) \right] - \mu_{2B}(xy) & \quad xy \in E_2 \\
&= \left[ \mu_{2A}(x) \land \mu_{2A}(y) \right] & \quad xy \notin E_2
\end{align*}
\]

\[
\begin{align*}
&= \mu_{1B}(xy) & \quad xy \in E_1 \\
&= \mu_{2B}(xy) & \quad xy \in E_2 \\
&= 0 & \quad xy \notin E_1 \text{ and } E_2
\end{align*}
\]

\[
.: (\mu_{1B} + \mu_{2B})(xy) = (\overline{\mu_{1B}} \cup \overline{\mu_{2B}})(xy)
\]

(ii) \((\nu_{1B} + \nu_{2B})(xy) = (\overline{\nu_{1B}} \cup \overline{\nu_{2B}})(xy)\) can be proved similarly.

Hence \(G_1 + G_2 \simeq \overline{G_1} \cup \overline{G_2}\).

Proof of (b), \(G_1 \cup G_2 \simeq \overline{G_1} + \overline{G_2}\) can be proceeded as in (a).

**Example : 4.3.7**

From the intuitionistic fuzzy graphs \(G_1 : (V_1, E_1)\) and \(G_2 : (V_2, E_2)\) given in Fig. 4.3 and 4.4, it can be obviously seen that \(\overline{G_1} \cup \overline{G_2} \simeq \overline{G_1} + \overline{G_2}\) and \(\overline{G_1} + \overline{G_2} \simeq \overline{G_1} \cup \overline{G_2}\). Hence “∪” and “+” are complementary to each other.
Fig. 4.7 $\overline{G}_1 : (\overline{V}_1, \overline{E}_1)$

Fig. 4.8 $\overline{G}_2 : (\overline{V}_2, \overline{E}_2)$

Fig. 4.9 $\overline{G}_1 \cup \overline{G}_2$

Fig. 4.10 $\overline{G}_1 + \overline{G}_2$

Fig. 4.11 $\overline{G}_1 \cup \overline{G}_2$

Fig. 4.12 $\overline{G}_1 + \overline{G}_2$
**Definition: 4.3.8**

Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be two intuitionistic fuzzy graphs. Then their Cartesian product $G_1 \times G_2$ is an intuitionistic fuzzy graph $G : (V, E)$ with $V = \{(x, y) \mid \text{for all } x \in V_1 \text{ and } y \in V_2\}$ and $E = \{(x_u, y_v), (x_v, y_u) \mid \text{for all } xy \in E_1 \text{ and } uv \in E_2\}$ where,

(i) $\mu_A(x_u) = \min\{\mu_A(x), \mu_A(u)\}$ and $\nu_A(x_u) = \max\{\nu_A(x), \nu_A(u)\}$

(ii) $\mu_B(x_u, y_v) = \mu_B(x_v, y_u) = \min\{\mu_B(xy), \mu_B(uv)\}$ and $\nu_B(x_u, y_v) = \nu_B(x_v, y_u) = \max\{\nu_B(xy), \nu_B(uv)\}$.

**Example: 4.3.9**

![Graphs](image)

Fig. 4.13 $G_1 : (V_1, E_1)$  
Fig. 4.14 $G_2 : (V_2, E_2)$  
Fig. 4.15 $G_1 \times G_2$

**Remark: 4.3.10**

(i) If $G_1$ and $G_2$ are two connected intuitionistic fuzzy graphs, then $G_1 \times G_2$ need not be a connected graph.
Example

Fig. 4.16  $G_1 : (V_1, E_1)$  

Fig. 4.17  $G_2 : (V_2, E_2)$

Fig. 4.18  $G_1 \times G_2$

It can be seen that for any two connected intuitionistic fuzzy graphs $G_1$ and $G_2$, $G_1 \times G_2$ is not a connected intuitionistic fuzzy graph.

(ii)  $\overline{G_1 \times G_2} \neq \overline{G_1} \times \overline{G_2}$

Example

The complement of above graph $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ in fig. 4.16 and fig. 4.17 are as follows.

Fig. 4.19  $\overline{G_1} : (\overline{V_1}, \overline{E_1})$

Fig. 4.20  $\overline{G_2} : (\overline{V_2}, \overline{E_2})$

Fig. 4.21  $\overline{G_1 \times G_2}$

Fig. 4.22  $\overline{G_1 \times G_2}$
Hence \( G_1 \times G_2 \neq G_1 \times G_2 \)

**Proposition : 4.3.11**

Cartesian product is distributive over union.

[i.e., \( (G_1 \cup G_2) \times G_3 \cong (G_1 \times G_3) \cup (G_2 \times G_3) \)].

**Proof**

i. Let \( x \in (G_1 \cup G_2) \times G_3 \)

\[ \Rightarrow x \in G_1 \cup G_2 \text{ and } y \in G_3 \]

\[ \Rightarrow x \in G_1 \text{ or } G_2 \text{ and } y \in G_3 \]

\[ \Rightarrow xy \in G_1 \times G_3 \text{ or } xy \in G_2 \times G_3 \]

\[ \therefore xy \in (G_1 \times G_3) \cup (G_2 \times G_3) \]

If \( xy \in (G_1 \times G_3) \cup (G_2 \times G_3) \)

\[ \Rightarrow xy \in (G_1 \times G_3) \text{ or } xy \in (G_2 \times G_3) \]

\[ \Rightarrow xy \in G_1 \text{ and } y \in G_3 \text{ or } xy \in G_2 \text{ and } y \in G_3 \]

\[ \therefore xy \in (G_1 \times G_3) \cup (G_2 \times G_3) \]

\[ \therefore (G_1 \cup G_2) \times G_3 \cong (G_1 \times G_3) \cup (G_2 \times G_3) \]

ii. Let \( (x_u, y_u) \in (G_1 \cup G_2) \times G_3 \)

\[ \Rightarrow xy \in G_1 \cup G_2 \text{ and } uv \in G_3 \]

\[ \Rightarrow xy \in G_1 \text{ or } xy \in G_2 \text{ and } uv \in G_3 \]

\[ \therefore (x_u, y_u) \in G_1 \times G_3 \text{ or } (x_u, y_u) \in G_2 \times G_3 \]

\[ \therefore (x_u, y_u) \in (G_1 \times G_3) \cup (G_2 \times G_3) \]

If \( (x_u, y_u) \in (G_1 \times G_3) \cup (G_2 \times G_3) \)

\[ (x_u, y_u) \in G_1 \times G_3 \text{ or } (x_u, y_u) \in G_2 \times G_3 \]

\[ \Rightarrow xy \in G_1 \text{ and } uv \in G_3 \text{ or } xy \in G_2 \text{ and } uv \in G_3 \]

\[ \therefore xy \in (G_1 \cup G_2) \text{ and } uv \in G_3 \]

\[ \therefore (x_u, y_u) \in (G_1 \cup G_2) \times G_3 \]

\[ \therefore (G_1 \cup G_2) \times G_3 \cong (G_1 \times G_3) \cup (G_2 \times G_3). \]
Example: 4.3.12

Consider the following intuitionistic fuzzy graphs $G_1$, $G_2$ and $G_3$ given below.

Fig. 4.23 $G_1 : (V_1, E_1)$

Fig. 4.24 $G_2 : (V_2, E_2)$

Fig. 4.25 $G_3 : (V_3, E_3)$

Fig. 4.26 $G_1 \cup G_2$

Fig. 4.27 $(G_1 \cup G_2) \times G_3$
From the above graphs it can be seen that

\[(G_1 \cup G_2) \times G_3 \cong (G_1 \times G_3) \cup (G_2 \times G_3).\]
4.4 APPLICATIONS

In this section, we present some of the applications of complement of intuitionistic fuzzy graphs in knowledge management system, which supports in decision making problems. Work allotment and replacement model can also be effectively managed with the help of complement intuitionistic fuzzy graphs.

4.4.1 Complement intuitionistic fuzzy graph in work allotment model

In higher educational institutions, transportation plays a vital role. A transport department authority has to ensure that the transport management system has been managed carefully to overcome practical difficulties. There are several problems in day today transport management system. In this part focus is given to alteration of duties for the drivers taking leave. Consider a higher educational organization which has 50 drivers on its roll and a transport manager to manage them. The problem is drivers are taking leave without proper intimation and alteration. In this situation the application of complement intuitionistic fuzzy graph effectively solves the problem by giving ultimate solution of identifying proper alternative arrangements. The analysis of the situation and formation of intuitionistic fuzzy graph with data available will lead to knowledge management system and the situation will be handled effectively.

In our example, we have collected sample data of attendance percentage of four drivers in an institution for a period of 30 days and with the available data, we have formed an intuitionistic fuzzy graph as shown in the fig below in which membership function of each vertices represent percentage of days present and non-membership function represent percentage of absence of each driver marked 1, 2, 3 and 4. Also the membership and non-membership function of edges represent “present percentage” and “absent percentage” of two drivers at a time. The following adjacency matrix shows the attendance percentage of all the drivers.
Table 4.2

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<tr>
<td>1</td>
<td>(0.6, 0.3)</td>
<td>(0.4, 0.25)</td>
<td>(0.2, 0.2)</td>
<td>(0.5, 0.1)</td>
</tr>
<tr>
<td>2</td>
<td>(0.4, 0.25)</td>
<td>(0.5, 0.35)</td>
<td>(0.3, 0.3)</td>
<td>(0.4, 0.1)</td>
</tr>
<tr>
<td>3</td>
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<td>(0.3, 0.3)</td>
<td>(0.35, 0.4)</td>
<td>(0.3, 0.15)</td>
</tr>
<tr>
<td>4</td>
<td>(0.5, 0.1)</td>
<td>(0.4, 0.1)</td>
<td>(0.3, 0.15)</td>
<td>(0.85, 0.15)</td>
</tr>
</tbody>
</table>

Fig. 4.31 $G : (V, E)$

Fig. 4.32 $\overline{G} : (\overline{V}, \overline{E})$

From the complement graph $\overline{G} : (\overline{V}, \overline{E})$ we can get an idea of replacement of a driver with another person in his absence. If driver 1 is on leave, he can be replaced with 2 by 10%, with 3 by 20% and with 4 by 10%.
Similarly driver 2 can be replaced with driver 1 by 10% etc. The result of this analysis is tabulated below.

<table>
<thead>
<tr>
<th>Replace 1</th>
<th>With 1</th>
<th>With 2</th>
<th>With 3</th>
<th>With 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replace 2</td>
<td>10%</td>
<td>-</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Replace 3</td>
<td>15%</td>
<td>5%</td>
<td>-</td>
<td>5%</td>
</tr>
<tr>
<td>Replace 4</td>
<td>20%</td>
<td>25%</td>
<td>25%</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.3 Replacement chart

From the above table, it can be viewed that replacement of driver 3 with driver 1 has more possibility than with driver 2 and 4. Similarly replacement of driver 1 with driver 3 is 100% more than with driver 2 and 4. Such type of survey and analysis done in higher institutions with more number of workers will be accurate, fruitful and reduces time in taking decisions.

4.4.2 Complement intuitionistic fuzzy graph in knowledge management system

A student preparing for some examination will use the knowledge sources such as

1. Prescribed textbooks (A)
2. Reference books in syllabus (B)
3. Other books from library (C)
4. Study materials from knowledgeable persons (faculties) (D)
5. E-gadgets and internet (E)

In the example we have conducted a survey among 500 students in an institution and recorded the datas of the study materials they use for preparation for their examinations and the datas are as follows. Here the membership function and non-membership function of vertices represent the proportion of students using the knowledge source and not using the knowledge source respectively. The membership and non-membership
function of each edge represents the proportion of students using and not using both the pair of knowledge sources (end vertices) respectively. The proportion of data collected and rounded to one decimal place is tabulated below.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – Textbooks</td>
<td>(0.8, 0.05)</td>
<td>(0.3, 0.2)</td>
<td>(0.1, 0.4)</td>
<td>(0.5, 0.1)</td>
<td>(0.5, 0.1)</td>
</tr>
<tr>
<td>B – Reference books in syllabus</td>
<td>(0.3, 0.2)</td>
<td>(0.4, 0.5)</td>
<td>(0.2, 0.3)</td>
<td>(0.3, 0.5)</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>C – Library books</td>
<td>(0.1, 0.4)</td>
<td>(0.2, 0.3)</td>
<td>(0.2, 0.6)</td>
<td>(0, 0)</td>
<td>(0.1, 0.1)</td>
</tr>
<tr>
<td>D – Study materials from faculties</td>
<td>(0.5, 0.1)</td>
<td>(0.3, 0.5)</td>
<td>(0, 0)</td>
<td>(0.6, 0.1)</td>
<td>(0.2, 0.1)</td>
</tr>
<tr>
<td>E – Internet and e-gadgets</td>
<td>(0.5, 0.1)</td>
<td>(0, 0)</td>
<td>(0.1, 0.1)</td>
<td>(0.2, 0.1)</td>
<td>(0.5, 0.1)</td>
</tr>
</tbody>
</table>

Table 4.4
From the analysis of membership values of the above complement graph we can arrive at a conclusion as given in the table below.

<table>
<thead>
<tr>
<th>Proportion of students not using</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-</td>
<td>30%</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
<td>-</td>
<td>30%</td>
<td>10%</td>
<td>40%</td>
</tr>
<tr>
<td>C</td>
<td>10%</td>
<td>0%</td>
<td>-</td>
<td>20%</td>
<td>10%</td>
</tr>
<tr>
<td>D</td>
<td>10%</td>
<td>0%</td>
<td>60%</td>
<td>-</td>
<td>0%</td>
</tr>
<tr>
<td>E</td>
<td>0%</td>
<td>50%</td>
<td>50%</td>
<td>30%</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4.5

From the above table it is obvious that the students who use the prescribed text book always use study materials from internet and e-gadgets (since using A and not using D is 0% and using E and not using A is again 0%). Also about 50% of students using internet does not use reference books and other books from library for study. Such type of analysis might be very useful for libraries in making decisions on purchase of books and improving their e-library system which benefits the students. From the above analysis it is obvious that more than 50% of students use online study materials along with their prescribed text book. Also the use of reference books and other library books has been reduced. The reason behind this is the time saving factor of search engine which provides them with solution for their problem within seconds rather than sending time between pages of library books and reference books.