Unsteady MHD flow through porous media with temporal variation in temperature and concentration at the plate.

5.0. Introduction

The MHD flow problems find applications in power generators, petroleum industries, cooling of nuclear reactors, plasma dynamics and the boundary layer control in aerodynamics and crystal growth. Abel et al.[78] have analyzed the effect of a magnetic field on the viscoelastic fluid flow and heat transfer over a stretching sheet with internal heat generation. Chamkha et al.[8] have studied the effects of thermal radiation on MHD forced convection flow of an electrically conducting and heat generating/absorbing fluid over a non-isothermal wedge. The influence of viscous dissipation and radiation on unsteady MHD free-convection flow past a heated vertical plate with time-dependent suction in an optically thin environment has been studied by Israel-Cookey et al. [25]. Sahoo et al. [129] discussed the hydromagnetic oscillatory flow and heat transfer of a viscous liquid past a vertical porous plate in a rotating medium.

Heat and mass transfer in various geometrical configurations being embedded in porous media find applications in engineering and geophysical systems such as drying of porous solids, thermal insulations and cooling of nuclear reactors. Sahoo et al. [131] studied the MHD mixed convection stagnation point flow and heat transfer in a porous medium. Das et al.[135] have discussed the mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Many researchers have focused their attention to the problem of combined heat and mass transfer in magnetohydrodynamics free convection flow due to the fact that natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial applications. Prasad et al.[146] have studied the radiation and mass transfer effects on two-dimensional flow past
an impulsively started infinite vertical plate. Reddy and Reddy [94] have studied finite difference analysis of radiation effects on unsteady MHD flow of a chemically reacting fluid with time-dependent suction. Sahoo [128] has studied the MHD flow of a viscoelastic fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with heat and mass transfer. Kar et al. [69] studied the heat and mass transfer effects on a radiative and dissipative viscoelastic magnetohydrodynamics flow over a stretching porous sheet. Again, Kar et al.[68] analyzed the effect of chemical reaction and Hall current on magnetohydrodynamics flow along an accelerated porous flat plate with internal heat absorption/generation.

Recently, Reddy and Reddy [94] have studied the effect of thermal radiation on the natural convective heat and mass transfer of an electrically conducting, incompressible, viscous, and chemically reacting fluid along a semi infinite vertical plate in the presence of time varying suction. They have made use of Crank-Nicolson method to solve the governing equations and interpret numerical results to bring out the effects of thermo physical parameters. The authors [94] have not considered flow in the presence of porous medium as well as volumetric heat source. The justification of considering these aspects runs as: flow through porous media finds application in geothermic, geophysics etc. and agricultural sciences. Mohapatra and Bora [138] presented the effect of scattering of internal waves in a two-layer fluid flowing through a channel with small undulations. Makinde [92] studied MHD mixed-convection interaction with thermal radiation and n\textsuperscript{th} order chemical reaction past a vertical porous plate embedded in a porous medium. He investigated the hydromagnetic mixed convection heat and mass transfer flow of an incompressible Boussinesq fluid past a vertical porous plate with constant heat flux in the presence of radiative heat transfer in an optically thin environment, viscous dissipation, and an n\textsuperscript{th} order homogeneous chemical reaction between the fluid and the diffusing species.

In the present analysis of flow through porous medium, Darcy’s law has been assumed to be of fundamental importance. This law is valid for low speed flows, but the speed in the filter or the flow in the region where the velocity changes abruptly, are not always small. The convective force may be important for high speed flow. We may consider the force acting on the fluid due to porous medium. The force may deviate from the usual Darcy drag which is directly proportional to the velocity. However, in case of flow through porous media such as filter, the deviation will be small enough, to be neglected. If we consider the convective force, we have to apply Brinkman nonlinear model to account for the
nonlinearity aspect. The Brinkman formula is indisputable as the experimental data supports the model.

Further, the effect of free convection in a flow through porous medium plays an important role in agricultural engineering and in petroleum industry in extracting pure petrol from the crude. The convection induced by the buoyancy effect resulting from vertical solutal and thermal concentration gradients, in a horizontal layer of saturated porous medium considering the general form of the law of Darcy was first analyzed by Nield [28].

Further, the inclusion of radiative heat flux, \( q \), and the rate of volumetric heat generation/absorption, \( Q_0 \), accounts for the work done due to internal heat generation/absorption and radiative heat flux.

The main objectives of the present study are to allow the flow to pass through a saturated porous medium which is accomplished by Darcy linear model and to account for the work done by internal heat generation/absorption modifying the energy equation.

Another aspect is the method of solution; Ready and Ready [94] have solved the governing equation by finite difference Crank Nicholson type but we have solved by an approximate analytical method as outlined in Schlichting and Gersten [47].

Besides the above two novel aspects we have also studied the buoyancy effects due to thermal buoyancy and mass buoyancy. We have shown the case of Reddy and Reddy [94] as a particular case.

5.1. Mathematical formulation

Consider the unsteady one dimensional laminar boundary layer flow of a viscous incompressible electrically conducting fluid along a semi-infinite vertical plate in the presence of thermal and concentration buoyancy effects. The fluid is assumed gray, absorbing and emitting but non-scattering. The x-axis is taken along the plate in the upward direction and the y-axis is taken normal to it. A time dependent suction velocity is assumed normal to the plate. A uniform magnetic field is applied in the direction perpendicular to the plate. Under the usual Boussinesq’s approximation, the governing boundary layer equations following Cortell [101], Vajravelu et al. [58] and Kar et al. [68]
\[ \frac{\partial v^*}{\partial y} = 0 \] (5.1.1)

\[ \frac{\partial u^*}{\partial t} + v^* \frac{\partial u^*}{\partial y} = \nu \frac{\partial^2 u^*}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u^* + g \beta (T^* - T_\infty) + g \beta (C^* - C_\infty) - \frac{\nu}{K} u^* \] (5.1.2)

\[ \frac{\partial T^*}{\partial t} + v^* \frac{\partial T^*}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^2} - \frac{1}{\rho C_p} Q_0 (T^* - T_\infty) \] (5.1.3)

\[ \frac{\partial C^*}{\partial t} + v^* \frac{\partial C^*}{\partial y} = D \frac{\partial^2 C^*}{\partial y^2} - K_l (C^* - C_\infty) \] (5.1.4)

The boundary conditions correspond to the above equations are

\[ y^* = 0: C^* = C_w + \varepsilon (C_w - C_\infty) A e^{\alpha r^*}, T^* = T_w + \varepsilon (T_w - T_\infty) A e^{\alpha r^*}, u^* = v_0 \] (5.1.5)

\[ y^* \rightarrow \infty: C^* = C_\infty, T^* = T_\infty, u^* = 0 \]

Using the Roseland approximation for radiation [76], we write

\[ q_r = -4\sigma_\nu \frac{\partial T^{*4}}{\partial y} \] (5.1.6)

On expanding \( T^{*4} \) about \( T_\infty \) and neglecting higher order terms, we get

\[ T^{*4} \equiv 4T_\nu^3 T^* - 3T_\nu^4 \] (5.1.7)

In view of equations (5.1.6) and (5.1.7), equation (5.1.3) reduces to
\[
\frac{\partial T^*}{\partial t} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_o}{\rho C_p} (T^* - T_\infty) + \frac{16 \sigma T_x^3}{3 \rho C_p k_e} \frac{\partial^2 T^*}{\partial y^{*2}}
\] (5.1.8)

5.2. Method of solution

The method of solutions to equations (5.2.2) – (5.2.4) with boundary conditions (5.2.5) is based upon the superimposed principle. Physically, it means that an exponentially accelerated unsteady flow is superimposed on the main steady flow. A regular perturbation method has been applied with small perturbation parameter as \( \varepsilon \), the suction parameter. This physically means the flow is subjected to small suction. The result here is valid only for small suction velocity not for strong suction. This method has been outlined in Schlichting and Gersten [47]. This method has been applied by Mahato [53]. We can derive steady state velocity, temperature and concentration in particular cases.

On integration equation (5.1.1) for variable suction velocity normal to the plate, we have

\[
v^* = -v_0 (1 + \varepsilon A e^{n^*})
\] (5.2.1)

where minus sign indicates that the suction is towards the plate. \( A \) is the suction parameter, so that \( \varepsilon A < 1 \). Here \( v_0 > 0 \) is the mean suction velocity. By introducing the non-dimensional quantities

\[
y = \frac{v_0 y^*}{v}, u = \frac{u^*}{v_0}, l = \frac{v_0^2 l^*}{v}, n = \frac{v_0 n^*}{v_0^2}, M = \frac{\sigma B_n^2 v}{\rho v_0^2}, Kp = \frac{K v_0^2}{\nu^2}, Pr = \frac{\nu p C_p}{k}, Sc = \frac{\nu}{D},
\]

\[
T = \frac{T^* - T_\infty}{T_\infty - T_\infty}, C = \frac{C^*-C_\infty}{C_\infty - C_\infty}, Gr = \frac{g \beta u (T_\infty - T_\infty)}{v_0^3}, Gm = \frac{g \beta u (C_\infty - C_\infty)}{v_0^3},
\]

\[
Kc = \frac{K c u}{v_0^2}, Rd = \frac{16 \sigma T_x^3}{3 k_e}, S = \frac{Q_o v}{v_0^2}
\]

in eqns. (5.1.2), (5.1.8) and (5.1.4) respectively, we have

\[
\frac{\partial u}{\partial t} (1 + \varepsilon A e^{n^*}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} -(M + \frac{1}{Kp})u + Gr T + Gm C
\] (5.2.2)

\[
\frac{\partial T}{\partial t} (1 + \varepsilon A e^{n^*}) \frac{\partial T}{\partial y} = \frac{1 + Rd}{Pr} \frac{\partial^2 T}{\partial y^2} - ST
\] (5.2.3)

\[
\frac{\partial C}{\partial t} (1 + \varepsilon A e^{n^*}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - Kc C
\] (5.2.4)
The corresponding boundary conditions are
\[
y = 0 : C = 1 + \varepsilon Ae^\mu, T = 1 + \varepsilon Ae^\mu, u = 1 \\
y \to \infty: C \to 0, T \to 0, u \to 0
\]
(5.2.5)

The system of partial differential equations (5.2.2)-(5.2.4) can be reduced to a system of ordinary differential equations by introducing
\[
u(y,t) = u_0(y) + A\varepsilon e^\mu u_1(y)
\]
(5.2.6)
\[
T(y,t) = T_0(y) + A\varepsilon e^\mu T_1(y)
\]
(5.2.7)
\[
C(y,t) = C_0(y) + A\varepsilon e^\mu C_1(y)
\]
(5.2.8)

Substituting eqns.(5.2.6), (5.2.7) and (5.2.8) in eqns.(5.2.2), (5.2.3) and (5.2.4) and equating constant and coefficient of \(\varepsilon Ae^\mu\), we have
\[
u_0'' + u_0' - (M + 1/Kp)u_0 = -GrT_0 - GmC_0
\]
(5.2.9)
\[
u_1'' + u_1' - (M + 1/Kp + n)u_1 = -u_0' - GrT_1 - GmC_1
\]
(5.2.10)
\[
T_0'' + (Pr/ (1 + Rd))T_0' - (S Pr/ (1 + Rd))T_0 = 0
\]
(5.2.11)
\[
T_1'' + (Pr/ (1 + Rd))T_1' - ((n + S) Pr/ (1 + Rd))T_1 = -(Pr/ (1 + Rd))T_0'
\]
(5.2.12)
\[
C_0'' + ScC_0' - KcScC_0 = 0
\]
(5.2.13)
\[
C_1'' + ScC_1' - Sc(Kc + n)C_1 = -ScC_0'
\]
(5.2.14)

The corresponding boundary conditions are
\[
u_0 = 1, u_1 = 0, T_0 = 1, T_1 = 1, C_0 = 1, C_1 = 1 \text{ at } y = 0 \\
u_0 = 0, u_1 = 0, T_0 = 0, T_1 = 0, C_0 = 0, C_1 = 0 \text{ as } y \to \infty
\]
(5.2.15)

\[
u(y,t) = e^{-\alpha_{1y}} + A_1(e^{-\alpha_{1y}} - e^{-\alpha_{2y}}) + A_2(e^{-\alpha_{1y}} - e^{-\alpha_{2y}}) + A_3(e^{-\alpha_{1y}} - e^{-\alpha_{2y}})
\]
\[
+ A\varepsilon e^\mu \left( A_3 e^{-\alpha_{1y}} + A_0(e^{-\alpha_{1y}} - e^{-\alpha_{2y}}) - A_1 e^{-\alpha_{1y}} - A_2 e^{-\alpha_{1y}} - A_1 e^{-\alpha_{1y}} - A_2 e^{-\alpha_{1y}} \right)
\]
\[
T(y,t) = e^{-\alpha_{1y}} + A\varepsilon e^\mu \left\{ e^{-\alpha_{1y}} + A_2(e^{-\alpha_{1y}} - e^{-\alpha_{2y}}) \right\}
\]
\[
C(y,t) = e^{-\alpha_{1y}} + A\varepsilon e^\mu \left\{ e^{-\alpha_{1y}} + A_2(e^{-\alpha_{1y}} - e^{-\alpha_{2y}}) \right\}
\]

The skin friction (\(\tau\)), the rate of heat transfer (\(Nu\)) and the rate of mass transfer (\(Sh\)) are given by
\[
\tau = \frac{\partial u}{\partial y} \bigg|_{y=0} = a_5 + A_3(a_5 - a_1) + A_2(a_5 - a_1) + A\varepsilon e^\mu(a_6A_5 + a_5A_0 - a_4A_1 + a_3A_2 - a_2A_3 - a_1A_4)
\]
\[ Nu = -\frac{\partial T}{\partial y} \bigg|_{y=0} = a_1 + A\varepsilon e^m \left( a_4 + A_2(a_3 - a_4) \right) \]

\[ Sh = -\frac{\partial C}{\partial y} \bigg|_{y=0} = a_1 + A\varepsilon e^m \left( a_2 + A_1(a_4 - a_2) \right) \]

5.3. Results and discussion

The effect of thermal radiation and porous matrix on unsteady MHD natural convective incompressible flow of a chemically reacting fluid with time dependent suction has been studied. The effect of different flow parameters on the velocity, temperature and concentration field have been studied and discussed with the help of graphs. Also, the effects of flow parameters on the skin friction, Nusselt number and Sherwood number have been discussed with the help of Tables 5.3, 5.4 and 5.5 respectively.

Fig. 5.2 exhibits the effect of Prandtl number \((\text{Pr})\), Magnetic parameter \((\text{M})\) and porosity parameter \((\text{Kp})\) on the velocity distribution. The analysis reveals that \(\text{Pr}\) and \(\text{M}\) reduces the velocity whereas \(\text{Kp}\) enhances it. This shows that higher Prandtl number fluid results in the decrease of velocity distribution everywhere in the flow domain. The increase of \(\text{Pr}\) means slow rate of thermal diffusion that results the low rate of momentum transport leading to decrease in velocity distribution. Similar explanations can be attributed to the effect of magnetic parameter; as the increase in magnetic field strength produces a greater resistive electromagnetic force which opposes the motion of fluid in the main direction whereas an increase in porosity parameter gives rise to increase in velocity because of higher the value of \(\text{Kp}\), lowers down the resistance owing to porous medium i.e. \(\text{Kp} \rightarrow \infty\) represents the case of without porous matrix giving rise to a clear flow. The hike of the profiles near the plate in a few layers may be attributed to the shearing effect due to the motion of the plate. After a few layers other resistive forces dominate over the shearing force, so that the velocity decreases.

From fig. 5.3 it is seen that an increase in volumetric heat source and destructive chemical reaction parameter decreases the velocity whereas thermal radiation enhances it. The enhancement of velocity due to increase in radiation parameter is evident from equation (5.2.3) as it appears in the diffusion term. An increase in thermal radiation \((\text{Rd})\), enhances the thermal diffusion causing a rise in temperature (fig. 5.5) of the flow domain. Consequently, the velocity increases.
Fig. 5.4 exhibits the velocity profiles indicating a sharper rise near the plate. The sharp rise is due to combined effect of buoyancy forces and shearing effect of the plate. We have studied the combined effects of both the buoyancy forces (Gr > 0, Gm > 0) which imply the cooling of the plate (\( T_u > T_w \)) with lowering of solutal concentration level (\( C_w > C_u \)). Another important aspect of the profile may be read as higher Sc i.e. heavier species, causes a fall in velocity distribution.

Fig. 5.5 is devoted to the temperature distribution. An increase in Pr decreases the temperature due to slow rate of thermal diffusion causing a decrease in velocity as discussed in fig. 5.2. The heat source lowers down the temperature but rise in radiation contributes to increase in temperature throughout the flow domain.

Fig. 5.6 presents the solutal concentration distribution in the flow domain. Asymptotic fall in solutal concentration is indicated in case of heavier species (i.e. in Sc, Schmidt number) and higher rate of destructive chemical reaction. The close observation of fig. 5.5 and 5.6 reveal that fall of temperature is sharper than the fall of solutal concentration indicating thinner thermal boundary layer.

In order to establish the consistency of the method applied for solution and the validity of the results obtained in the present study we have prepared Table-5.1. Comparison of some of the entries of the forming table obtained earlier by Ready and Ready [94] with the present result in the absence of porous matrix and heat source, has been made.

Surface conditions at the boundary plays an important role in augmenting the flow, heat and mass transfer phenomena as well as their boundary layers.

Table-5.2 presents the skin friction for various values of the physical and material parameters. It is observed that Prandtl number (Pr), porosity parameter (Kp), heat source (S), thermal Grashof number (Gr) and mass Grashof number (Gm) reduce the skin friction whereas in case of Schmidt number (Sc) reverse effect is experienced, destructive reaction parameter (Kc), magnetic parameter (M) and radiation parameter (Rd). Physical interpretation of the effect agrees with velocity, temperature and concentration distribution. For a particular parameter M, it is evident from Table-5.2 that the magnetic parameter increases the skin friction as the electromagnetic resistive force which opposes the motion of the fluid and hence greater resistance is experienced at the bounding surface. But an increase
in porosity of the medium (Kp), decreases the skin friction because for the higher the value of Kp, the flow of fluid experiences less resistance at the surface.

Tables-5.3 and 5.4 show the variation of Nusselt number and Sherwood number in respect of pertinent parameters affecting the surface criteria respectively. It is observed that an increase in Pr and S enhance the rate of heat transfer at the surface. Moreover, heavier species (Sc) and rate of destructive reaction accelerates the rate of solutal concentration at the surface. The inferences arrived at, are trivial as those depend upon the material property of the fluid and ongoing physical processes.

The related explanations are; increase in Pr leads to low rate of diffusion. Consequently, transport of thermal energy in the flow domain decreases, hence rate of heat transfer increases at the surface. On the other hand, an increase in radiative heat leads to decrease the temperature of the fluid and hence rate of heat transfer at the surface increases. Further, the heavier foreign species deplete the concentration in the flow domain and hence rate of solutal concentration enhances at the surface. The similar explanation can be attributed to the rate of destructive reaction parameter also.

5.4. Conclusion

The present study gives the following results related to the flow field of physical interest.

- High Prandtl number fluids with externally imposed magnetic field, volumetric heat source and destructive chemical reaction give rise to reduction in velocity and producing thinner boundary layer.
- An increase in radiation contributes to the growth of velocity boundary layer and thermal boundary layer also.
- Heavier diffusing species reduce the velocity in the flow domain.
- Thinning of the thermal boundary layer occurs due to high Prandtl number fluids but the radiation has a thickening effect.
- The skin friction does not change sign indicates that no flow reversal occurs.
- Higher Prandtl number fluids and heat source enhance the rate of heat transfer at the surface and in case of concentration distribution, Schmidt number and destructive reaction rate accelerates the solutal concentration at the surface causing a fall in concentration in the flow domain.
Fig. 5.2 Effect of Pr, M and Kp on velocity profile

Fig. 5.3 Effect of S, Rd and Kc on velocity profile
<table>
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<th>Curve</th>
<th>Sc</th>
<th>Gr</th>
<th>Gm</th>
</tr>
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<tr>
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<td>4.0</td>
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<tr>
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<td>4.0</td>
<td>4.0</td>
</tr>
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<td>III</td>
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<td>IV</td>
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</tr>
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</table>

Pr = 0.63 M = 0.5 Kp = 0.5
S = 0.5 Rd = 0.5 Kc = 0.5
t = 1.0, n = 0.1, A = 0.3, p = 0.02

Fig. 5.4 Effect of Sc, Gr and Gm on velocity profile

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Fig. 5.5 Effect of Pr, S and Rd on temperature profile
Fig. 5.6 Effect of Sc and Kc on concentration profile
Table- 5.1 (Comparison Table)

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<th>Kp</th>
<th>S</th>
<th>U</th>
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Table- 5.2 (Skin friction)

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Table- 5.3(Nusselt number)

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