CHAPTER 2

STOCHASTIC ANALYSIS OF A COLD STANDBY SYSTEM WITH PREVENTIVE MAINTENANCE AND SERVER FAILURE DURING REPAIR

2.1 Introduction

The studies on stochastic modeling of repairable systems have suggested that the availability of any operating system can be increased by providing unit wise and component wise redundancy. And, the cold standby redundancy has been considered as one of the best method for enhancing the performance of such systems for a reasonable duration. Murari and Goyal (1984), Cao and Wu (1989) and Malik (2013) have analyzed cold standby systems using different repair mechanisms. But, in most of these systems it is assumed that repair facility neither fails nor deteriorates during jobs. In fact, this assumption seems to unrealistic when repair facility meets with an accident due to the reasons like mishandling of the functioning of the systems, electric shocks and carelessness on the part of the server, etc. In such a situation, treatment may be given to the server in order to resume the job. Dhankhar and Malik (2010) studied a single unit system with server failure during repair. Also, the existing literature on reliability modeling of repairable systems indicate that preventive maintenance is helpful in reducing the deterioration rate of the systems working in different environmental conditions. Malik (2013) discussed reliability model of a computer system with cold standby redundancy under preventive maintenance and repair.

The concentration of the study is on the stochastic analysis of a cold standby system of two identical units by introducing simultaneously the ideas of preventive maintenance and failure of server. The operative and complete failure modes of the system are considered. A single server is called immediately to handle the faults as and when appeared during operation of the system. The provision of preventive maintenance is made before failure of the unit. The repair of the unit is done by the server at its complete failure. The server may fail while conducting repair of the failed unit. And, treatment is given to the failed server in order to resume the jobs. The statistical independent approach is considered for different points related to failure time of the unit, the rate by which unit under goes for PM, repair time of the unit, PM time of the unit, failure and treatment times of the server. The distributions for failure rate of the unit and server,
the rate by which unit undergoes for preventive maintenance are taken as negative exponential while the distributions for preventive maintenance of the unit, treatment of the server and repair of the unit are assumed as arbitrary. The SMP is adopted to evaluate reliability characteristics including mean time to system failure (MTSF), availability, but period of the server due to repair and preventive maintenance, expected number of repairs, treatments and preventive maintenances and finally the profit function. The results for a particular case are obtained to depict the behavior of some important reliability characteristics. The profit analysis of the system model has been made.

2.2 System Description

A stochastic model of a system consisting two identical units with preventive maintenance and server failure during repair has been developed. The possible transition diagram of the system model is shown in Fig.:2.1
The system model (figure 2.1) has the following transition states:

- Regenerative States: \( S_0, S_1, S_2 \) and \( S_7 \)
- Non-regenerative: \( S_3, S_4, S_5, S_6, S_8, S_9, S_{10}, S_{11}, S_{12} \) and \( S_{13} \)

The following are the possible transition states of the system:

\[
\begin{align*}
S_0 &= (O, \text{Cs}), & S_1 &= (O, \text{UPm}), & S_2 &= (O, \text{FUr}), \\
S_3 &= (\text{WPM}, \text{UPm}) & S_4 &= (\text{UPM}, \text{FWr}) & S_5 &= (\text{WPM}, \text{FWr}) \\
S_6 &= (\text{FWr}, \text{FUR}) & S_7 &= (O, \text{FWr}, \text{SFUt}) & S_8 &= (\text{WPM}, \text{FWr}, \text{SFUt}) \\
S_9 &= (\text{FWr}, \text{FWr}, \text{SFUt}) & S_{10} &= (\text{FWr}, \text{FWr}, \text{SFUt}) & S_{11} &= (\text{WPM}, \text{FWr}, \text{SFUt}) \\
S_{12} &= (\text{WPM}, \text{FUr}, \text{SG}) & S_{13} &= (\text{Fur}, \text{FWR}, \text{SG})
\end{align*}
\]

2.3 Transition Probabilities (TP)

The differential transition probabilities are given by

\[
\begin{align*}
dQ_{01}(t) &= \alpha_0 e^{-(\alpha_0+\lambda)t}dt \\
dQ_{10}(t) &= e^{-(\alpha_0+\lambda)t}h(t)dt \\
dQ_{14}(t) &= \lambda e^{-(\alpha_0+\lambda)t}H(t)dt \\
dQ_{25}(t) &= \alpha_0 e^{-(\alpha_0+\lambda+\mu)t}G(t)dt \\
dQ_{27}(t) &= \mu e^{-(\alpha_0+\lambda+\mu)t}G(t)dt \\
dQ_{42}(t) &= h(t)dt \\
dQ_{58}(t) &= \mu e^{-\mu t}G(t)dt \\
dQ_{69}(t) &= \mu e^{-\mu t}G(t)dt \\
dQ_{7,10}(t) &= \lambda e^{-(\alpha_0+\lambda)t}F(t)dt \\
dQ_{8,12}(t) &= f(t)dt \\
dQ_{10,13}(t) &= f(t)dt \\
dQ_{12,1}(t) &= e^{-\mu t}g(t)dt \\
dQ_{13,2}(t) &= e^{-\mu t}g(t)dt \\
dQ_{02}(t) &= \lambda e^{-(\alpha_0+\lambda)t}dt \\
dQ_{13}(t) &= \alpha_0 e^{-(\alpha_0+\lambda)t}H(t)dt \\
dQ_{20}(t) &= e^{-(\mu+\alpha_0+\lambda)t}g(t)dt \\
dQ_{26}(t) &= \lambda e^{-(\alpha_0+\lambda+\mu)t}G(t)dt \\
dQ_{31}(t) &= h(t)dt \\
dQ_{51}(t) &= e^{-\mu t}g(t)dt \\
dQ_{62}(t) &= e^{-\mu t}g(t)dt \\
dQ_{72}(t) &= e^{-(\alpha_0+\lambda)t}f(t)dt \\
dQ_{7,11}(t) &= \alpha_0 e^{-(\alpha_0+\lambda)t}F(t)dt \\
dQ_{9,13}(t) &= f(t)dt \\
dQ_{11,12}(t) &= f(t)dt \\
dQ_{128}(t) &= \mu e^{-\mu t}G(t)dt \\
dQ_{139}(t) &= \mu e^{-\mu t}G(t)dt
\end{align*}
\]
We have the following expressions for transition probabilities:

\[ p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t) \, dt \]

\[ p_{01} = \frac{a_0}{\lambda + a_0} \quad p_{02} = \frac{\lambda}{\lambda + a_0} \]

\[ p_{10} = h^*(\lambda + a_0) \quad p_{13} = \frac{a_0}{\lambda + a_0} \left[ 1 - h^*(\lambda + a_0) \right] \quad p_{14} = \frac{\lambda}{\lambda + a_0} \left[ 1 - h^*(\lambda + a_0) \right] \]

\[ p_{20} = g^*(\mu + \lambda + a_0) \]

\[ p_{25} = \frac{a_0}{\mu + \lambda + a_0} \left[ 1 - g^*(\mu + \lambda + a_0) \right] \quad p_{26} = \frac{\lambda}{\mu + \lambda + a_0} \left[ 1 - g^*(\mu + \lambda + a_0) \right] \]

\[ p_{27} = \frac{\mu}{\mu + \lambda + a_0} \left[ 1 - g^*(\mu + \lambda + a_0) \right] \quad p_{31} = p_{42} = h^*(0) \quad p_{51} = \frac{\lambda}{\lambda + a_0} \left[ 1 - f^*(\lambda + a_0) \right] \]

\[ p_{62} = p_{12,1} = p_{13,2} = g^*(\mu) \quad p_{58} = p_{69} = p_{12,8} = p_{13,9} = 1 - g^*(\mu) \]

\[ p_{72} = f^*(\lambda + a_0) \quad p_{7,10} = \frac{\lambda}{\lambda + a_0} \left[ 1 - f^*(\lambda + a_0) \right] \]

\[ p_{7,11} = \frac{a_0}{\lambda + a_0} \left[ 1 - f^*(\lambda + a_0) \right] \quad p_{8,12} = p_{9,13} = p_{10,13} = p_{11,12} = p_{12,1} = p_{12,8} = p_{13,2} + p_{13,9} = 1 \]

(2.3)

For a perfect distribution

\[ p_{01} + p_{02} = p_{10} + p_{13} + p_{14} = p_{20} + p_{25} + p_{26} + p_{27} = p_{31} = p_{42} = p_{51} + p_{58} = p_{62} + p_{69} = p_{72} + p_{7,10} + p_{7,11} = p_{8,12} = p_{9,13} = p_{10,13} = p_{11,12} = p_{12,1} + p_{12,8} = p_{13,2} + p_{13,9} = 1 \]

(2.4)

2.4 Mean Sojourn Times (MST)

\[ \mu_0 = m_{01} + m_{02} \quad \mu_1 = m_{10} + m_{13} + m_{14} \quad \mu_2 = m_{20} + m_{25} + m_{26} + m_{27} \]

\[ \mu_3 = m_{31} \quad \mu_4 = m_{42} \quad \mu_5 = m_{51} + m_{58} \]

\[ \mu_6 = m_{62} + m_{69} \quad \mu_7 = m_{72} + m_{7,10} + m_{7,11} \quad \mu_8 = m_{8,12} \]

\[ \mu_9 = m_{9,13} \quad \mu_{10} = m_{10,13} \quad \mu_{11} = m_{11,12} \]

\[ \mu_{12} = m_{12,1} + m_{12,8} \quad \mu_{13} = m_{13,2} + m_{13,9} \quad \mu_1 = m_{10} + m_{11,3} + m_{12,4} \]

\[ \mu_2 = m_{20} + m_{21,5} + m_{21,5(8,12)^n} + m_{22,6} + m_{22,6(9,13)^n} + m_{27} \]

\[ \mu_3 = m_{7,11,12} + m_{7,11(12,8)^n} + m_{72} + m_{72,10,13} + m_{72,10(13,9)^n} \]

(2.5)
2.5 Reliability Measures

The following reliability measures have been evaluated for the system model

2.5.1 Reliability and Mean Time To System Failure (MTSF)

The expressions for \( \pi_i(t) \) are given as under:

\[
\begin{align*}
\pi_0(t) &= Q_{01}(t) \& \pi_1(t) + Q_{02}(t) \& \pi_2(t) \\
\pi_1(t) &= Q_{10}(t) \pi_0(t) + Q_{13}(t) + Q_{14}(t) \\
\pi_2(t) &= Q_{20}(t) \pi_0(t) + Q_{27}(t) \pi_7(t) + Q_{25}(t) + Q_{26}(t) \\
\pi_7(t) &= Q_{72}(t) \& \pi_2(t) + Q_{710}(t) + Q_{711}(t)
\end{align*}
\]

Let us take LST of (2.6) and solving for \( \pi_0^*(s) \)

We have

\[
R^*(s) = \frac{1-\pi_0^*(s)}{s}
\]

And,

\[
MTSF = \lim_{s \to 0} \frac{1-\pi_0^*(s)}{s} = \frac{N_1}{D_1}
\]

Where, \( N_1 = (1 - p_{27}p_{72})(\mu_0 + p_{01}\mu_1) + p_{02}(\mu_2 + p_{27}\mu_7) \)

And, \( D_1 = (1 - p_{01}p_{10})(1 - p_{27}p_{72}) - p_{02}p_{20} \)

2.5.2 Long Run (Steady State) Availability

The expressions for \( A_i(t) \) are given as under:

\[
\begin{align*}
A_0(t) &= M_0(t) + q_{01}(t)A_1(t) + q_{02}(t)A_2(t) \\
A_1(t) &= M_1(t) + q_{10}(t)A_0(t) + q_{113}(t)A_1(t) + q_{124}(t)A_2(t) \\
A_2(t) &= M_2(t) + q_{20}(t)A_0(t) + \left(q_{215}(t) + q_{215(8,12)}A_1(t) + \left(q_{226}(t) + q_{226(9,13)}A_2(t) + q_{27}(t)A_7(t)ight)ight)A_1(t) + \left(q_{72}(t) + q_{711(12,8)}A_1(t) + \left(q_{72}(t) + q_{7210(3,9)}A_1(t) + \left(q_{72}(t) + q_{7210(13,9)}A_7(t) + q_{7210(13,9)}A_2(t)\right)\right)\right)
\end{align*}
\]

Where,

\[
\begin{align*}
M_0(t) &= e^{-(\lambda + \alpha_0)t} \\
M_2(t) &= e^{-(\mu + \lambda + \alpha_0)t}G(t) \\
M_7(t) &= e^{-(\lambda + \alpha_0)t}H(t) \\
M_1(t) &= e^{-(\lambda + \alpha_0)t}G(t) \\
M_7(t) &= e^{-(\lambda + \alpha_0)t}F(t)
\end{align*}
\]
Let us take Laplace Transform of relations (2.9) and solving for $A_0(s)$.
The steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2}$$

Where,

$$N_2 = [p_{20}(p_{10} + p_{14}) + p_{10}(p_{25} + p_{27}p_{7,11}]\mu_0 + [p_{20}p_{01} + p_{25} + p_{27}p_{7,11}]\mu_1 + (p_{02}p_{10} + p_{14})(\mu_2 + p_{27}\mu_7)$$

$$D_2 = [p_{20}(p_{10} + p_{14}) + p_{10}(p_{25} + p_{27}p_{7,11}]\mu_0 + [p_{20}p_{01} + p_{25} + p_{27}p_{7,11}]\mu_1 + (p_{02}p_{10} + p_{14})(\mu_2 + p_{27}\mu_7)$$

### 2.5.3 Busy Period of the Server Due to Repair in the Long Run

The expressions for $B^R(t)$ are given as under:

$B^R_0(t) = q_{01}(t)\circ B^R_1(t) + q_{02}(t)\circ B^R_2(t)$

$B^R_1(t) = q_{10}(t)\circ B^R_0(t) + q_{11.3}(t)\circ B^R_1(t) + q_{12.4}(t)\circ B^R_2(t)$

$B^R_2(t) = W^R_2(t) + q_{20}(t)\circ B^R_0(t) + \left(q_{21.5}(t) + q_{21.5(8,12)}^n(t)\right)\circ B^R_1(t) + 
\left(q_{22.6}(t) + q_{22.6(9,13)}^n(t)\right)\circ B^R_2(t) + q_{27}(t)\circ B^R_7(t)$

$B^R_7(t) = \left(q_{71.11,12}(t) + q_{71.11(12,8)}^n(t)\right)\circ B^R_1(t) + 
\left(q_{72}(t) + q_{72.10,13}(t) + q_{72.10(13,9)}^n(t)\right)\circ B^R_2(t)$

(2.10)

Where $W^R_2(t) = e^{-(\alpha_0+\mu+\lambda)t} \frac{G(t)}{G(t)} + (\alpha_0e^{-\alpha_0t}\mu e^{-\mu t}\circ f(t)e^{-\mu t}\circ 1) \frac{G(t)}{G(t)}$

Let us take LT of relations (2.10) and solving for $B^R_0(s)$.

We get, $B^R_0(\infty) = \lim_{s \to 0} s B^R_0(s) = \frac{N_3}{D_2}$

$N_3 = W^R_2(0)[p_{14} + p_{02}p_{10}]$ and $D_2$ is already specified.

### 2.5.4 Busy Period of the Server due to Preventive Maintenance (PM) in the Long Run

The expressions for $B^P(t)$ are given as under:

$B^P_0(t) = q_{01}(t)\circ B^P_1(t) + q_{02}(t)\circ B^P_2(t)$

$B^P_1(t) = W^P_1(t) + q_{10}(t)\circ B^P_0(t) + q_{11.3}(t)\circ B^P_1(t) + q_{12.4}(t)\circ B^P_2(t)$

$B^P_2(t) = q_{20}(t)\circ B^P_0(t) + \left(q_{21.5}(t) + q_{21.5(8,12)}^n(t)\right)\circ B^P_1(t) + 
\left(q_{22.6}(t) + q_{22.6(9,13)}^n(t)\right)\circ B^P_2(t) + q_{27}(t)\circ B^P_7(t)$
\[ B_p^I(t) = \left( q_{71.11,12}(t) + q_{71.11(12,8)}^n(t) \right) \mathcal{B}_1^P(t) + \\
\quad \left( q_{72}(t) + q_{72.10,13}(t) + q_{72.10(13,9)}^n(t) \right) \mathcal{B}_2^P(t) \]

Where, \( W_p^P(t) = e^{-\left(\alpha_0 + \lambda \right)t} \mathcal{H}(t) + (\lambda e^{-\lambda t} \mathcal{C}1) \mathcal{H}(t) + (\alpha_0 e^{-\alpha_0 t} \mathcal{C}1) \mathcal{H}(t) \)

Let us take LT of relations (2.11) and solving for \( B_0^P(s) \).

We get, \( B_0^P(\infty) = \lim_{s \to 0} sB_0^P(s) = \frac{N_4}{D_2} \)

\( N_4 = W_1^{P_0}(0)[p_{01}p_{20} + p_{25} + p_{27}p_{7,11}] \) and \( D_2 \) is already specified.

2.5.5 Expected Number of Repairs of the Unit in the Long Run

The expressions for \( R_i(t) \) are given as under:

\[
R_0(t) = Q_{01}(t) \mathcal{S} R_1(t) + Q_{02}(t) \mathcal{S} R_2(t) \\
R_1(t) = Q_{10}(t) \mathcal{S} R_0(t) + Q_{11.3}(t) \mathcal{S} R_1(t) + Q_{12.4}(t) \mathcal{C} A_2(t) \\
R_2(t) = Q_{20}(t) \mathcal{S} (1 + R_0(t)) + \left( Q_{21.5}(t) + Q_{21.5(8,12)}^n(t) \right) \mathcal{S} (1 + R_1(t)) \\
\quad + \left( Q_{22.6}(t) + Q_{22.6(9,13)}^n(t) \right) \mathcal{S} (1 + R_2(t)) + Q_{27}(t) \mathcal{S} R_7(t) \\
R_7(t) = \left( Q_{71.11,12}(t) + Q_{71.11(12,8)}^n(t) \right) \mathcal{S} (1 + R_1(t)) + Q_{72}(t) \mathcal{S} R_2(t) + \left( Q_{72.10,13}(t) + Q_{72.10(13,9)}^n(t) \right) \mathcal{S} (1 + R_2(t))
\]

Let us get LT of relations (2.12) and solving for \( R_0^*(s) \).

We get, \( R_0 = \lim_{s \to 0} sR_0^*(s) = \frac{N_5}{D_2} \)

Where, \( N_5 = (1 - p_{27}p_{72})(p_{14} + p_{25}p_{10}) \) and \( D_2 \) is already specified.

2.5.6 Expected Number of Preventive Maintenances (PM) in the Long Run

The expressions for \( PM_i(t) \) are given as under:

\[
PM_0(t) = Q_{01}(t) \mathcal{S} PM_1(t) + Q_{02}(t) \mathcal{S} PM_2(t) \\
PM_1(t) = Q_{10}(t) \mathcal{S} (1 + PM_0(t)) + Q_{11.3}(t) \mathcal{S} (1 + PM_1(t)) + Q_{12.4}(t) \mathcal{S} (1 + PM_2(t)) \\
PM_2(t) = Q_{20}(t) \mathcal{S} PM_0(t) + \left( Q_{21.5}(t) + Q_{21.5(8,12)}^n(t) \right) \mathcal{S} PM_1(t) \\
\quad + \left( Q_{22.6}(t) + Q_{22.6(9,13)}^n(t) \right) \mathcal{S} PM_2(t) + Q_{27}(t) \mathcal{S} PM_7(t) \\
PM_7(t) = \left( Q_{71.11,12}(t) + Q_{71.11(12,8)}^n(t) \right) \mathcal{S} PM_1(t) + \left( Q_{72.10,13}(t) + Q_{72.10(13,9)}^n(t) \right) \mathcal{S} PM_2(t)
\]

Let us take LT of relations (2.13) and solving for \( PM_0^*(s) \).
We get, \( PM_0 = \lim_{s \to 0} sPM_0^* (s) = \frac{N_6}{D_2} \)

Where, \( N_6 = p_{01}p_{20} + p_{25} + p_{27}p_{7,11} \) and \( D_2 \) is already specified.

### 2.5.7 Expected Number of Treatments (ENT) Given to the Server in the Long Run

The expressions for \( T_i(t) \) are given as under:

\[
\begin{align*}
T_0(t) &= Q_{01}(t) \bullet T_1(t) + Q_{02}(t) \bullet T_2(t) \\
T_1(t) &= Q_{10}(t) \bullet T_0(t) + Q_{11.3}(t) \bullet T_1(t) + Q_{12.4}(t) \bullet T_2(t) \\
T_2(t) &= Q_{20}(t) \bullet T_0(t) + Q_{21.5}(t) \bullet T_1(t) + Q_{21.5(8,12)}(t) \bullet (1 + T_2(t)) + Q_{22.6}(t) \bullet T_2(t) \\
& \quad + Q_{22.6(9,13)}(t) \bullet (1 + T_2(t)) + Q_{27}(t) \bullet T_7(t) \\
T_7(t) &= (Q_{71.11,12}(t) + Q_{71.11(12,8)}(t) + Q_{72}(t) + Q_{72.10,13}(t) + \\
& \quad Q_{72.10(13,9)}(t)) \bullet (1 + T_2(t)) \\
& \quad (2.14)
\end{align*}
\]

Let us take LST of relations (2.14) and solving for \( T_0^{**}(s) \).

We get, \( T_0 = \lim_{s \to 0} sT_0^{**}(s) = \frac{N_7}{D_2} \)

Where, \( N_7 = (p_{14} + p_{10}p_{02})(p_{25}p_{58} + p_{26}p_{69} + p_{27}) \) and \( D_2 \) is already specified.

### 2.6 Profit Analysis

The profit of the system model can be obtained as:

\[ P = K_9A_0 - K_1B_0^R - K_4B_0^P - K_2R_0 - K_5P_0 - K_3T_0 \]

Where, the variables \( P, K_0 \) to \( K_5 \) are defined under the common notations.

### 2.7 Particular Cases

Let us take \( g(t) = \beta e^{-\beta t} \), \( f(t) = \alpha e^{-\alpha t} \), \( h(t) = \gamma e^{-\gamma t} \)

\[
\begin{align*}
p_{01} &= \frac{\alpha_0}{\lambda + \alpha_0} & p_{02} &= \frac{\lambda}{\lambda + \alpha_0} & p_{10} &= \frac{\gamma}{\gamma + \lambda + \alpha_0} \\
p_{13} &= \frac{\alpha_0}{\gamma + \lambda + \alpha_0} & p_{14} &= \frac{\lambda}{\gamma + \lambda + \alpha_0} & p_{20} &= \frac{\beta}{\beta + \mu + \lambda + \alpha_0} \\
p_{25} &= \frac{\alpha_0}{\beta + \mu + \lambda + \alpha_0} & p_{26} &= \frac{\lambda}{\beta + \mu + \lambda + \alpha_0} & p_{27} &= \frac{\mu}{\beta + \mu + \lambda + \alpha_0} \\
p_{31} &= p_{42} = p_{8,12} = p_{9,13} = p_{10,13} = p_{11,12} = 1 \\
p_{51} &= \frac{\beta}{\beta + \mu} & p_{58} &= \frac{\mu}{\beta + \mu} & p_{62} &= \frac{\beta}{\beta + \mu} \\
p_{69} &= \frac{\mu}{\beta + \mu} & p_{72} &= \frac{\alpha}{\alpha + \lambda + \alpha_0} & p_{7,10} &= \frac{\lambda}{\alpha + \lambda + \alpha_0}
\end{align*}
\]
\[ p_{7,11} = \frac{\alpha_0}{\alpha + \lambda + \alpha_0} \quad p_{12,1} = \frac{\beta}{\beta + \mu} \quad p_{12,8} = \frac{\mu}{\beta + \mu} \]

\[ p_{13,2} = \frac{\beta}{\beta + \mu} \quad p_{13,9} = \frac{\mu}{\beta + \mu} \quad \mu_0 = \frac{1}{\lambda + \alpha_0} \]

\[ \mu_1 = \frac{1}{\gamma + \lambda + \alpha_0} \quad \mu_2 = \frac{1}{\beta + \mu + \lambda + \alpha_0} \quad \mu_3 = \mu_4 = \frac{1}{\gamma} \]

\[ \mu_7 = \frac{1}{\alpha + \lambda + \alpha_0} \quad \mu_8 = \mu_9 = \mu_{10} = \mu_{11} = \frac{1}{\alpha} \quad \mu_6 = \mu_5 = \mu_{12} = \mu_{13} = \frac{1}{\beta + \mu} \]

\[ MTSF = \frac{N_1}{D_1} \quad \text{Availability}(A_0) = \frac{N_2}{D_2} \]

\[ B_0^R = \frac{N_3}{D_2} \quad B_0^P = \frac{N_4}{D_2} \quad R_0 = \frac{N_5}{D_2} \]

\[ P_0 = \frac{N_6}{D_2} \quad T_0 = \frac{N_7}{D_2} \]

Where,

\[ N_1 = (\gamma + \lambda + 2\alpha_0)[(\beta + \lambda + \mu + \alpha_0)(\alpha + \lambda + \alpha_0) - \mu \alpha] + \lambda(\gamma + \lambda + \alpha_0)(\alpha + \lambda + \mu + \alpha_0) \]

\[ D_1 = [(\lambda + \alpha_0)(\gamma + \lambda + \alpha_0) - \gamma \alpha_0][(\beta + \lambda + \mu + \alpha_0)(\alpha + \lambda + \alpha_0) - \mu \alpha] \]

\[ -\lambda \beta(\gamma + \lambda + \alpha_0)(\alpha + \lambda + \alpha_0) \]

\[ N_2 = \frac{(\beta + \lambda + \mu + \alpha_0)[\beta(\alpha + \lambda + \alpha_0) + \alpha_0(\alpha + \lambda + \mu + \alpha_0)] + \lambda(\alpha + \lambda + \mu + \alpha_0)}{(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)((\beta + \lambda + \mu + \alpha_0))^2} \]

\[ \alpha \beta \gamma[\beta(\gamma + \lambda)(\alpha + \lambda + \alpha_0) + \alpha_0 \gamma(\alpha + \lambda + \mu + \alpha_0)] + \alpha \beta \alpha_0(\gamma + \lambda + \alpha_0) \]

\[ \beta(\alpha + \lambda + \alpha_0) + (\lambda + \alpha_0)(\alpha + \lambda + \mu + \alpha_0)] + \gamma \lambda(\gamma + \lambda + \alpha_0) \]

\[ D_2 = \frac{[(\alpha + \lambda + \alpha_0)(\alpha \beta(\lambda + \alpha_0)(\alpha + \mu)) + \mu(\alpha \beta + (\lambda + \alpha_0)(\alpha + \mu + \beta))]}{\alpha \beta \gamma(\lambda + \alpha_0)(n + \lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\beta + \lambda + \mu + \alpha_0)} \]

\[ N_3 = \frac{\lambda[\beta \mu + (\lambda + \alpha_0)(\beta + \lambda + \mu + \alpha_0)]}{\beta(\lambda + \alpha_0)(\alpha_0 + \mu + \lambda)(\beta + \lambda + \mu + \alpha_0)} \]

\[ N_4 = \frac{\alpha_0[\beta(\alpha + \lambda + \alpha_0) + (\lambda + \alpha_0)(\alpha + \lambda + \mu + \alpha_0)]}{\gamma(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\beta + \lambda + \mu + \alpha_0)} \]

\[ N_5 = \frac{\lambda[(\beta + \lambda + \mu + \alpha_0)(\alpha + \lambda + \alpha_0) - \mu \alpha]}{(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\beta + \lambda + \mu + \alpha_0)} \]

\[ N_6 = \frac{\alpha_0[\beta(\alpha + \lambda + \alpha_0) + (\lambda + \alpha_0)(\alpha + \lambda + \alpha_0 + \mu)]}{(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\beta + \lambda + \mu + \alpha_0)} \]

\[ N_7 = \frac{\mu \lambda}{(\lambda + \alpha_0)(\beta + \mu)} \]
2.8 Numerical and Graphical Representation of Reliability Measures

Table 2.1: MTSF Vs Failure Rate

<table>
<thead>
<tr>
<th>λ</th>
<th>$\beta=3.1, \alpha=2.1$</th>
<th>$\alpha=3.1$</th>
<th>$\beta=4.1$</th>
<th>$\mu=0.4$</th>
<th>$\alpha_0=2.6$</th>
<th>$\gamma=2.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.2725</td>
<td>1.2727</td>
<td>1.2773</td>
<td>1.27179</td>
<td>1.0326</td>
<td>1.3820</td>
</tr>
<tr>
<td>0.2</td>
<td>1.2086</td>
<td>1.2088</td>
<td>1.2172</td>
<td>1.2073</td>
<td>0.9890</td>
<td>1.3058</td>
</tr>
<tr>
<td>0.3</td>
<td>1.1503</td>
<td>1.1506</td>
<td>1.1621</td>
<td>1.1486</td>
<td>0.9487</td>
<td>1.2369</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0971</td>
<td>1.0974</td>
<td>1.1112</td>
<td>1.0950</td>
<td>0.9114</td>
<td>1.1746</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0482</td>
<td>1.0485</td>
<td>1.0644</td>
<td>1.0459</td>
<td>0.8766</td>
<td>1.1178</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0032</td>
<td>1.0036</td>
<td>1.0209</td>
<td>1.0008</td>
<td>0.8444</td>
<td>1.0659</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9616</td>
<td>0.9621</td>
<td>0.9807</td>
<td>0.9592</td>
<td>0.8142</td>
<td>1.0184</td>
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<tr>
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<td>0.9236</td>
<td>0.9432</td>
<td>0.9206</td>
<td>0.7861</td>
<td>0.9747</td>
</tr>
<tr>
<td>0.9</td>
<td>0.8875</td>
<td>0.8879</td>
<td>0.9083</td>
<td>0.8849</td>
<td>0.7596</td>
<td>0.9344</td>
</tr>
</tbody>
</table>

Fig. 2.2: MTSF Vs Failure Rate

Table 2.2: Availability Vs Failure Rate

<table>
<thead>
<tr>
<th>λ</th>
<th>$\beta=3.1, \alpha=2.1, \mu=0.2$</th>
<th>$\alpha=3.1$</th>
<th>$\beta=4.1$</th>
<th>$\mu=0.4$</th>
<th>$\alpha_0=2.6$</th>
<th>$\gamma=2.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.6294</td>
<td>0.6298</td>
<td>0.6329</td>
<td>0.6281</td>
<td>0.5748</td>
<td>0.708</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6088</td>
<td>0.6096</td>
<td>0.6157</td>
<td>0.6062</td>
<td>0.5569</td>
<td>0.684</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5891</td>
<td>0.5902</td>
<td>0.5996</td>
<td>0.5854</td>
<td>0.5398</td>
<td>0.662</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5703</td>
<td>0.5717</td>
<td>0.5832</td>
<td>0.5655</td>
<td>0.5235</td>
<td>0.64</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5523</td>
<td>0.5539</td>
<td>0.5677</td>
<td>0.5466</td>
<td>0.5078</td>
<td>0.619</td>
</tr>
</tbody>
</table>
0.6 | 0.5351 | 0.5369 | 0.5534 | 0.5286 | 0.4928 | 0.6  
0.7 | 0.5186 | 0.5207 | 0.5388 | 0.5114 | 0.4784 | 0.581  
0.8 | 0.5028 | 0.5051 | 0.5252 | 0.4951 | 0.4646 | 0.563  
0.9 | 0.4877 | 0.4902 | 0.5120 | 0.4794 | 0.4514 | 0.545

Fig.2.3: Availability Vs Failure Rate

Table 2.3: Profit Vs Failure Rate

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\beta=3.1, \alpha=2.1, \mu=0.2, \gamma=2.1, \alpha_0=2.2$</th>
<th>$\alpha=3.1$</th>
<th>$\beta=4.1$</th>
<th>$\mu=0.4$</th>
<th>$\alpha_0=2.6$</th>
<th>$\gamma=2.7$</th>
</tr>
</thead>
<tbody>
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<td>8771.0681</td>
<td>8831.672</td>
<td>8745.488</td>
<td>7914.4773</td>
<td>9921.2986</td>
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<tr>
<td>0.2</td>
<td>8343.5918</td>
<td>8353.9676</td>
<td>8471.021</td>
<td>8305.891</td>
<td>7544.8470</td>
<td>9441.5607</td>
</tr>
<tr>
<td>0.3</td>
<td>7939.5832</td>
<td>7954.1692</td>
<td>8123.681</td>
<td>7886.419</td>
<td>7190.3127</td>
<td>8981.4853</td>
</tr>
<tr>
<td>0.4</td>
<td>7552.5772</td>
<td>7570.7945</td>
<td>7788.945</td>
<td>7485.951</td>
<td>6850.0934</td>
<td>8540.2608</td>
</tr>
<tr>
<td>0.5</td>
<td>7181.6955</td>
<td>7203.0156</td>
<td>7466.161</td>
<td>7103.433</td>
<td>6523.4553</td>
<td>8117.0871</td>
</tr>
<tr>
<td>0.6</td>
<td>6826.1106</td>
<td>6850.0519</td>
<td>7154.726</td>
<td>6737.874</td>
<td>6209.7092</td>
<td>7711.1814</td>
</tr>
<tr>
<td>0.7</td>
<td>6485.0426</td>
<td>6511.1666</td>
<td>6854.079</td>
<td>6388.342</td>
<td>5908.2069</td>
<td>7321.7814</td>
</tr>
<tr>
<td>0.8</td>
<td>6157.7562</td>
<td>6185.6648</td>
<td>6563.699</td>
<td>6053.964</td>
<td>5618.3392</td>
<td>6948.1482</td>
</tr>
<tr>
<td>0.9</td>
<td>5843.5592</td>
<td>5872.8904</td>
<td>6283.1</td>
<td>5733.918</td>
<td>5339.5321</td>
<td>6589.5682</td>
</tr>
</tbody>
</table>

Fig.2.4: Profit Vs Failure Rate
2.9 Conclusion

The trend of some important reliability measures has been observed for arbitrary values of various parameters including the costs: $K_0 = 15000$, $K_1 = 3000$, $K_2 = 1000$, $K_3 = 500$, $K_4 = 400$ and $K_5 = 200$ as shown respectively in figures 2.2, 2.3 and 2.4. It is observed that MTSF, availability and profit keep on decline with the increase of the failure rate of the unit and server, the rate at which unit goes on Preventive Maintenance while they move up with the increase of treatment rate of the server ($\alpha$), repair rate ($\beta$) and preventive maintenance rate of the unit ($\gamma$).