Conclusion and future outlook of this thesis

In this work, we have studied the structure of proton at small $x$ by constructing the proton structure functions based on self-similarity.

In Chapter 1, we have introduced the definition of self-similarity and its application in Unintegrated Parton Distribution Functions (uPDF), Parton Distribution Functions (PDFs) and structure function of proton. We have also given outlines of the following topics in this chapter:

1. QCD evolution equations: DGLAP and BFKL with the kinematics of DIS and SIDIS.
2. Transverse Momentum Dependent Parton Distributions Functions (TMD) with the kinematics of DIS and SIDIS and its evolution equation.
(3) Froissart bound and its applicability in DIS.

In Chapter 2, we have introduced the model of proton structure function suggested by Lastovicka and its phenomenological ranges of validity in $x$ and $Q^2$, which was fitted by HERA data [57, 58]. We have refitted the model with recently complied HERA data [59] and found the phenomenological ranges of validity nearly equal to the earlier ones. The analysis does not make any big difference with the previous one except the phenomenological range of validity enhance upto $Q^2 \leq 150$ GeV$^2$ instead of $Q^2 \leq 120$ GeV$^2$. Both the two models have got singularities at $x_0 \sim 0.019$ and $x''_0 \sim 0.012$ respectively, even though each one is outside its phenomenological range of validity.

In Chapter 3, we have removed the singularities in the models discussed in chapter 2 within the entire $x$-range: $0 < x < 1$ by putting an extra condition on model parameters that it should be positive definite. But an effort to make a model singularity free reduces its phenomenological range of validity drastically; $Q^2 \leq 10$ GeV$^2$.

In Chapter 4, we have therefore improved the earlier versions of proton structure function by generalizing the definitions of defining magnification factors in uPDF such that it has power law rise in $\log Q^2$, closer to QCD. We have found that if the defining parameters satisfy certain conditions among themselves, linear rise in $Q^2$ and singularity free in structure functions in models of chapter 2 and 3 can be avoided.

In Chapter 5, we have incorporated the Froissart saturation bound in the self-similarity based models of proton structure functions discussed in chapter 4. Our analysis indicates that a self-similarity based model of proton structure function with power law growth in $\frac{1}{x}$ has a wider phenomenological range of validity than one with a Froissart bound compatible slower $\log^2 \frac{1}{x}$ growth suggesting that asymptotic energy scale is not yet been reached at HERA regime.
In chapter 6, we have calculated the momentum fractions carried by quarks and gluons in the models described in chapters 2 to 5. We have also compared the results with the predictions of Perturbative QCD, Lattice QCD and Ads/QCD models.

In Chapter 7, we outline how Transverse Momentum Dependent Parton Distribution Functions (TMDs) can be introduced in the self-similarity based models of proton structure functions discussed in chapters 2 and 4. Limitations of this approach are also discussed.

In Chapter 8, we have studied how the imposition of Froissart bound on uPDF changes the corresponding TMDs form. We have also shown the $k_t^2$-fall in TMDs w.r.t. $x$ and $Q^2$. The difference between the power law rise in $\left(\frac{1}{x}\right)$ in TMDs as discussed in chapter 7 and that of $\log \frac{1}{x}$ discussed in this chapter has been studied.

Let us end this section with the theoretical limitation of the present work. Although fractality in hadron-hadron and electron-positron interactions has been well established experimentally [69], self-similarity itself is not a general property of QCD and is not yet established, either theoretically or experimentally. In this work, we have merely used the notion of self-similarity to parametrize PDFs as a generalization of the method suggested in Ref. [1] and have shown that under specific conditions among the defining parameters, logarithmic rise in $Q^2$ of structure function is achievable even in such an approach and has wider phenomenological range of validity in $x$ and $Q^2$. However, the model based on fractal inspired parametrization of PDFs are not comparable to QCD. Modern analyses of PDFs in perturbative QCD are carried out upto Next-to-Next-to-leading order (NNLO) [137, 138] with and without Froissart saturation using standard QCD evolution equation and corresponding calculable splitting functions in several orders of strong coupling constant and compare with QCD predictions. Instead, the present work is carried out only at the level of a parton model. In this way, the models merely parametrize the input parton distributions and their evolution in a self-similarity based compact form, which contains both perturbative
and non-perturbative aspects of a formal theory, valid in a finite $x - Q^2$ range of data. It presumably implies that while self-similarity has not yet been proved to be a general feature of strong interactions, under specific conditions, experimental data can be interpreted with this notion as has been shown in the present chapter. To prove it from the first principle is the future course of the present approach.