CHAPTER 4

MEASUREMENT OF CAPACITY UTILIZATION: A THEORATICAL APPRAISAL

A measure of Capacity Utilization (CU) is necessary to know the levels of the utilization of existing production capacity in the production process. The economic theory also favours the estimation of CU because an accurate assessment of a production function requires ‘capital in use’ not ‘capital in place’ (Solow, 1957). The later represents the total capital stock available for use in the production process, whereas the earlier reflects the proportion of the total available capital stock actually utilized. In the studies like ours, where the estimation of production function is required to access the technical efficiency and productivity performance of the Indian sugar industry, an estimation of the index of CU is significant to adjust the existing capital stock according to the capacity utilization level using the following relation:

\[ \text{Capital in Use} = \text{Capital in Place} \times \text{Capacity Utilization} \]

The capital in place can be obtained by using the usual techniques such as perpetual inventory method, etc. However, measuring the rate of capacity utilization requires identifying the capacity output \( Y^* \) and then, the capacity utilization rate is defined as the ratio of the actual output \( Y_0 \) to capacity output (Kirkley et al., 2002) i.e.,

\[ CU = \frac{Y_0}{Y^*} \]

However, the notion of capacity output has been defined in two alternative ways; i) primal-technology based or engineering concept; and ii) an economic based concept. As per the primal technology based engineering concept, the potential output may be technologically derived and hence defined relative to the maximum possible physical output that the fixed inputs are capable of supporting when the variable inputs are fully utilized (Johanson, 1968). Alternatively, full capacity output is that level of output which the existing stock of equipment is intended to produce under normal conditions with respect to the use of variable inputs (Smithies, 1957). In contrast, economists define that full capacity output level is associated with full competitive equilibrium and for an individual firm this point occurs at the minimum of the average cost curve (Chamberlin, 1947). Thus, from the hindsight of an economist, the potential output can be defined relative to an economic optimum such as the level of output which minimizes cost or
maximizes revenue or profits (Gréboval and Munro, 1998). Figure 4.1, presents two notions of capacity output elaborated through the engineering and economics based definitions of capacity.

Panel A, explains \(OY_o\) level of capacity output as per the engineering concept of capacity whereas, \(OY_E\) is the economic based measure of capacity. As the engineering measure of capacity is a physical measure, its estimation doesn’t require information regarding input prices. Alternatively, economic measure entails the information regarding the prices of factor inputs to estimate a cost-function. Thus, the engineering measure of capacity has found to be more operational than the economists’ concept (Budin and Paul, 1961). Most of the managers and technical experts prefer to operate with the engineering definition of capacity and incidentally the same definition is the basis of the capacity definition of central statistical organization (CSO), Ministry of Statistics and Program Implementation, India (Paul, 1974a, 1974b). Further, empirical determination of the economists’ version of capacity output is indeed difficult especially in the context of multiproduct firm. However, if most cost curves are L-shaped the economic concept can also be approximated by the engineering concept of capacity (Johanston, 1960).

**Figure 4.1: Two Concepts of Capacity Output**

Source: Gréboval, (2002)

Diverse measures such as, engineering measure of capacity, electricity consumption method, maximum achieved output approach, survey approach, Wharton or peak to peak index of capacity utilization, the RBI index, time intensity approach, *Production Function* or cost function based approach, and Minimum Capital Output Ratio methods etc., are available to quantify capacity utilization levels. All of these methods are discussed as follows:
4.2 Measures of CU based on Engineering Concept

Under engineering measure the center of the focus is capital. The level of output which optimize the use of the fixed factor of production i.e., capital is the capacity output. The following measures are based upon the engineering concept of CU:

4.2.1 Engineering Measure of Capacity

According to engineering concept, capacity is maximum potential output attainable from given capital equipment in a given period of time, and utilization of capacity means the actual output of output in relation to technically possible level of output. Therefore, it is ratio between actual output and maximum possible output. If, $Y_h$ is the hourly output from given equipment at 100 percent efficiency, $T$ is total hours in a year (i.e. $24 \times 365 = 8760$), then total annual maximum output $\hat{Y} = Y_h \times T$. If $Y_A$ is actual output during the year then capital utilization ($K_U$) can be defined as in (4.1),

$$K_U = \frac{Y_A}{\hat{Y}} \quad (4.1)$$

In practice, it is impossible for an item of equipment to operate for 24 hours, of 365 days. This is because, some loss of the equipment’s time can occur due to the wastage in process, machine breakdowns, and normal stoppages for maintenance. Therefore, feasible technically maximum output is more relevant measure of capacity (rated capacity). It is calculated on the basis of available time for the actual use of equipment. If, actual time available for the equipment is $T$ where $T < \hat{T}$ then technically feasible output ($Y_f$) and utilization ($K_f$) can be defined as in equation (4.2),

$$K_f = \frac{Y_A}{Y_h \times T} = \frac{Y_A}{\hat{Y}_f} \quad (4.2)$$

Where, $Y_f = Y_h \times \hat{T}$

The method given above is appropriate for micro level studies. The degree and level of aggregation permissible in these measures will depend upon the following conditions: a) Output should be homogeneous; and b) technology and size of the plants are also homogeneous.
4.2.2 Electricity Consumption Method

This method has been developed by Foss (1963) and later used by Jorgenson (1966), Griliches (1967) for U.S.A. and by Heathfield (1972) for U.K. This method is based on the proposition that like output and labour, CU is related to the flow of capital services. Electricity is perfectly homogenous product of uniform quality, and it is easier to aggregate electricity consumption data without any aggregation problem. The electric motor utilization rate is the ratio of actual consumption of electricity (khw) to the technically possible consumption by installed electric motors. The technically possible consumption of electricity measured on the basis of 24 hours use of electricity for 365 days, adjusted for the energy loss of 10 percent due to dissipation in the form of heat. If \( AE_i \) is actual consumption of electricity by the motors in the plant in the \( i^{th} \) industry, \( R_i \) is the rated capacity of the electric motors, 8760 are the total hours in a year, and .90 is the efficiency of electric motors then electricity based measure of utilization of capital \( (M_i) \) can be written as in equation (4.3),

\[
M_i = \frac{AE_i}{(R_i \times 8760)/0.90} \times 100
\]  

(4.3)

This method is more useful where electricity is the main source of energy; it will be better proxy for the intensity with which the machines are being driven by electric motors.

It is however felt that this measure underestimates the level of capital utilization in case of underdeveloped countries, because in these countries due to shortages in the supply of electricity, plants may be operated by other prime movers such as steam engine, turbines and gasoline engines. It is also an important fact that some of the equipments may need input of electricity not only as prime movers, but also to use it to generate heat and coldness. Under these conditions, if we use this method, it would tend to understate the extent of utilization. It does not take into account time of repair, maintenance and adjustments. It does not have proper system of weights when different sections of plants operate at different rates this result in the downward bias in the measurement of utilization rates.

4.2.3 Production Function Approach

The method of estimating capacity by Production Function is an engineering estimate of capacity for a firm or industry. In this approach several inputs can be implicitly included in a capacity utilization index by combining them in Production Function. It has been developed and
used by the eminent authors' viz., Ball and Smolensky (1961), Klein and Preston (1967), Briscoe et al. (1970), and Harris and Taylor (1985). In their seminal work Klien and Preston (1967) estimated capacity through Production Function for 30 U.S. manufacturing industries for 1957-60. The estimated Production Function captures the long-run relationship between the inputs and output (e.g., capital stock, labour and output). For any period, the figures of actual output are compared to the capacity output estimated at full employment of resources to determine capacity utilization series. Thus the notion of capacity used is the economists’ notion of maximum output corresponding to all factors of production, including capital stock and estimation technique is an analytical one based on Production Function. Suppose the relationship between output and inputs at any period $t$ can be stated as follows:

$$Y_t = \phi(L_t, K_t, t, \varepsilon_t)$$  \hspace{1cm} (4.4)

Where, $Y_t$ denotes real output, $L_t$ denotes labour input expressed in terms of man-hours, $K_t$ the flow of capital services, $t$ is a proxy for technical change and $\varepsilon_t$ is an error term. If $\phi$ can be specified (e.g. Cobb Douglas) and its parameters are estimated, then we can write the relationship for output at peak (full capacity) periods as:

$$Y^c_t = \phi(L^f_t, K^f_t, t)$$  \hspace{1cm} (4.5)

Where, $L^f_t$ and $K^f_t$ refer to full employment supply of man-hours and available flow of capital services. The index of capacity utilization is then computed as $U_t = Y_t / Y^c_t$ for any period $t$.

The problems with this method relate to the measurement of capital stock series as well as measurement of full employment levels of man-hours and flow of capital services. The measurement of the two inputs are interrelated and alternative assumptions have been made in estimating $L^f_t$ and $K^f_t$. Klein and Preston (1967) relate capital utilization to manpower and assumed that:

$$\frac{K^f_t}{K_t} = \frac{L^f_t}{L_t}$$  \hspace{1cm} (4.6)

If the Production Function is Cobb Douglas type, then:

$$\frac{Y_t}{Y^c_t} = \left(\frac{L_t}{L^f_t}\right) \times \varepsilon_t$$  \hspace{1cm} (4.7)
Full employment labour force is obtained by specifying it as a function of employed labour force, involuntary unemployed labour force and frictionally unemployed labour force. Full employment man-hours are then obtained by an adjustment factor for actual man-hours used. Thus the rate of unemployment is directly related to the rate of capacity utilization. The index of utilization is obtained as:

\[ \text{Log} Y_t - \text{Log} Y_t^e = (\alpha + \beta) \times (\text{Log} L_t - \text{Log} L_t^e) + \text{Log} \varepsilon_t \]  

(4.8)

The method used by Klein and Preston (1967) assumes that the output observed in any time period is the equilibrium level for the observed rates of utilization of the inputs. If this is not an appropriate assumption as pointed out by Briscoe et al. (1970) then the divergence of actual output from full capacity is composed of two elements. First, there is the divergence due to insufficient utilization of inputs by the industries and second, there is divergence due to insufficient production with the inputs actually observed. This would be because labour hoarding is occurring within the industry or because demand for output is fluctuating regularly so that the firms are repeatedly in disequilibrium with respect to output levels (or employment levels for inputs). It is important to distinguish between these components since they have different implications for policies concerned with the elimination of excess capacity. Specifically, we might find inputs fully utilized for an aggregate of industry but total output below the level it could attain since the resources are not correctly allocated between the industries. Given a proper functioning of the price system and sufficient time, this situation will be corrected. On the other hand, each industry could be at its equilibrium output level for the optimum allocation of resources but resources, as a whole may not be fully employed. To remedy this situation may require a deliberate policy designed to simulate overall demand, employment of resources and hence supply of output. Briscoe et al. (1970) use the simple dynamic adjustment mechanism of Koyck to show the influence of lagged output term on the estimation of capacity utilization. If the Cobb Douglas Production Function is assumed, the equilibrium level of output is:

\[ Y_t^* = A e^{\beta_t} L_t^{\beta_t} K_t^{\beta_t} Z_t \]  

(4.9)

and,

\[ \frac{Y_t}{Y_{t-1}} = \left( \frac{Y_t^*}{Y_{t-1}} \right)^q \]  

(4.10)
with restrictions: $0 < \eta < 1$ and $Y_t^* \neq Y_t \neq Y_{t-1}$. Here $A$, $\beta_1$, $\beta_2$, and $\beta_3$ are parameters to be estimated. $Y_t^*$ is the equilibrium level of output for observed factor inputs and $\eta$ is the speed of response while $Z_t$ is the error term. From (4.9) and (4.10) we get:

$$\ln Y_t = \eta \ln Y_t^* + (1 - \eta) \ln Y_{t-1}$$

$$= \eta [\ln A + \beta_1 t + \beta_2 \ln L_t + \beta_3 \ln K_t + \ln Z_t] + (1 - \eta) \ln Y_{t-1}$$

(4.11)

Once the parameters are estimated, then capacity utilization series is obtained as:

$$U_t = \ln Y_t - \ln Y_t^c$$

(4.12)

The approach brings out clearly the influence of lagged output on the estimated capacity. Thus, the Production Function approach is analytically better than simple statistical constructs.

**4.2.4 Measure based on Data Envelopment Analysis**

Färe et al. (1994) used the relationship between technical efficiency and capacity utilization to develop a DEA based model to quantify capacity measure using the definition of capacity output given by the Johansen (1968): capacity is the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factor of production is not restricted. Thus, capacity utilization is the degree to which the DMU is achieving its potential (capacity) output given its physical characteristics (i.e. fixed inputs such as fixed capital in our case). In contrast, technical efficiency is related to the difference between the actual and potential output given both fixed and variable input use. A DMU may be operating at below its capacity level due to underutilization of the fixed inputs, or the inefficient use of these inputs, or some combination of the two. The two concepts are illustrated in Figure 4.2, in which a DMU of a given size is observed to be producing $O_o$ level of output as a result of using $V_o$ levels of inputs. If all inputs were fully utilized (i.e. using $V_c$ rather than $V_o$ variable inputs), and the DMU was operating at full efficiency, then the potential (capacity) output would be $O_c$. Even at the lower level of input usage, if the DMU was operating efficiently it would be expected to produce $O_e$ level of output. Hence, the difference $O_c-O_e$ is due to capacity underutilization; and the difference $O_e-O_o$ is due to inefficiency.

To model capacity measure, the input vector is separated into a sub vector of fixed inputs, and a sub vector of variable inputs and a scalar measure of capacity can be obtained for each decision making unit $i$ at time period $t$. The sub vector of fixed inputs is bounded by observed
values, while the bounds on the variable inputs are allowed to vary, which essentially constraints production by the fixed factors, consistent with the Johansen definition.

**Figure 4.2: Capacity Utilization and Technical Efficiency**

![Diagram showing capacity utilization and technical efficiency](source: Food and Agriculture Organization, 2008)

The mathematical model to compute capacity measure, proposed by the Färe et al. (1994) can be defined as follows:

\[
\text{Maximize } \phi_i^t \quad (4.13)
\]

Subject to:
\[
\phi_i y_i^t \leq \lambda Y, \\
x_m^t \geq \lambda' X_m, \quad m \in F_X \\
\mu_m x_m = \lambda' X_n, \quad n \in V_X \\
\lambda, \mu_t \geq 0.
\]

Where, \( \phi_i^t \) = capacity measure at time \( t \) for \( i^{th} \) decision making unit (DMU). Assume there are \( m \) fixed inputs, \( n \) variable inputs and \( k \) outputs, then \( x_{im}, x_m, \) and \( y_{ik} \) denotes, respectively, the fixed input, variable inputs and output vectors for the \( i^{th} \) year. Thus, \( x_{im} \) is a \((m \times 1)\) column vector, \( x_m \) is a \((n \times 1)\) column vector and \( y_{ik} \) is a \((k \times 1)\) column vector. Moreover, \( X_m = (x_1, x_2, ..., x_m) \) is the \((m \times T)\) matrix of fixed inputs, \( X_n = (x_1, x_2, ..., x_T) \) is the \((n \times T)\) matrix of variable inputs and \( Y = (y_1, y_2, ..., y_T) \) is the \( k \times T \) output matrix. Further, \( \lambda \) is vector of intensity variable of order \( T \times 1 \) and \( \mu_m \) represents input utilization rate of variable input \( n \) at time \( t \) and defined as the ratio of the optimal use of each input to its actual usage. However, capacity utilization (CU) generally refers to the proportion of potential capacity that is used, and is typically measured as the ratio of actual output to capacity output (Kirkley and Squires, 1999). This ratio generally cannot exceed
unity. Färe et al. (1989) proposed that CU be measured as the ratio of output oriented technical efficiency to the capacity measure i.e.,

\[ (CU_{DEA})_i^t = \frac{\theta^i_t}{\phi^i_t} \]  

(4.14)

Where, \( \theta^i_t \) = Technical efficiency score for the \( i^{th} \) DMU at time \( t \) and \( \phi^i_t \) = capacity measure for the \( i^{th} \) DMU at time \( t \). The \( \theta^i_t \) can be defined from the following model which is popularly known as output-oriented CCR model.

Maximize \( \theta^i_t \)  
Subject to: \( \theta^i_t y_i \leq \lambda^t Y \), \( x_i \geq \lambda^t X \), \( \lambda \geq 0 \).

In model (4.15) the output constraint is same as given in model (4.13) whereas, the handling of input constraints differs to some extent. In model (4.15), each input acquires same treatment and no differences exist between fixed and variable inputs. Thus, \( X = (x_1, x_2, ..., x_T) \) becomes a matrix of order \((m + n) \times T\). It is evident from relation (4.14) that capacity utilization and technical efficiency are related with each other.

4.3 Measure based on Operational/Managerial Concept

The problem with the engineering measure and production function based approaches relates to the measurement capital input and the full employment resources of both labour and capital. Another major problem with this approach is that unless the parameters of the Production Function are re-estimated sufficiently frequently, utilization rates will become increasingly biased. Furthermore, whenever the parameters are re-estimated, the entire historical series has to be revised. Thus, the following attempts have been made to make the measure of CU more operational and overcome the weaknesses of engineering methods:

4.3.1 Survey Approach

The most direct and obvious means of obtaining numerical CU levels is to ask firms for their own assessment of the extent to which they are using available capacity. Almost all industrialized countries now include this question in monthly surveys of business. For Ireland, the IBEC-ESRI Monthly industrial Survey (MIS) undertaken on behalf of the EU provides
information on capacity utilization for a number of industrial sector and sub-sector classification. The two questions relating to capacity utilization are: i) For the coming year do you consider your present capacity is: Excessive (+) or Insufficient (-); and ii) During the month, you were operating at about what percent of capacity-please indicate to the nearest 10 percent, 50 percent, 60 percent, 70 percent, etc.

While the responses to question (i) cannot yield exact CU ratio, they provide an overall indication of the direction of change in CU. The number of negative responses gives an indication of the percentage of firms operating at or above full capacity. As a result, the trend in capacity utilization can be inferred from the month-on-month change in the balance of positive over negative (or vice-versa). Question (i) is forward looking in its orientation. On the other hand question (ii) is retrospective in that it refers to the previous month operating period. The responses (b) can also provide a numerical CU ratio, which can be aggregated to provide industry or pectoral measures.

Apart from the normal sampling and measurement error associated with the any survey there are some other important difficulties, which relate specifically to survey CU. One problem relates to how individual respondents will choose to define the terms “capacity” in both questions. Firms may interpret it in the narrow sense of capital utilization or in the broader sense which includes all inputs (labour and materials). Similarly, respondents may confuse the concept of economic capacity with their own subjective or preferred capacity level of operations. Christian (1981) has found that surveyed measures tend to indicate a higher level of excess capacity than do data based measures. He also notes that respondents tend to find capacity when demand is strong and lose it when demand is weak.

**4.3.2 Maximum Achieved Output Approach**

This approach relates to output achieved to a given period of time. The methods included in this approach are simple in calculation as they are based on the factual observations of maximum output achieved at a point of time. The methods compare actual output with estimates of maximum attainable output. The important of these methods are: 1) The Wharton Index of Capacity utilization or Trend through Peak method; and 2) The R.B.I Index of potential utilization.
4.3.2.1 The Wharton Index of Capacity Utilization or Peak-to-Peak Method

Klein and summers (1966) evolved an output time series method known as Wharton Index of capacity utilization. It is the simplest procedure of generating indices of capacity utilization and is also the most common one. Under this method seasonally adjusted monthly values for each of the sub-divisions of the Federal Reserve Board Index of Industrial production are averaged into quarterly figures. The quarterly figures are then charted and peak in each of the series are then chosen by inspection. Each peak is called capacity and is defined as one where output (measured at constant prices) exceeds the level of the immediately preceding quarter and the two succeeding quarters. A straight line from peak to peak describes capacity during the intervening period. After the linear interpolation between output peaks is made, the utilization rate is taken as the ratios of actual output to points on this line segment. The picture of the graph so obtained by joining the successive peak in a straight line is with several kinks. For a period after a peak but before another has been reached, the last straight line is extrapolated with the same slope until the current production intersects the extrapolated line. After intersection, capacity is taken to be the line connecting the last peak and the most recent output figure until a new peak is reached. To arrive at a capacity measure for the manufacturing sector as a whole, the individual industries are aggregated with the help of value added weight.

Despite of various advantages of this method, the procedure is still not free from criticism. Briscoe et al. (1970) Paul (1974a), Bhagwati (1961) and Phillips (1963) are among those who have discussed its disadvantages. It is argued that: i) this measure treats output as a single function of time and does not relate output to input; ii) the assumption that capacity grows at a constant rate is not reliable; iii) this procedure considers capacity growth to be smooth between the two peaks, irrespective of the increase in labour and capital availability; and iv) it does not take into account the supply constraints and other rigidities of underdeveloped economies.

4.3.2.2 The R.B.I. Index of Potential Utilization

The index of Potential utilization, which is estimated by Reserve Bank of India, is a modified version of Wharton School measure of capacity. However, some differences exist between the two measures. The important ones are: a) The RBI index of Potential utilization makes use of monthly output; b) No attempts are made here in this measure to connect successive peaks by linear interpolation; and c) The RBI monthly indices of output are not
desesionalised. Despite these differences the RBI Index of potential Utilization is very much in the intellectual tradition of the Wharton school procedure. Some points may be raised about the RBI Index. First of all, it is very unlikely that monthly indices meet the sustainability criterion. Secondly, capacity expansion may take discrete jumps as implied by the RBI measure which is perhaps a more faithful description of reality at the individual firm level. But this may not be so when several firms are aggregated at the industry level. Capacity expansion may look like a smooth curve as suggested by the Wharton measure. Thirdly, there is no reason to believe that seasonality is confined only to sugar, tea and salt industries.

**4.3.2.3 Time Intensity Approach**

The time measure devised by Winston (1974), measure the number of hours the capital plant is utilized in a year as a percentage of 8760 hours (the number of hours available in a year) and then adjusting the time measure to the intensity of use. This approach relates 24 hours a day and 365 days a year with “full capacity”. Most machines can be operated at different speeds though there is probably only one optimal speed, which corresponds to the least wear tear. Production managers desired this rate, when it is realized it is said to operate at 100% and there is no need to adjust $U_t$. If, on the other hand, the actual speed of operation is only 50% of “optimal” speed, then the intensity of use only 50% and $U_t$ has to be adjusted downward by half. Therefore, there is necessity to have an additional Time and intensity measure of capital utilization $U_{ti}$. The Time-Intensity measure $U_{ti}$ survey based approach. The manager of plant is asked to state the number of operation hours and the approximate percentage of total capital contained in the section. In cases, where different sections of plants have different production schedules and therefore capital utilization rates, the share of each section in the total replacement value of the plant is used as weight in calculating the Time intensity capital utilization rate for the plant as whole. The same procedure is for the sample sector as whole. There are certain advantages in using this approach to estimate the level of capital utilization and also there doesn’t arise the problem of measuring the variables used in the output-based approach of aggregating *Production Function* and of specifying the relationships between capital utilization and its proxy.
4.4 Economic Measure of Capacity Utilization

The rate of capacity utilization inherently assumes that the industry is not in a position to adjust the level of capital to its equilibrium level in the short-run. Thus in the case of shortfall in demand and consequent decline in output, the level of capital stock cannot be reduced immediately. This results in under-utilization of the existing capacity. Similarly when increased demand warrants expansion of capital stock but short-run rigidity does not allow it, the industry has to manage with overutilization of existing capital stock. Thus CU can be on either side of unity, which is in sharp contrast to traditional CU measures which are always less than unity. Moreover, the problem of CU in this framework is essentially a short-run phenomenon.

4.4.1 A Cost Function Based Approach

Klein (1960), Morrison (1985a) and Berndt (1989) among others, have the intuitively appealing suggestion that capacity output is inherently a short-run concept conditional on the level of quasi-fixed input available to producers. Capacity output can then be defined as the optimum level of output for given levels of quasi-fixed factors. In this context, the producer’s technology can be represented by the Production Function \( f \) below in model (4.16),

\[
Y = F(V, X, t)
\]  

(4.16)

Where, \( V \) is \( n \times 1 \) vector of variable inputs and \( X \) a \( j \times 1 \) vector of quasi-fixed inputs. Time, \( t \) is included as an argument in the Production Function to act as a proxy for technological advance. Subject to certain regularity conditions (Diewert, 1974), if costs are minimized with respect to the variable inputs \( V \) conditional on the level of output (\( Y \)) the quasi-fixed output (\( X \)), then there exist a variable or restricted cost function (\( C_v \)) which is dual to (4.16)

\[
C_v = G(Y, P_v, X, t)
\]  

(4.17)

Where, \( P_v \) is the vector of prices of variable inputs of order \((1 \times n)\). Thus, the Short-run average total cost (\( SRAC \)) can be defined as the sum of average variable costs and average fixed costs in equation (4.18),

\[
SRAC = \frac{C_v}{Y} + \frac{P^*_X}{Y} X
\]  

(4.18)

Where \( P^*_X \) is \( 1 \times j \) price vector for the quasi-fixed inputs. Capacity output, defined as optimal level of output for given level of the quasi-fixed factors, is that level of output which minimized \( SRAC \). Thus, at point where actual and capacity output are equal, i.e. \( Y = Y^* \), equation (4.18) is
minimized. Differentiating (4.18) with respect to $Y$ and setting equal to zero yields equation (4.19),

$$\frac{\delta SRAC}{\delta Y^*} = \left( \frac{1}{Y^*} \right) \left( \frac{\delta C_v}{\delta Y^*} \right) - \left( \frac{C_v}{Y^*} \right) - \left( \frac{P_x X}{Y^*} \right) = 0$$ (4.19)

For many functional forms for $C_v$, an exact analytical solution can be obtained for $Y^*$ from (4.16). However, by simple inversion, it is clear that $Y^*$ will depend on the argument of the variable cost function $(P_v, X, t)$ and the price vector of the quasi-fixed factor as shown in model (4.20),

$$Y^* = Y^*(P_v, X, P_x, t)$$ (4.20)

In this setting, capacity output is therefore directly related to variable input prices, the level of fixed factors, the prices of fixed factors and state of technology. From the solution to (4.19), the rate of capacity utilization can be defined as actual output, $Y$, over capacity output, $Y^*$ i.e.,

$$CU = \frac{Y}{Y^*}.$$

**A diagrammatic Analysis**

This section presents a simple diagrammatic exposition of above CU measure. We assume, i) a single quasi-fixed factor and (e.g. capital); and ii) long run constant return to scale. Under long run constant return to scale, the firms long run cost function is homogeneous of degree one in output. As a result long run average costs don’t depend on the level of output and are represented by the horizontal line $LRAC$ in Figure-4.3. We can also define two short –run average cost curves ($SRAC_0$ and $SRAC_1$) for levels of quasi-fixed capital stock ($K_0$ and $K_1$). For $K_1 > K_0$, $SRAC_1$ will lie to the right of $SRAC_0$. Since average fixed cost tends to fall with output and average variable costs tend to raise, both $SRAC_0$ and $SRAC_1$ will be $U$-Shaped as depicted in Figure 4.3.

Capacity output (i.e., the value of $Y$ which solves the equation (4.11)) for different levels of quasi–fixed capital stock is given by minimum points on the $SRAC_0$ and $SRAC_1$. Furthermore, the long run cost function is just the short run function evaluated at the optimum level of the quasi-fixed factors and therefore, capacity output will also correspond to the point of tangency between $SRAC$ and $LRAC$. With only one quasi-fixed factor, this measure of CU strictly corresponds to the narrower concept of CU which refers to the utilization rate of all inputs.
(labour and materials). Under this theoretical framework, there is no rational for variation in the utilization rates of the other factors of production since they will always adjust instantaneously to their optimal levels. Furthermore, under this short run interpretation, measure of CU can be either greater than equal to or less than one. If the firm has $SRAC_0$ and if actual output is given by $Y'$, then short run CU given by $\frac{Y'}{Y_0}$ is greater than 1. If such a firm expect output to remain at $Y'$ it would have an incentive to invest in productive capital in order to lower average costs. Alternatively, if the firms short run cost curve is given by $SRAC_1$, then CU given by $\frac{Y'}{Y_1}$ is less than one and the firm has an incentive to disinvest if it expects that output remain at $Y'$.

**Figure 4.3: Diagrammatic Measurement of CU**

### 4.4.2 Minimum Capital Output Ratio Approach

The method of minimum capital output ratio, as suggested by the National Conference Board of the United States, estimate capacity on the basis of capital output ratio. Fixed capital output ratios are estimated in terms of constant prices. A benchmark year is then selected on the basis of the observed lowest capital output ratio. In choosing the benchmark year other independent evidence is also taken into consideration. The lowest observed capital output ratio is considered as capacity output. The estimate of capacity is obtained from real fixed capital stock deflated by minimum capital output ratio. The utilization rate is given by actual output as a proportion of the estimate of capacity.

Thus,

$$\left( CU_T \right)_t = \left( \frac{Y_t}{K} \right) \times 100$$

(4.21)
\begin{align*}
\hat{K} &= \text{Min} \left[ \frac{K_t}{\left( \frac{K_t}{Y_t} \right)} \right]
\end{align*}

Where, \((CU_T)_i\) is capacity utilization by \(i^{th}\) state at time \(t\), \(Y_t\) is gross output, \(\hat{K}\) is the estimate of capacity, \(K\) represents real gross fixed capital, and \(\left( \frac{K_t}{Y_t} \right)\) represents capital output ratio. Although, this method provides useful measure of capacity utilization, the problems of measurement of capital are formidable. Capital is even more difficult to measure than capacity. Needless to say, the usefulness of this method depends critically on accuracy of the measurement of capital.

### 4.5 Choice of the Technique for CU Measurement

It is evident from the previous chapters that DEA based non-parametric methods are to be applied for efficiency and productivity measurements. Thus, the *Production Frontier* based data envelopment analysis method has also been preferred to develop a CU index for analyzing the inter-temporal and inter-state variations in CU of Indian sugar industry. The same index (i.e., \(CU_{DEA}\)) has also been utilized to adjust the variable of gross fixed capital (i.e., a proxy of capital in place) as per the utilization of the plant capacity to obtain capital in use. The choice of the non-parametric DEA based measures of CU over the other production and cost function estimation based parametric techniques has been governed by its ability to handle small samples. Further, it does not require any pre-supposition regarding the technical nature of production or cost functions (i.e., Cobb-Douglas, CES or Translog, etc.). However, to check the robustness of the CU results, an additional approach i.e., minimum capital output ratio method has also been utilized to obtain CU measures for Indian sugar industry.

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