CHAPTER – 2

REVIEW OF LITERATURE
The purpose of literature review is to create familiarity with current thinking and research on a particular topic. Many studies have been conducted over the years to explain about the special functions. In this chapter researchers and their respective research work are highlighted which relates to the topic. A number of research papers have been studied for review purpose. Some of the authors and their research works are summarized below.

**Oparnica (2001)** developed a mathematical model for formulating and analyzing the viscoelastic rod to described constitutive equation which contain fractional derivatives. The outcomes are obtained as particular cases.

**Agrawal & Yuan (2002)** established a mathematical method for dynamic investigation of mechanical systems which are comparative to fractional derivatives of displacements, a fractional differential equation leading the dynamic system is distorted into a set of differential equations with no fractional derivative expressions. This set is converted to a set of first order ordinary differential equations, which are included with a mathematical method to obtain the retort of the method when using Laguerre integral formula.

**Bahuguna (2003)** considered a group of integro differential equations in an arbitrary Banach space and found the existence, uniqueness, regularity and continuation of solutions to these integrodifferential equations using the theory of analytic semigroups.

**Kumar & Suresh (2003)** evaluated the Mittag-Leffler distribution, and studied different distributional properties and characterizations associated to the Mittag-Leffler allocation and revised semi-Mittag Leffler distribution, Generalized positive Linnik distribution and other related distributions. Obtained several innovative outcomes associated to
the distributional properties of the semi-Mittag Leffler distribution and examined the outcome associated to evaluation of parameters in Mittag-Leffler distribution.

Mainardi & Pagini (2003) studied the fractional calculus and its estimated discretization. Presented two direct discretization methods useful in control and digital filtering for discretizing the fractional-order differentiator or integrator and specified comprehensive mathematical formulae and tables. Offered to demonstrate the basically utility of the two planned discretization methods as an illuminating illustration.

Agrawal (2004) introduced a Fractional derivative approach for thermal analysis of disk brakes. The formulation urbanized involve fractional semi integral and derivative expressions, which present an uncomplicated approach to calculate friction plane temperature and heat volatility as functions of moment. The formulation provides a means to compute the surface when given the temperature fluctuation.

Bahuguna (2004) evaluated a semilinear nonlocal functional differential problem in a Banach space and found the existence, uniqueness and continuation of an easygoing explanation. Proved a number of promptness consequences for this easygoing explanation beneath dissimilar circumstances. Lastly think about a number of applications of the conceptual outcomes.

Goyal & Goyal (2004) created an extremely broad and beneficial theorem which communicates the Laplace transform as well as the universal Weyl fractional integral operator concerning the multivariable H-function of related functions of numerous variables.

kilabas et al (2004) studied the classical Mittag-Leffler function $E_{\rho,\mu}(z)$ and the Kummer confluent hypergeometric function $\Phi(\gamma, \mu; z)$and proved the properties of Mittag-Leffler function. They recognized Riemann–Liouville fractional integration and differentiation operators and presented for the integral operators containing the Mittag-Leffler and Kummer functions, $E_{\rho,\mu}(z)$ and $\Phi(\gamma, \mu; z)$, in the kernels.
Saxena et al (2004) investigate the solution of a unified form of fractional kinetic equation in which the free term contains any integrable function \( f(t) \), which provides the unification. The solution has been developed in terms of the Wright function in a closed form by the method of Laplace transform. Derived a closed-form solution of a fractional diffusion equation. The results obtained are in a form suitable for numerical computation.

Suthar et al (2004) discussed the descriptions of the product of H-function, in which its argument comprises a factor \( ( \) \( k \) \( k \) \( z \) \( r \) \( t \) \( c \) \), and a common group of polynomials beneath the numerous Erdelyi-Kober operators. The outcome obtained are common in quality and includes as particular cases.

Saxena et al (2005) showed that the fractional integration which is basis of the Kober operator, the base analogue of the H function and after deducing positive outcome imply \( G_q (\cdot) \). The basic Bessel equation \( (\cdot) \) basic functions such as main result applications. The results are seen as an innovative involvement to the assumption of fraction q-calculation.

Saxena & Kalla (2005) solved a fractional integro-differential equation concerning a comprehensive hypergeometric function in several complex variables and contains a continuous function and used properties of fractional calculus and the standard Laplace transform. Considered a Cauchy-type problem concerning the Caputo fractional derivatives and a comprehensive Volterra integral equation.

Kilbas et al (2006) studied the nonsequential linear fractional differential equations with invariable coefficients concerning the Caputo fractional derivatives. The equations are studied in terms of Mittag-Leffler functions and generalized Wright functions to obtain the general
explicit solutions by the Laplace transform. Linearly independent solutions can be obtained by the given conditions which form a primary classification of solutions.

Yadav et al (2008) identified Fractional q-integral operators of generalized Weyl type, which is containing universal basic hyper-geometric functions in addition to basic analogue of Fox's H- function. Numerous integrals containing various q-functions which have been evaluated as an applications of main result and they have also discussed some special cases.

Chourasia & Pandey (2009) developed a generalized fractional kinetic equation involving the Lorenzo–Hartley function, a generalized function for fractional calculus. The fractional kinetic equation can be used to examine an extensive division of identified fractional kinetic equations. An explanation is recognized in expressions of the Lorenzo–Hartley function. Some special cases have been measured.

Prajapati & Nathwani (2009) presented a recurrence relation and an integral representation of a additional generalization of a generalized Mittag-Leffler function. Some properties of a special case of this function are also studied by the means of fractional calculus.

Srivastava & Tomovski (2009) introduced a fractional calculus through the integral operator, which contains a group of generalized Mittag-Leffler functions for the Kernel. Some special cases are discussed in the literature.

Bhatter & Shekhawat (2010) found a pair of multi-dimensional operators integrated rationales whose cores refer to the product of multifunctional polynomial function H. Get first images of the two functions as operator of the study. We then establish two theorems that give generalized multidimensional integrated fractional operators and vice versa, fractional integrals of generalized multidimensional transform. Finally, the fraction Integral operators studied by us
are quite general in nature and can be considered extensions and the unification of a series of results for simple single fractional operators.

Chaurasia & Pandey (2010) developed a comprehensive fractional kinetic equation involving the Lorenzo-Hartley function. It is discussed for using to consider a spacious group of recognized fractional kinetic equations, until then sprinkled in the text and recognized a solution in expressions of the Lorenzo-Hartley function. Considered the Special cases concerning the comprehensive Mittag-Leffler function.

Chourasia & Saxena (2010) evaluate some relationships involving triple integral H function and multivariate function H. Three theorems that contain the creation of H function, through our core results and the use of integral transformation Mellin. The outcomes are relatively common in nature owing to the occurrence of a fundamental functions of nature.

Gupta & Agrawal (2010) create a common theorem that provides the representation of a improved H-transform in the fractional integral operator containing the multi-variable H-function. They assume two essential corollaries relating to universal Bessel function, Mittag-Leffler function.

Purohit & Yadav (2010) introduced two operators hypergeometric integration q fractional, which can be considered extensions q-fractional integral operators. By defining q-extensions of the similar, the authors investigated their fundamental properties, such as integration via parts and the relation theorem through Mellin q-analog Transform.

Torres et al (2010) demonstrate the optimization conditions for different variational functions that contains Caputo left and right fractional derivatives. The fractional
isopheremetric problem is then formulated with an integral strain also containing Caputo derivatives.

**Changpin & Qian (2011)** studied the main characteristics of the Riemann-Liouville derivative, which has been frequently used in fractional derivatives. A number of essential properties of the Caputo derivative which have not been deliberated in another place are at the same time mentioned. They presented the partial fractional derivatives. These deliberations are advantageous in thoughtful fractional calculus and exhibiting fractional equations in science and engineering.

**Haubold et al (2011)** presented a comprehensive account or relative to briefly depiction of the Mittag-Leffler function, generalized Mittag-Leffler functions, Mittag-Leffler type functions, and their remarkable and valuable properties. Applications of Generalized. Mittag-Leffler functions in certain regions of physical and applied sciences which are moreover established in this research paper publication, almost every kind of Mittag-Leffler type functions obtainable in the text are existing.

**Purohit & Kalla (2011)** considered the solutions of generalized fractional partial differential equations concerning the Caputo time-fractional derivative and the Liouville space-fractional derivatives. The explanation of these equations are attained by applying the joint Laplace and Fourier transforms. Several special cases as solutions of one-dimensional non-homogeneous fractional equations occurring in quantum mechanics are presented in the concluding section.

**Saha et al (2011)** presented certain integrals including product of the I-function through exponential function, Gauss's hypergeometric function & Fox's H-function. The outcomes resulting are simple in nature and can contain a number of different outcome as particular cases.
Sharma & Dhakad (2011) showed a different function K2 which has the expansion of the function well-defined by Miller and Ross and its associations through additional special functions. Some relations are too resulting those occurs connecting the K2 – function and the operators of Riemann Liouville fractional calculus.

Srivastava et al (2011) established relationship between Zeta function and the H-function. Derive a analytic continuation formula which provides an elegant extension of the well-known systematic extension formula for the Gauss hypergeometric function. Fractional derivatives associated with the generalized Hurwitz–Lerch Zeta functions are obtained.

Agrawal (2012) studied and developed the generalized fractional integral operators specified by Saigo. Author established two theorems which has given the descriptions of the product of $H$-function and the common group of polynomials in Saigo operators. Along with discussion of several special cases.

Ahmed (2012) introduced a new Mittag multivariable generalized Mittag Leffler function, certain properties of the new Mittag Leffler generalized multivariate function associated with the fractional calculation points compounded by Riemann's Liouville integral fractional operators associated with multivariate Mittag Leffler function is obtained in the nucleus.

Ansari (2012) provided definite innovative outcome in the theory of fractional derivative for which these outcome shows the flexible operational techniques which can be made applicable in the integral transforms to obtain the formal solutions.

Chand & chourasia (2012) developed a finite integral pertaining to two H-functions with extended Jacobi-polynomial. Discussed the integration of product of a certain class of Feynman integral and application of the main result of the Riemann-Liouville type fractional integral operator. The results derived are basic in nature and they are likely to be useful applications into quite a few fields particularly electromagnetic hypothesis, statistical mechanics and probability hypothesis.

Chaurasia & Singh (2012) modify a pathway fractional integral operator concerning with the pathway representation and pathway probability solidity of a number of product of special functions. The outcomes resultant here are relatively general in nature, and hence encompass several cases of interest.

Chaurasia & Kumar (2012) given the solution of a unified fractional Schrödinger equation. The solution is obtained by applying the Laplace and Fourier transform closed form according to the Mittag-Leffler function. The result here is common within natural world and is accomplished of producing a huge amount of scattered results in the literature. the majority of the outcomes are in a form appropriate for numerical calculations.

Hassan et al (2012) revised the correlation involving Sumudu and Laplace transforms and create several assessment on the solutions and presented a number of examples of solution of differential equations with Laplace transform though the examples point out that if the solution of differential equation by Sumudu transform exists then the solution necessarily exists by Laplace transform.
**Prajapati & Shukla (2012)** studied some special properties of the function. The authors define the decomposition of the function in the farm truncated series of powers as equation and its various properties, including the integral representation, the derivative, we obtain inequality and cases.

**Purohit et al (2012)** deduced the generalized operators of fractional integration involving Appell's function $F_3(.)$ due to Marichev-Saigo-Maeda, to the Bessel’s function of first kind and communicate the representation in conditions of the generalized Wright and hypergeometric functions.

**Salim et al (2012)** studied a new generalized function of Mittag-Leffler type. Investigated a variety of properties including differentiation, Laplace transform, Beta transform, Mellin transform, Whittaker transform, generalized hypergeometric series form, Mellin-Barnes integral illustration along with its connection through Fox’s $H$-function along with Wright hypergeometric function. properties of generalized Mittag-Leffler function connected with fractional differential and integral operators are measured.

**Saxena et al (2012)** introduced the Laplace Sumudu transformations and the Alef function and, for comparative purposes, the implementation of both Alef-function transformations to see the differences and similarities and more were discussed special cases.

**Sharma (2012)** presented a solution of an integral equation of the fractional generalized K4 function Volterra involving the use of Sumudu transformation. It is expected that some of the results could have applications in the solution of some fractional order differential and integral equations.
Singh & Mehta (2012) presented a mathematical model through taking into consideration that the rate of growth for types of species and carrying capability of its situation are unswervingly exaggerated through contamination and reduce as the pollutants attentiveness increases. After that a explanation of this accurate mathematical equation it can also attained through the facilitate of generalized hypergeometric function.

Torres et al (2012) studied a proper fractional extension of the classical calculus of variations by considering variational functionals through a Lagrangian counting on a collective Caputo partial derivative as well as the conventional derivative and verified Euler–Lagrange equations to the fundamental and isoperimetric problems.

Ahmad et al (2013) studied the Dirichlet mean of the Hartley R function and were introduced by Lorenzo. The demonstration of these dealings is obtained in conditions of fractional operators incorporated by Riemann - Liouville and several individual cases were considered.

Benson et al (2013) studied the fractional derivatives derived from the governing equations of stable Lévy motion, and that fractional integration is the correspondently inverse operator. Fractional integration, and its multi-dimensional extensions resulting by this steps, which are familiarly secured with fractional Brownian motions and noises basically. Represents a study of fractional calculus methods in hydrology through a small number of twists.

Choi & Agarwal (2013) presented two generalized integral formulas involving the Bessel function of the first type, which are articulated in conditions of generalized hypergeometric functions (Wright). And several individual cases were discussed. also point out that the results presented here, being of general disposition, are without difficulty reducible to
give up a lot of miscellaneous original and recognized integral formulas involving simpler functions.

Faraj et al (2013) explained more properties for the generalized Mittag-Leffler function which are $E\alpha, \beta, p\gamma, \delta, (z)$ with different type of fractional calculus operators and the same was called Weyl fractional integral & differential operators. A newly integral operator which is depending on Weyl fractional integral operator and having $E\alpha, \beta, p\gamma, \delta, (z)$ in its kernel is demarcated and studied.

Kumar & Daiya (2013) given the two theorems of generalized fractional derivative operators specified by Saigo-Maeda. The theorems are established in conditions of the product of $H$-function and a common group of polynomials with the support of Saigo-Maeda power function formulae.

Singh & Kumar (2013) obtained new outcomes for the product of the general class of multi-variable polynomials and the multi-variable H-function including the Riemann-Liouville, the Weyl, and similar other fractional-calculus operators, The fractional derivative is considered in the Caputo sense and numerous special cases also have been discussed.

Purohit & Raina (2013) obtained certain new integral inequalities involving the Saigo fractional integral operator. It is also shown how the various inequalities considered themselves of q-extensions which are capable of yielding various results in the theory of q-integral inequalities.
Ahmed et al (2014) introduced a new generalization of M-series. Investigated various properties of M series together with differentiation, Laplace transform, Mellin transform, recurrence relation, Beta transform, Mellin Barnes integral representation and its relationship with Fox's H-function and Wright hypergeometric function. The integral operator containing the generalized M-Series considered its boundedness on L(a,b) and established Hilfer's fractional derivative.

Gaboury & Rathie (2014) presented a generalization of hypergeometric function $pF_q$ with $p$ numerator parameters and $q$ denominator parameters involving hypergeometric identities. The result is obtained by suitably applying fractional calculus method to a generalization of the hypergeometric transformation formula.

Gaboury et al (2014) derived several new expansion formulas involving a generalized Hurwitz-Lerch zeta function. These explanations are collected by means of a number of fractional calculus theorems comparatively the generalized Leibniz rules for the fractional derivatives and the Taylor-like expansions in terms of different functions. Several special cases are also considered.

Garg et al (2014) considered linear fractional derivative as time derivative and Riesz-Feller fractional derivative with skewness zero as space derivative. Concerned Laplace and Fourier transforms to obtain its solution. Reaction-Diffusion equations have found numerous applications in space-time fractional reaction diffusion equation.
**Gupta (2014)** consequented the solution of generalized kinetic equations of fractional order concerning the Wright generalized Bessel function. The obtained outcome, involve the identified outcome of further specifically in terms of $K_4$–function introduced Special case, concerning the F-function is measured.

**Kumari & Nambisan (2014)** found some differentiation formulas for the I-function of two variables and an amount of particular cases of the outcome are also discussed and obtained differentiation formulas including elementary functions.

**Ozarslan (2014)** presented the function Mittag Leffler extended the use of the extended beta function and obtaining a plurality of integral representation of them. The Mellin Transform of this function is given in terms of Wright general hypergeometric series. In addition, it is shown that the extended fractional derivative the usual Mittag Leffler function allows the extended Mittag Leffler function.

**Ahmed Shakeel (2014)** used the generalized fractional calculus operators on the generalized Mittag Leffler function. A number of outcomes will be obtained with generalized Wright function. The current outcomes are obtained as particular cases.

**Teodoroa & Oliveira (2014)** introduced the Mittag-Leffler function as a simplification of the exponential function. Special cases are recovered. The exponential function is the explanation of a linear differential equation through invariable coefficients and the Mittag-Leffler function is a resolution of the fractional linear differential equation through invariable coefficients. Use infinite series and the Laplace transform,

**Yasar & Ozarslan (2014)** defines the generalized Mittag Leffler function through the general pochhammar symbol and presents some relation of recurrence, derived properties, obtain
integral representation and obtained a relation involving wright hypergeometric function and the generalized Mittag-Leffler function.

**Bhatter & Faisal (2015)** prepared a new feature called Mittag Leffler function that unifies many forms of Mittag Leffler function and many other special features, including a newly defined sinusoidal function, and then draw their relationship with some Known special functions.

**Grothaus et al (2015)** developed the Mittag Leffler analyzes convergent sequences that characterize the distribution space. Then apply the structures and analysis techniques Mittag Leffler to show that the function of a to the time fractionated heat equation can be constructed by means of a Brownian motion incorporating the generalized fractional Feynmankas formula.

**Gill & Modi (2015)** introduced generalized fractional integral and derivative operators concerning the Appell function $F_3$. Presented unified fractional integral and derivative formulas which are involved in a product of Aleph functions. A large number of outcomes of fractional integral and derivative concerning simpler functions are obtained.

**Prajapati & Nathwani (2015)** introduced an operator in the space of Lebesgue measurable real or complex functions. several properties of the Riemann–Liouville fractional integrals and differential operators connected by means of the function $E_{a,b,c;\cdot}(cz; s, r)$ are considered and the integral representations are obtained. Some properties of a special case of this function are studied of fractional calculus.

**Saxena et al (2015)** studied an operator of the generalized Mittag Leffler function. The Laplace and Mellin transformations of this new operator are studied. The result is useful when
the Mittag Leffler function occurs naturally. Established the limiting the properties and the
composition of this operator. The importance of the results of the derivatives also lies in the
fact that the results of the generalization Mittag Leffler function are also given.