ABSTRACT

Approximation Theory is an old and rich branch of analysis and a large number of researchers have studied this subject. The theory has many applications in mathematical analysis, nonlinear problems arising in physical sciences, engineering and social sciences. Since the particular examples of approximation often arise from problems of Science and Technology, they provide proper motivation for the subject of Approximation Theory.

Fixed point theory is another most dynamic area of research, with a lot of applications in various fields of pure and applied mathematics, as well as, in physical, chemical, life and social sciences. This theory is of interest in itself as it provides ways to establish the existence of solutions of algebraic, differential and integral equations.

The problem of solving the equation \( T(x) = 0 \) is equivalent to finding fixed point of the mapping \( x \mapsto x - T(x) \). Since finding an exact solution of the equation \( T(x) = 0 \) is not always possible, approximation theory comes to our rescue and we try to find an approximate solution (which is best possible subject to given constraints). Thus the two subjects of fixed point theory and approximation theory are closely related.

Most of the results in Approximation Theory and Fixed Point Theory are available in Hilbert spaces and normed linear spaces and the consideration of approximation and fixed point problems in more general spaces viz. \( p \)-normed spaces, convex metric linear spaces, metric linear spaces, convex metric spaces and metric spaces is quite challenging. Since the results available in these more general spaces do not constitute a unified theory, we make an attempt in this direction. Also most of the results of approximation theory using fixed point theorems are available in normed linear spaces, so another aim of this project is to discuss such applications when the underlying spaces are spaces more general than normed linear spaces.

In the first chapter, we start with a brief historical background of the subject, give certain notations, necessary definitions and terminology frequently used in the thesis, and also chapterwise summary of the results proved in the subsequent chapters.

In the second chapter, we discuss the existence of fixed points and common fixed points of single-valued self mappings satisfying some contractive or nonexpansive type conditions in the setting of convex metric spaces and metric spaces. As applications we
discuss some results on the existence of invariant points from a set of best approximation of a single-valued mapping in the third chapter. We generalize and extend some of the known results of Brosowski-Meinardus types on invariant approximation for a single-valued mapping in metric linear spaces, convex metric spaces and metric spaces in this chapter. We also discuss the existence of invariant points from a set of best simultaneous approximation and deduce some results on invariant points from a set of best approximation at the end of this chapter.

Fourth chapter is concerned with the existence of invariant points from a set of best approximation for a pair of commuting mappings in the framework of convex metric spaces. We also generalize, unify and extend some of the known results of Brosowski-Meinardus types on invariant approximation for a pair of commuting mappings when the underlying spaces are metric spaces.

Fifth chapter is concerned with the continuity of the best approximation map and the existence of invariant points of a nonexpansive mapping from the set of $\varepsilon$-approximation when the underlying spaces are convex metric spaces and metric spaces.

Last four chapters are concerned with some results on the existence of common fixed points for some noncommuting mappings viz. $R$-subweakly, pointwise $R$-subweakly, $C_q$-commuting and for Banach operator pairs respectively. In these chapters, as applications various results on invariant points from a set of best approximation for these classes of mappings are also obtained in normed linear spaces, convex metric spaces and metric spaces.

*******