A controlled approach to plan the experiments is a necessity for an efficient manner of experiments. By the statistical design of experiments, the process of planning the experiments is carried out, so that appropriate data will be collected and analysed by statistical methods resulting is valid and objective conclusions. When the problem involves data that is subjected to experimental error and statistical methodology is the only objective approach to the analysis. Thus, the design of the experiments and the statistical analysis of the data are the characteristics of an experimental problem.

These two points are closely related to the method of analysis depends directly on the design of experiments employed. The advantages of the design of experiments are as follows:

- Numbers of trials are significantly reduced.
- Important decision variables of the process can be easily identified.
- Optimal setting of the parameters can be found.
- Qualitative estimation of parameters can be made.
- Experimental error can be estimated.
- Inference regarding the effect of parameters on the characteristics of the process can be made.

Several innovative techniques have been used to extract maximum usage from early developments in the field design of experiments. Commonly applied experimental design methods include the single factor at a time approach that is, one factor is changed at a time for a given experimental runs and all other factors remained constant (Kumar et al. 2003), Full factorial (Cochran et al. 1962; Montgomery et al. 2001), Taguchi’s method (Taguchi, 1986; Ross et al. 1996) etc. In the following sections, the analysis procedure for Taguchi’s analysis is explicated. In the present work, the Taguchi’s methodology has been used to plan the experiments and subsequent analysis of the data are collected.

4.1 Taguchi’s Experimental Design and Analysis

The traditional factorial design has the disadvantage of including all the possible combination setting of the parameters involved in the study, resulting in a very large number of experiments and considerable time required to accomplish the task. In the fractional factorial design of experiments, only a small fraction of settings
from all possible combination is selected, which reduces the number of tests to the reasonable level. This method is well known, but the only main shortcoming is non-availability of general accepted standards and guidelines for both the design of experiments and the analysis of the results (Jiang et al. 1997; Creese, 2001). Taguchi’s orthogonal arrays provide a substitute to standard factorial design, which is a powerful and efficient method for process design that operates consistently and optimally over a variety of situations. The primary goal of robust parameter design is to find factor setting that minimizes response variation while adjusting the process on the objective. The setting of controllable factors is made after determining the factors affecting variations. A product designed with this goal will deliver more consistent performance irrespective of the environment in which it is used.

In the Taguchi’s method, the experimental results are analysed to achieve one or more of the following objectives to the optimum conditions are identified with the help of the main effect of each of the parameters. Two approaches are suggested by Taguchi to carry out the analysis of the experiments. In the first approach, the results of single run are processed through the main effect. The multiple run, the second approach, where S/N ratio is used. To accommodate wide ranging data and for suitability of linearity, a logarithmic transformation of mean squared deviation (signal to noise ratio) are used for analysis of results. The optimum conditions identified from such analysis are likely to produce the consistent performance. The loss function is linked with the quality matrix (Roy et al. 1990). The loss associated with the process the of product can be minimized by maximizing the S/N ratio. Ascertain the optimum condition for a product or process.

1. To estimate the contribution of the individual variables.
2. Estimation of the optimum condition response.

### 4.1.1 Loss Function

The loss is measured in terms of monetary units and is related to quantifiable product characteristic. Taguchi defines quality loss via his “loss function” (Braker et al. 1986). He unites the financial loss with the functional specification through a quadratic relationship that comes from a Taylor series expansion. The quadratic function takes the form of a parabola. Taguchi defines the loss function as a quantity proportional to the deviation from the nominal quality characteristic (Roy et al. 1990).
Figure 4.2 presents the quadratic loss function. The following quadratic form to be a useful workable function (Roy et al. 1990):

\[ L(y) = k(y - m)^2 \]  

(4.1)

Where,

- \( L \) = Loss in monetary units
- \( m \) = value at which the characteristic should be set
- \( y \) = actual value of the characteristic
- \( k \) = constant depending on the magnitude of the characteristic and the monetary unit involved.

**Figure 4.1: Step Function**

**Figure 4.2: Quadratic Loss Function**
The loss function represented in equation 1 and is graphically present in figure 4.1. The characteristics of the loss function are (Roy et al. 1990):

1. The product’s characteristic varies from the target value, the greater is the loss.
2. The loss must be zero when the quality characteristic of a product meets its target value.
3. The loss is a continuous function and not a sudden step as in the case of traditional approach (figure 4.2). This consequence of the continuous loss function illustrates the point that merely making a product within the specification limits does not necessarily mean that product is of good quality.

4.2 Signal to Noise (S/N) Ratio

In Taguchi’s Methodology, a loss function is used to calculate the deviation between the experimental value and the desired value (Ross 1996). Taguchi created a transformed form of the loss function which is called Signal to Noise (S/N) ratio. The S/N ratio is a quality indicator by means of which the evaluation of the effects of change of particular process parameter on the response can be determined. The S/N ratio is also identified as the logarithm transformation of the mean squared deviation has a multiplier of -10. The multiplier 10 is a scaling factor. This negative multiplier changes the desirability of smaller is better for mean squared deviation to bigger is better for S/N ratio (Roy, 2001). Taguchi proposed a number of performance measures in the context of quality engineering based on the response of interest (Taguchi, 1986; Taguchi, 1987). These performance measures referred as Signal to Noise (S/N) ratios are also discussed (Leon et al. 1987; Phadke et al. 1989; Ross, 1996). A higher value of S/N ratio indicates that the signal is much higher than the random effects of noise factors. However, the procedure consistent with higher S/N ratio value always yields ideal quality with minimum variation. The equations for calculating S/N ratios for Smaller the Better (SB), Larger the Better (LB) and Nominal the Best (NB) along with their graphical representations (Ross, 1988). Figure 4.3 and figure 4.4 presents the quality loss characteristics of smaller the better and quality loss characteristics of larger the better respectively.
A. Smaller the Better (SB):

\[
\left( \frac{S}{N} \right)_{SB} = -10 \log_{10} \left[ \frac{1}{n} \sum_{j=1}^{n} Y_j^2 \right]
\]

(4.2)

Where,

\( Y_j \) = value of the characteristic in an observation \( j \)
\( n \) = number of repetitions in a trial

Alternately,

\[
\left( \frac{S}{N} \right)_{SB} = -10 \log (\text{MSD}_{SB})
\]

(4.3)

Figure 4.3: Quality loss characteristic: Smaller the-better (SB)

where, \( \text{MSD}_{SB} = \frac{y_{12} + y_{22} + \cdots + y_{n2}}{n} \) (4.4)

B. Larger the Better (LB):

\[
\left( \frac{S}{N} \right)_{LB} = -10 \log \left[ \frac{1}{n} \sum_{j=1}^{n} 1/Y_j^2 \right]
\]

(4.5)

Where,

\( Y_j \) = value of the characteristic in an observation \( j \)
\( n \) = number of repetitions in a trial
Alternately,

\[
\left( \frac{S}{N} \right)_{LB} = -10 \log (\text{MSD}_{LB}) \quad (4.6)
\]

\[
\text{MSD}_{LB} = \frac{1}{n} \left( y_{12} + y_{22} + \ldots + y_{n2} \right)
\quad (4.7)
\]

Here the target value \((t) = 0\)

![Diagram showing quality loss characteristic: Larger the better (LB)](image)

**Figure 4.4:** Quality loss characteristic: Larger the-better (LB)

C. **Nominal-the-Best (NB):**

\[
\left( \frac{S}{N} \right)_{NB} = -10 \log \left[ \frac{1}{n} \sum_{j=1}^{n} (y_j - y_0)^2 \right] \quad (4.8)
\]

where,

\( y_j \) = value of the characteristic in an observation \( j \)

\( n \) = number of repetitions in a trial

\( y_0 \) = nominal value of the characteristic
Alternately,
\[
\left(\frac{S}{N}\right)_{Nb} = -10\log (\text{MSD}_{NB})
\]  

where,
\[
\text{MSD}_{NB} = \frac{(y_1 - y_2)^2 + (y_2 - y_2)^2 + \ldots + (y_n - y_0)^2}{n}
\]

The Mean Squared Deviation (MSD) is a statistical quantity that replicates the deviation from the target value. The expressions for MSD are different for different quality characteristics. The S/N ratio is treated as a response to the experiment, which is a measure of the variation within a trial when noise factors are present. A standard ANOVA can be conducted on the S/N ratio which will identify the significant parameters (mean and variance). Quality loss characteristics: Nominal the Best (NB) is shown in figure 4.5.

![Figure 4.5: Quality loss characteristic: Nominal the best (NB)](image)

4.3 Taguchi’s Procedure for Experimental Design and Analysis

The step wise procedure for Taguchi’s experimental design and analysis is shown in figure 4.6. It is described in the following section.

4.3.1 Role of Orthogonal Array’s (OA’s)

Taguchi’s methodology utilizes a special design known as orthogonal array to examine the effects of the process parameters through the small number of
Design of Experiments

experiments (Lin et al., 2009). An orthogonally designed parametric optimization experiment identifies and separates the effect of each significant design on the performance measure. The orthogonal arrays are the set of matrices that specify the exact combinations of factor behaviours with which one conducts different experimental trials. Tables of orthogonal arrays are accessible in several books for selecting factors and factor levels (Hedayat et al., 1999; Czitrom et al. 2003). Taguchi’s orthogonal arrays analysis are used to give out best parametric settings for the optimum design process.

The fundamental kinds of OA’s developed by Taguchi are either two level arrays or three level arrays or mixed array which indicate the number of the trials in that array. The total Degree of Freedom (DF) available in an OA’s is equal to the number of trials minus one (Ross, 1996).

\[
\text{Degree of freedom, } df = n - 1
\]

where,
\[
df = \text{Total degree of freedom of an OA}
\]
\[
Ln = \text{Orthogonal array designation}
\]
\[
n = \text{Number of the trials}
\]

When a particular OA’s is selected for an experiment, the following inequality must be satisfied:
\[
fLn \geq \text{Total df required for parameters and interactions.}
\]

Depending on the number of levels of the process parameters and total degree of freedom required for the experiments, an appropriate orthogonal array is selected. In Taguchi’s method OA’s have number of columns available for assignment of process parameters and some columns subsequently can also estimate the effect of interactions of these process parameters. 2 level, 3-level, mixed-arrays are some of the orthogonal arrays, used for experimental studies. Figure 4.6 presents the Taguchi’s experimental design and flow diagram of work.
The following prerequisites are required in selecting appropriate OA.
1. Number of parameters and/or their interactions to be studied
2. Selection of number of levels

The proper orthogonal array can be selected by knowing the number of parameters and number of their levels involved in the study. To minimize the size of experiments, two levels of each parameter are recommended. Three levels are required for higher order polynomial relation (Baker, 1990). A number of standard OA’s like two level arrays (L4, L8, L12, L16, and L32) and three level arrays (L9, L18, L27) have been developed by Taguchi to facilitate Design of Experiments (DOE). The following equality should be satisfied while selecting a particular OA.

4.3.2 L18 Orthogonal array

In Taguchi’s methodology, L18 is a mixed level Orthogonal Array (OA’s) that comprises of one factor having 2 levels and a maximum of seven factors having 3 levels. Taguchi’s method L18 orthogonal array allows estimation of only one interaction effect between factors in the first column (2 level factor) and the second column (one of the seven three-level factors). However, the design matrix provides 18-1=17 degree of freedom for studying the effect of factors on the performances.

Following methods have been used in the present work:
1. ANOVA for raw data
2. ANOVA for S/N data

ANOVA is used to investigate and model the relationship between a response characteristic and one or more predictor variables. ANOVA is conducted in a similar manner as other structured experiments (Ross, 1996). A better feel for the relative effect of the different process parameters can be obtained by the disintegration of variance which is commonly called Analysis of Variance (ANOVA) (Phadke, 1989). The motive of using ANOVA is to breakdown the variability of the experimental results into components of variance and then the optimum level of the significant process parameter is found by examining the level averages of the process parameter (Montgomery, 1997; Logothetis, 2004).
Figure 4.6: Taguchi’s experimental design and flow diagram
ANOVA studied the S/N ratios determined from the experimentally observed values, in order to study the effect of each process parameters on the performance characteristics. ANOVA is useful in determining the influence of any given input process parameter from a series of experimental results using Design of Experiment (DoE). This is a considerable capability for a test because variability may be caused by one or several independently influencing factors, and by their interaction (Phadke, 1989; Bagchi, 1993). The S/N ratio is the measure of the variation within the trial when the noise factors are present and it identifies the significant parameters. ANOVA of raw data and S/N ratio’s identifies the Noise and control factors, which are classified into four groups:

A - Parameters, which affect both mean and variations
B - Parameters, which affect variation only
C - Parameters, which affect mean only
D - Parameters, which affects nothing

The appropriate parametric design strategy is adopted by selecting suitable groups to reduce the variations or adjust the average values to the target value.

4.4 Analysis of Variance (ANOVA)

4.4.1 Degree of freedom & Mean Square

The number of independent parameters linked with an entity like a matrix experiment, or a factor or a sum of squares is called its degree of freedom.

ANOVA is a popular statistical approach, which is able to infer some key conclusions and utilized to analyse the experimental results. Mean Square (MS) is calculated by using Degree of Freedom. It is used to examine the rank of significance of factor or interaction of input variables on a particular response variable. Mean square (MS) deviation in an ANOVA table is defined as:

$$MS = \frac{SS (Sum \ of \ squared \ deviation)}{Degree \ of \ freedom \ (DOF)} \quad (4.13)$$

The number of independent parameters linked with an entity like a matrix experiment, or a factor or a sum of squares is called its degree of freedom.
4.4.2 F-test

A better feel for the relative effect of the different process parameters can be obtained by the decomposition of variance which is commonly called Analysis of Variance (ANOVA) (Phadke, 1989). The motive of using ANOVA is to breakdown the variability of the experimental results into components of variance and then the optimum level of the significant process parameter is found by examining the level averages of the process parameter (Montgomery, 1997; Logothetis, 2004). ANOVA studied the S/N ratios determined from the experimentally observed values, in order to study the effect of each process parameters on the performance characteristics.

Fisher’s ratio or Variance ratio (F-value) is defined as:

\[
F = \frac{MS \text{ of a term}}{MS \text{ for the error term}}
\]  

(4.14)

The Probability of significance (P-value) depends upon the F-value. The effect of the factors of the input factors is significant if P-value for a factor is less than 0.05 considering 95% confidence level.

ANOVA is useful in determining the influence of any given input process parameter from a series of experimental results using DoE’s. This is a considerable capability for a test because variability may be caused by one or several independently influencing factors, and by their interaction (Phade, 1989; bagchi 1993). The following equations are used to prepare ANOVA:

\[
S_M = \frac{(\text{sum total of all observation})}{\text{total number of all observation}} \sum \frac{N_i^2}{N}
\]  

(4.15)

\[
(SS_T) = \sum N_i^2 - S_M
\]  

(4.16)

\[
(SS_A) = \frac{\Sigma N_i^2 A_i}{N} - S_M
\]  

(4.17)

\[
(MSS_A) = \frac{(SS_A)}{(df)}
\]  

(4.18)

\[
(S_E) = (SS_T) - \Sigma SS_A
\]  

(4.19)

\[
(MSS_E) = \frac{S_E}{df}
\]  

(4.20)

\[
(F_A) = \frac{MSS_A}{MSS_E}
\]  

(4.21)
where:
\[ S_M = \text{Sum of squares due to mean} \]
\[ (SS_T) = \text{Total sum of squares} \]
\[ (SS_A) = \text{Sum of squares due to parameter} \]
\[ N = \text{Repeating number of each level of parameter} \]
\[ \text{df} = \text{Degree of freedom} \]
\[ (MSS_A) = \text{Mean sum of squares due to parameter} \]
\[ (MSS_E) = \text{Mean sum of square due to error} \]
\[ F_A = \text{F test value} \]

4.5 The Problem under Investigation

The following steps for Design of Experiments (DOE) are employed in the present work dealing with (Phadke, 1989):

1. Planning: To evaluate the objectives, process parameters involved in the project
2. DoE: Based on process parameters and levels identified in the planning phase
3. Running Experiments: The experimental run is performed on the basis of designed experiment
4. Analysis of Results: The experimental results are analysed to provide information in the following steps:
   a) Response tables
   b) Main Effects plots for S/N ratio,
   c) ANOVA (Analysis of Variance)
   d) Optimal parametric setting
   e) Conclusions

4.6 Confirmation Experiments

The final step is to verify the conclusions drawn from the experiments. The optimum conditions are set for the significant parameters and a selected number of tests are run under constant specified conditions. The average of the confirmation experiment results is compared with the anticipated average based on the parameters and levels tested. The predicted S/N ratios using the optimal levels of machining are calculated. (Lin et al., 2009; Phadke, 1989). The significant parameters are set at the
recommended optimum levels and selected numbers of test runs are conducted. The average values of the test runs are compared with the predicted values and these values should lie within the 95% confidence intervals. However, these values may or may not lie within the 95% confidence interval. (Ross, 1996; Phadke, 1989).

4.7 Concluding Remarks

Taguchi’s experimental design is adopted to prepare the L18 Orthogonal Array (OA’s) and based on this, OA’s experiments are performed. The effect of process parameter on the responses i.e. material removal rate & surface roughness are analysed. Analysis of Variance (ANOVA) is used to find out the significant and non-significant parameters that affect the response characteristics. Based on ANOVA results the significant parameters are identified by using F value and P value. The test of significance is applied to check the significance of variables and their interactions at 95% confidence level. The confirmation experiments are planned to justify the conclusions drawn from different set of experiments.