Chapter 1

INTRODUCTION
a number of graph theorists Konig [22], Ore [30], Bauer Harary [6], Laskar [26], Berge [7], Cockayne [10], Hedetniemi [18], Alavi[2], Allan [3], Chartrand [2], Kulli [23], Sampathkumar [23], Walikar [39], Arumugam [5], Acharya [1], Neeralgi [34], Nagaraja Rao [28] and many others have done very interesting and significant work in the domination numbers and other related topics. Cockayne [10] and Hedetniemi [18] gave an exhaustive survey of research on the theory of dominating sets in 1975 and it was updated in 1978 by Cockayne [18]. A survey on the topics on domination was also done by Hedetniemi and Laskar recently.

A domination number is defined to be the minimum cardinality of all dominating sets in the graph G and a set $S \subseteq V$ is said to be a dominating set in a graph, if every vertex in $V \setminus S$ is adjacent to some vertex in S.

In this thesis, we have defined two new domination parameters viz., matching domination number and Balanced domination number.
INTRODUCTION

The theory of domination has been the nucleus of research activity in Graph Theory in recent times. This is largely due to a variety of new parameters, that can be developed from the basic definition of domination. The NP-completeness of the basic domination problems and its close relationship to the other NP-completeness problems have contributed to the enormous growth of research activity in domination theory. It is clearly established from the exclusive coverage of the "Topics on domination in graph" in the 86th issue of the Journal of Discrete Mathematics (1990), that the theory of domination is a very popular area for research activity in Graph Theory.

The study on dominating sets was initiated as a problem in the game of chess in 1850. It is about the placement of the minimum number of queens/rooks/horses, in the game of chess so as to cover every square in the chess board. However a precise notion of a dominating set is said to be given by Konig [22], Berge [7] and Ore [30]. Berge [7], Ore [30], Vizing [38] were the first to derive some interesting results on dominating sets. Since then
The matching dominating is defined as follows:

Let $G : <V, E>$ be a finite graph without isolated vertices. Let $S \subseteq V$. A dominating set $S$ of $G$ is called a matching dominating set if the induced subgraph $<S>$ admits a perfect matching. The cardinality of a minimum matching dominating set is called the matching domination number.

In Chapter 2, we have studied some properties of matching domination number. We have also obtained the results on certain classes of graphs for which the domination number will be the same as the matching domination number.

In Chapter 3, we have defined three types of product graphs viz.

2. Cartesian product of two graphs.
3. Lexicograph product of two graphs.

We have obtained the matching domination of the product of two graphs $G_1$ and $G_2$ in each type and obtained an expression for this number in terms of matching domination number of $G_1$ and $G_2$. While obtaining these results, we have obtained several other
interesting results on these product graphs relevant to our study of matching domination.

In any applications of matching domination number, it becomes imperative to construct the graph with a given matching domination number. As the perfect matching is involved in the definition of matching domination number, it is necessary that the matching domination number must be even.

The construction of a graph with a given domination number was accomplished by Vasumathi and Vangipuram [36]. It has also been established that for the construction of a graph with a given condition, number theoretic tools will be of great help by Vijayasagaradhi and Vangipuram [37].

In Chapter 4, using this method, we have constructed a graph with a given matching domination number. The construction turns out to be a simple and elegant method as in [36] and [37].

In Chapter 5, we have defined a signed vertex graph similar to the signed graph defined by Cartwright and Harary [9]. An
interesting treatise on the contributions to the theory of signed graph and its applications was given by Gill [13].

Interest in signed graphs was stimulated by Harary's works [15,16,17] who was motivated to introduce signed graphs into the literature of graph theory from a study of a paper of Heider [19]. Heider's paper dealt with the analysis of the patterns of cognitive consistency in a triad consisting of two human individuals (P,O) and an object (X) subject to their observation and opinion expression. a cognitive structure we mean the individual's organization of the world into a unified system of beliefs, concepts, attitudes and expectations. A diagram in which group interactions are analysed on the basis of mutual attractions or antipathies between group members is referred as sociogram. A social system is the organization of a group in terms of the stratification of persons, interpersonal relationships and any other factors which differentiate one group from the other groups. Note that, we can represent a social system by a signed graph S whose vertices correspond to the individuals. A positive edge joins two vertices if there is a positive relation between the corresponding individuals
and a negative edge joins the two vertices if there is a negative relation between the corresponding individuals. Hence, in sociological literature, signed graphs are referred as sociograms. They have assigned +ve sign to some edges of the graph, the remaining edges being assigned -ve sign. Analogous to this concept we have assigned +ve sign to some vertices of an Arithmetic graph using an Arithmetic function, viz., Liouville's function. The remaining vertices are being assigned the negative sign.

We have considered another new domination parameter called the Balanced domination number for signed vertex graph.

A graph is called a signed vertex graph if we assign + or - sign to the vertices of the graph. i.e., an ordered pair $S = (G, s)$ is called a signed vertex graph whenever $G = (V, E)$ is a graph called the underlying graph of $S$ and $s : V(G) \rightarrow \{+, -\}$ is sign assigning function defined on vertex set $V(G)$ of $G$. We let $V^+(S) = \{v \in V(G)/ s(v) = + \}$ and $V^-(S) = \{v \in V(G)/ s(v) = - \}$. Hence the set $V^+(S) \cup V^-(S)$ is the vertex set of signed graph $G$. A set of vertices $S_1$ of the signed graph is said to be a positive set if the product of signs of all the vertices in $S_1$ is positive i.e. if it contains even number of
vertices of \( V \setminus S \). A balanced dominating set is defined as a positive dominating set. The cardinality of a minimal balanced dominating set is called the balanced domination number.

In Chapter 5, we have studied some interesting properties of the balanced domination number of a graph. We have also established a method of constructing a signed vertex graph with a given balanced domination number using once again the tools of number theory.

The terminology and notations used in this thesis are the same as in Apostol [4], Bondy and Murty [8].

We have made a modest attempt in this thesis to reaffirm that the domination theory in graph theory subscribes to the creation of new parameters giving a great fillip to the research activity. We hope that we have enthused the researchers in this area that the tools of number theory can be usefully explored in the matter of construction of graphs with required conditions in a simple and elegant manner.