Summary and Conclusion

Chapter 7

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In the present work, we have introduced bivariate Mittag-Leffler (BML $(\mu_1, \mu_2, \alpha_1, \alpha_2, \theta)$) distribution, its generalizations, discrete analogues and studied their properties. BML $(\mu_1, \mu_2, \alpha_1, \alpha_2, \theta)$ distribution gives a generalization to the well known Moran’s bivariate exponential distribution. Distributional properties of BML $(\mu_1, \mu_2, \alpha_1, \alpha_2, 1)$ like, distribution function, density function, product moments, etc are obtained. The distribution is characterized using a random summation in which the number of summands have geometric or bivariate geometric distribution. Using these compoundings, we have obtained BML $(\mu_1, \mu_2, \alpha_1, \alpha_2, 1)$ as the distribution of sum of random vectors in which the components are independently distributed as Mittag-Leffler. Estimates of the parameters are obtained using log moments of the distribution. First order stationary autoregressive processes with BML $(\mu_1, \mu_2, \alpha_1, \alpha_2, 1)$ marginals are developed. The bivariate Mittag-Leffler forms of different important bivariate exponential distributions like Marshall-Olkin’s bivariate exponential, Hawkes’
bivariate exponential and Paulson's bivariate exponential are also introduced. As a generalization to the BML \((\mu_1, \mu_2, \alpha_1, \alpha_2, \theta)\) distribution, bivariate quasi factorial gamma distribution is introduced. This distribution is a generalization to the Moran's bivariate gamma distribution. Distributional properties of bivariate quasi factorial gamma distribution are studied. We have obtained the bivariate quasi factorial gamma as the negative binomial compound of independently and identically distributed random vectors. Time series models with bivariate quasi factorial gamma marginals are developed. As a generalization to bivariate quasi factorial gamma, bivariate semi quasi factorial gamma distribution is introduced and studied.

Bivariate discrete Mittag-Leffler distribution and bivariate discrete Linnik distributions are introduced as the discrete analogues of BML \((\mu_1, \mu_2, \alpha_1, \alpha_2, \theta)\) and bivariate quasi factorial gamma distributions. As a special case of the bivariate discrete Mittag-Leffler distribution, a bivariate geometric distribution is studied. Characterizations of bivariate discrete Mittag-Leffler distribution are obtained using the geometric compounding. Autoregressive processes with bivariate discrete Mittag-Leffler distribution marginals are developed. Using bivariate geometric compounding, discrete analogues of the bivariate Mittag-Leffler distributions that generalize Marshall-Olkin's bivariate exponential distribution and Hawkes' bivariate exponential distribution are introduced. Bivariate discrete Linnik distribution is introduced as a generalization to the bivariate discrete Mittag-Leffler distribution and its properties are obtained. As a special case of bivariate discrete Linnik distribution, bivariate negative binomial
distribution is studied. Characterization of bivariate discrete Linnik distribution is obtained using the negative binomial compounding. First order stationary autoregressive models with bivariate discrete Linnik marginals are developed. Bivariate tailed Mittag-Leffler distribution and bivariate tailed discrete Mittag-Leffler distribution are introduced and the corresponding autoregressive models are developed.

Random summation arises in many contexts. It is mainly applied in modeling practical problems that deal with certain phenomena in which the respective mathematical models are sums of random number of independent random variables. Gnedenko and Korolev (1996) gave a number of situations where we usually come across random summation, especially geometric summation and describe the modeling of such situations with respective physical terminology.

Geometric summation arises naturally in many applied problems. Kozubowski and Panorska (1999a) established the applications of geometric summation in financial portfolio modeling. Kozubowski and Rachev (1994) used geometric random sums as an adequate device to model the foreign currency exchange rate data. Distribution of geometric sums appear in queuing theory and reliability in connection to 'regenerating processes with rare events' (see Gertsbakh (1984) and Jacobs (1986)). Gnedenko and Korolev (1996) expressed the waiting time of a telephone customer calling at an arbitrary time to talk to the operator as a compound of geometric distribution. The applications of random summation in insurance are discussed in Rolski et al. (1999). The probability distributions, we have introduced and studied in this work
may be appropriate in modeling bivariate data sets which are closed under geometric summation.

Even though the constant hazard rate and memoryless property causes much applications of exponential distribution in reliability studies and renewal theory, it is inadequate to model heavy tailed data. The Mittag-Leffler distribution being heavy tailed as compared to exponential distribution is a most suitable fit in such situations. Jayakumar (2003) used Mittag-Leffler distribution to model the rate of flow of water in Kallada river, Kerala, India. The semi Mittag-Leffler distribution and semi quasi factorial gamma distributions are useful in modeling data sets that exhibit periodic movements. Recently much focus is given on developing different time series models with non Gaussian marginals (see Balakrishna and Jayakumar (1997) and Block et al. (1988)). The first order stationary autoregressive processes having bivariate Mittag-Leffler and bivariate discrete Mittag-Leffler marginals are developed in Mundassery and Jayakumar (2006, 2007b)