Chapter 3

MRAS based Sensorless control of Induction Motor Drive

3.1 Introduction

In the previous chapter the problems associated with the conventional MRAS scheme are presented and different methods to overcome these shortcomings are also given. Before going for analysis the mathematical model of the system is to be developed. In this chapter a brief procedure for developing the mathematical model of induction motor and principle of MRAS scheme is presented.

To analyze any machine it is very much required to obtain the machine in terms of equivalent mathematical equations. Traditionally, per phase equivalent circuits were used for steady state analysis of machines. The steady state model and per phase equivalent circuit of the machine are helpful in studying the operation of the drive under steady state region but was not able to predict the dynamic performance of the drive. To assess the adequacy of the converter, converter switches and their interaction for a given motor it is required to evaluate the dynamics of converter-fed variable-speed drives. This evaluation is also helpful in determining the excursions of current and torque in the converter and the motor [1], [3], [18]. An accurate dynamic model of the drive is very much necessary which is useful in understanding the dynamic behavior of the motor under transient and steady state conditions. The dynamic model considers
the instantaneous effects of varying voltages or currents, stator frequency and torque disturbances.

The three phase rotor windings move with respect to three phase stator windings as shown in Fig 3.1.

![Fig. 3.1 Coupling effect in three-phase windings of Induction motor](image)

### 3.2 Dynamic Modeling of Induction Motor

For deriving the dynamic model of an induction motor following assumptions are made which are summarized as [3].

- Air gap is assumed to be uniform,
- Rotor and stator windings are assumed to be balanced with sinusoidally distributed emf,
- Changes due to parameter and saturation are neglected and
- Inductance vs. rotor positions is assumed sinusoidal.

A machine model can be described by differential equations with time varying mutual inductances, but this model will be a little complex [1]. It is well known that a three phase induction machine
can be represented by an equivalent two phase machine as shown in Fig. 3.2

In Fig 3.2 $d_s$, $q_s$ corresponds to stator direct and quadrature axes and $d_r$, $q_r$ corresponds to rotor direct and quadrature axes respectively. This model is simple but the problem of time varying is still present. Hence to overcome this problem R.H. Park proposed, usually known as Park’s Transformation, a new theory of electric machine analysis in which he formulated a change of variables which replace the variables, such as voltages, currents and flux linkages, associated with the stator windings of a machine with variables associated with imaginary windings rotating with rotor at synchronous speed.

![Fig. 3.2 Equivalent two-phase machine](image)

To summarize it can be said that the induction motor can be modeled in three different reference frames as

- Rotor Reference Frame
- Synchronously Rotating Reference Frame and
• Stationary or Stator Reference Frame

Among the various reference frames sensorless control utilizes stationary reference frame. Hence, in this thesis attention is given to motor model in stationary reference frame.

The induction motor in stationary or stator reference frame can be given by (3.1) [1], [3], [18].

\[
\begin{bmatrix}
    v_{sq} \\
    v_{sd} \\
    0 \\
    0
\end{bmatrix}
= 
\begin{bmatrix}
    R_s + L_s p & 0 & L_m p & 0 \\
    0 & R_s + L_s p & 0 & L_m p \\
    L_m p & -\omega_r L_m & R_r + L_r p & -\omega_r L_r \\
    \omega_r L_m & L_m p & \omega_r L_r & R_r + L_r p
\end{bmatrix}
\begin{bmatrix}
    i_{sq} \\
    i_{sd} \\
    i_{rd}
\end{bmatrix}
\]

Where \( \omega_r = \frac{d\theta}{dt} \) and \( p = \frac{d}{dt} \)

Among the different reference frame models available, the stationary frame of reference is used in this work since this model allows elegant simulation of stator controlled induction motor drives such as inverter controlled or phase controlled induction motor drives [3], [18]. The input variables are well defined and can be used to find the stator \( d \) and \( q \) axes voltages through a set of simple algebraic equations given by (3.3).

In stator frame of reference the stator and rotor flux linkages can be given as [3].

\[
\psi_{sq} = L_s i_{sq} + L_m i_{rq}
\]  \hspace{1cm} (3.2)

\[
\psi_{sd} = L_s i_{sd} + L_m i_{rd}
\]  \hspace{1cm} (3.3)

\[
\psi_{rq} = L_r i_{rq} + L_m i_{sq}
\]  \hspace{1cm} (3.4)
\[ \psi_{rd} = L_r i_{rd} + L_m i_{sd} \quad (3.5) \]

\[ \psi_{mq} = L_m (i_{sq} + i_{rq}) \quad (3.6) \]

\[ \psi_{md} = L_m (i_{sd} + i_{rd}) \quad (3.7) \]

In the same way the stator and the rotor \(d\) and \(q\) axes voltages can be given as

\[ v_{sd} = R i_{sd} + \frac{d\psi_{sd}}{dt} \quad (3.8) \]

\[ v_{sq} = R i_{sq} + \frac{d\psi_{sq}}{dt} \quad (3.9) \]

\[ 0 = R i_{rd} + \omega \psi_{rq} + \frac{d\psi_{rd}}{dt} \quad (3.10) \]

\[ 0 = R i_{rq} - \omega \psi_{rd} + \frac{d\psi_{rq}}{dt} \quad (3.11) \]

From (3.10) and (3.11) the equations for \(i_{rd}\) and \(i_{rq}\) are given as

\[ i_{rd} = \frac{-p\psi_{rd} - \omega \psi_{rq}}{R_r} \quad (3.12) \]

\[ i_{rq} = \frac{-p\psi_{rq} - \omega \psi_{rd}}{R_r} \quad (3.13) \]

From the above equations \(\psi_{sd}\) and \(\psi_{sq}\) are obtained as

\[ \psi_{sd} = \int \left( v_{sd} - R i_{sd} \right) dt \quad (3.14) \]

\[ \psi_{sq} = \int \left( v_{sq} - R i_{sq} \right) dt \quad (3.15) \]

\[ \psi_{rd} = \frac{-L_r \omega \psi_{rq} + L_m i_{sd} R_r}{R_r + sL_r} \quad (3.16) \]
\[ \psi_{rq} = \frac{L_r \omega \psi_{rd} + L_m R_f i_{sq}}{R_r + sL_r} \]  
(3.17)

The dynamic \( d-q \) model of the induction motor under stationary or stator frame of reference is shown in Fig. 3.3

Fig. 3.3 (a)

Fig. 3.3 (b)

Fig. 3.3 (a) \( q \)-equivalent circuit (b) \( d \)-equivalent circuit

In Fig. 3.3 \( L_{ls} \) and \( L_{lr} \) are stator and rotor leakage inductances respectively.

The electromagnetic torque equation under stationary frame of reference can be given as

\[ T_e = \frac{3}{2} \left( \frac{P}{2} \right) \left( \psi_{sd} i_{sq} - \psi_{sq} i_{sd} \right) \]  
(3.18)

or
The electro-mechanical relation for induction motor can be given as

\[ T_e = \frac{3}{2} \left( \frac{p}{2} \right) L_m \left( i_{sq} r d - i_{sd} r q \right) \]  
(3.19)

or

\[ T_e = \frac{3}{2} \left( \frac{p}{2} \right) \frac{L_m}{L_r} \left( i_{sq} \psi r d - i_{sd} \psi r q \right) \]  
(3.20)

The electro-mechanical relation for induction motor can be given as

\[ T_e - T_L = J \frac{d\omega_m}{dt} = \frac{2}{P} J \frac{d\omega_r}{dt} \]  
(3.21)

where \( T_L \) = Load torque, \( J \) = Rotor inertia and \( \omega_m \) = mechanical speed.

By using the above procedure the induction motor model is developed in stationary reference frame and utilized in this work.

### 3.3 Model Reference Adaptive System

Sensorless vector control, as the name implies, is nothing but vector without speed or position sensors. In sensorless vector control the speed encoder is not utilized for speed measurement which not only reduces the cost but also improves the reliability of the drive. As the speed encoder is not utilized for speed measurement, the speed is estimated by using the machine terminal voltages and currents. The estimation of the speed is normally a complex phenomenon and is heavily dependent on machine parameters. It is seen that the speed estimation of motor using Model Referencing Adaptive System is one of the robust techniques, which mainly concentrates on the reduction of error between two models, and focus is on improving the performance of this method. The different MRAS techniques for speed estimation were discussed which mainly includes rotor flux based
MRAS, back EMF based MRAS or reactive power based MRAS. At low frequencies the EMF and reactive power itself die out hence the MRAS utilizing rotor flux is considered to be the best among the different MRAS schemes. Hence, rotor flux based MRAS scheme is used in this research work. The main focus of this section is to utilize rotor flux based MRAS scheme which is the most conventionally used scheme.

### 3.3.1 Principle of MRAS

The main essence of MRAS is the reduction of error between two models such that it tends to zero. Among the two models, the one that does not involve quantity to be estimated, the rotor speed in this case, is known as the reference model and the model that has the quantity to be estimated involved is known as the adjustable model or adaptive model. The output of the adaptive model is compared with the output of the reference model and the error hence obtained is minimized by using an adaptation mechanism conventionally employing a PI controller.

The basic principle of rotor flux based MRAS speed observer for estimation of speed can be easily understood from model shown in Fig 3.4. As shown in Fig. 3.4 it consists of two models, reference model and adaptive model and an adaptation mechanism which minimizes the error between the two models. The reference model also known as voltage model represents the stator equations. It generates the reference value of the rotor flux components in the stationary frame of reference from the monitored stator voltage and current signals. This model is derived as below.
From equations (3.3), (3.5), (3.8) and proper rearrangement we have

\[ v_{sd} = \frac{L_m}{L_r} \frac{d}{dt} \psi_{rd} + R_i s_{sd} + \sigma L_s p_i s_{sd} \]  

(3.22)

Rearranging equation (3.22) we get

\[ p \psi_{rd} = \frac{L_r}{L_m} \left( v_{sd} - \left( R_i s_{sd} + \sigma L_s p_i s_{sd} \right) \right) \]  

(3.23)

In the similar way from equations (3.2), (3.4), (3.9) and proper rearrangement we have

\[ p \psi_{rq} = \frac{L_r}{L_m} \left( v_{sq} - \left( R_i s_{sq} + \sigma L_s p_i s_{sq} \right) \right) \]  

(3.24)

**Fig. 3.4 Block diagram of speed estimation using MRAS**
The equations (3.23) and (3.24) do not contain rotor speed and therefore does not get affected by rotor speed and hence it is taken as reference model. These equations can be written in matrix format as

\[
\begin{bmatrix}
    \frac{p\psi_{rd}}{L_r} \\
    \frac{p\psi_{rq}}{L_m}
\end{bmatrix}
= \begin{bmatrix}
    v_{sd} \\
    v_{sq}
\end{bmatrix}
- \begin{bmatrix}
    R_s + \sigma L_s p & 0 \\
    0 & R_s + L_s p
\end{bmatrix}
\begin{bmatrix}
    i_{sd} \\
    i_{sq}
\end{bmatrix}
\]

(3.25)

By using the equations from (3.2) to (3.5), (3.10) and (3.11) the current model consisting of rotor is obtained and is given in equations (3.26) and (3.27)

\[
\frac{p\dot{\psi}_{rd}}{T_r} = \frac{1}{T_r} (L_m i_{sd} - \dot{\psi}_{rd}) - \omega_r \dot{\psi}_{rq}
\]

(3.26)

\[
\frac{p\dot{\psi}_{rq}}{T_r} = \frac{1}{T_r} (L_m i_{sq} - \dot{\psi}_{rq}) + \omega_r \dot{\psi}_{rd}
\]

(3.27)

The above equations can be written in the matrix form as

\[
\begin{bmatrix}
    \frac{p\dot{\psi}_{rd}}{T_r} \\
    \frac{p\dot{\psi}_{rq}}{T_r}
\end{bmatrix}
= \begin{bmatrix}
    -\frac{1}{T_r} & -\omega_r \\
    \omega_r & -\frac{1}{T_r}
\end{bmatrix}
\begin{bmatrix}
    \dot{\psi}_{rd} \\
    \dot{\psi}_{rq}
\end{bmatrix}
+ \frac{L_m}{T_r}
\begin{bmatrix}
    i_{sd} \\
    i_{sq}
\end{bmatrix}
\]

(3.28)

In the above equations \( p = \frac{d}{dt} \), \( T_r = \frac{L_r}{R_r} \) is rotor time constant and \( \sigma = 1 - \frac{L_m^2}{L_s L_r} \) is leakage coefficient.

The current model given in equations (3.28) consists of rotor speed and hence gets affected by it. Therefore this model is known as adaptive model or adjustable model. The reference model and the adaptive model are used to estimate the rotor flux linkages. The
angular difference of the outputs of the two estimators known as error is given as in equation (3.29).

\[ \xi = X - X^1 = \dot{\psi}_{rd} \psi_{rq} - \dot{\psi}_{rq} \psi_{rd} \]  

(3.29)

This is used as speed tuning signal that is given as input to adaptation mechanism, which forms the main part of the MRAS scheme. The adaptation mechanism conventionally utilizes a PI controller. When the error between the two models is not zero then the adaptation mechanism tunes the rotor speed. In other words when the error between the rotor flux linkages \((X - X^1)\) is not zero, the PI controller tunes the rotor speed. The rotor speed identification algorithm is actuated by the error signal generated when the error is not zero, making the error converge to zero. The estimated speed by using MRAS scheme is given as

\[ \dot{\omega}_r = \left( \frac{K_p}{p} + \frac{K_i}{p} \right) \xi \]  

(3.30)

where \(K_p\) and \(K_i\) are proportional and integral gain constants respectively

3.3.2 Results and Discussions

Simulations are carried out for conventional MRAS based control of induction motor drive. The simulation parameters, specifications of induction motor used are given in Appendix – I. Different conditions such as no load, on load, step change in load, step change in speed, load torque disturbance are considered. The various speeds considered are set as reference speeds during the simulation.
Fig. 3.5 shows the response of induction motor drive using conventional MRAS at 1300 rpm on no-load. The response of speed, torque and stator current are shown and it can be seen that ripples are present in both torque and current waveforms. In Fig. 3.6 the response of conventional MRAS with step change in load when a load of 10 N-m applied at 1 sec are shown. It can be seen that the ripples are present both in current and torque waveforms and also the speed is getting affected by the application of load. In Fig. 3.7 the response of the motor with load torque disturbances is shown from which it can be seen that with load variations the speed is getting varied frequently. In Fig. 3.8 the response of the motor with step change in speed from 800 rpm to 1300 rpm is shown where the speed change command is applied at 1 sec. During low speeds the ripples are high and hence 800 rpm is also considered. Fig. 3.9 shows the response of conventional MRAS on no load at 800 rpm and the on load response when a load of 10 N-m applied at 1 sec is shown in Fig. 3.10. Fig. 3.11 shows the response of the motor with load torque disturbance and it can be seen here that the speed is getting affected by the frequent change in load. The difference in the reference speed and the estimated speed is higher when the applied is load of large magnitude while this difference is lower when the applied load is small in magnitude. In Fig. 3.12 response of the motor is shown when there is a step change in speed from 1300 rpm to 800 rpm. Fig. 3.13 shows the response of the motor at 100 rpm on no load.
As can be seen from the above waveforms the speed control during low speed region is not perfect and there are ripples present in the current and torque waveforms. Also the speed of the motor is getting affected by the load variations.

Fig. 3.5 Response of Conventional MRAS at 1300 rpm on no load
Fig. 3.6 Response of conventional MRAS at 1300 rpm with step change in load (Load of 10 N-m applied at 1 Sec)
Fig. 3.7 (a) Response of conventional MRAS at 1300 rpm with external load torque disturbance
Fig. 3.7 (b) External load torque disturbance
Fig. 3.8 Response of conventional MRAS with step change in speed from 800 rpm to 1300 rpm at 1 sec
Fig. 3.9 Response of Conventional MRAS at 800 rpm on no load
Fig. 3.10 Response of conventional MRAS at 800 rpm with step change in load (Load of 10 N-m applied at 1 Sec)
Fig. 3.11 (a) Response of conventional MRAS at 800 rpm with external load torque disturbance
Fig. 3.11 (b) External load torque disturbance
Fig. 3.12 Response of conventional MRAS with step change in speed from 1300 rpm to 800 rpm
Fig. 3.13 Response of conventional MRAS at 100 rpm on no load
3.4 Conclusions

In adjustable speed drives the transient behavior of the induction motor is to be considered. Hence, an accurate mathematical model is obtained in stationary frame of reference. The stationary frame of reference is simple when compared to synchronously rotating reference and is used in the proposed model reference adaptive system scheme. Also the principle of conventional MRAS speed observer is presented in detail and the equations are derived for reference model, adaptive model, adaptation mechanism and estimated speed.

Simulations are performed for conventional MRAS speed observer and it is found that even though the MRAS is simple and robust when compared to other estimation schemes but it suffers from drawbacks such as difficulty in speed control during low speed region, presence of ripples in current and torque and the speed not remaining constant during external load torque variations.

In this research work a two stage HPF with feed forward control of stator flux for speed control during low speed region, SVPWM technique for reducing the ripples present in current and torque are proposed. For further improvement in quality of waveform and to reduce THD, HPWM technique utilizing imaginary switching times and to improve the robustness of the drive during load torque variations FLC and IFLC are also proposed.

3.5 Sensorless control of Induction motor using MRAS with HPF

In the previous sections the mathematical model of induction motor in stationary frame of reference was derived. Also the principle
of Model Reference Adaptive System (MRAS) and its mathematical model was given and explained in detail. The simulation results of conventional MRAS speed observer were presented and analyzed. In this section the mathematical models developed are used and different methods are proposed to improve the performance of sensorless control of induction motor drive using MRAS.

The squirrel cage induction motors are very economical, rugged, reliable and are available in fractional horse powers to multi megawatt capacity. Hence these motors have become the workhorse for the industrial applications where variable speed drives are the main requirement. Out of the different control strategies MRAS based sensorless control technique is one of the best technique available.

The sensorless control of induction motor based on the conventional model reference adaptive system was discussed and derived in section 3.3. It is seen that the rotor flux based MRAS speed observer is one of the best scheme for sensorless control of induction motor drive due to its robustness. Although it has many advantages, control during the low speed and very low speed region is difficult due to the initial condition and integration problems.

3.5.1 Design of two stage High Pass Filter for MRAS

To improve the performance of this scheme during low and very low speed operations a two stage high pass filter is proposed which is cascaded to the reference model. With the introduction of filter a time delay is introduced in the development of stator flux which generates torque jerk at starting during transition from standstill to sensorless
vector control mode. To overcome the problem of torque jerk at starting, it is very much required that the time delay in the development of the stator flux should be avoided. Hence, to achieve this, feedforward control of stator flux is proposed which compensates the time delay produced in the development of the stator flux [48].

The rotor flux linkages equation for two axes stationary frame of reference induction motor are expressed as

\[ p\psi_{rd} = \frac{1}{T_r}(L_m i_{sd} - \psi_{rd}) - \omega \psi_{rq} \]  
\[ p\psi_{rq} = \frac{1}{T_r}(L_m i_{sq} - \psi_{rq}) + \omega \psi_{rd} \]  

(3.31)  
(3.32)

The stator and rotor flux linkages equations are given by equations (3.2) to (3.5). At standstill the rotor speed is zero, hence substituting \( \omega = 0 \) in the above equations and after necessary simplifications the rotor flux linkages as a function of stator currents are expressed as

\[ \psi_{rd} = \frac{L_m}{T_r p + 1} i_{sd} \]  
\[ \psi_{rq} = \frac{L_m}{T_r p + 1} i_{sq} \]  

(3.33)  
(3.34)

The stator flux linkages may be expressed in terms of stator current and rotor flux linkages as

\[ \psi_{sd} = L_a i_{sd} + \frac{L_m}{L_r} \psi_{rd} \]  
\[ \psi_{sq} = L_a i_{sq} + \frac{L_m}{L_r} \psi_{rq} \]  

(3.35)  
(3.36)
where \( L_a = \frac{L_sL_r - L_m}{L_r} \)

By using equations (3.33) to (3.36) the stator flux can be estimated at standstill where the d and q axes flux components are calculated by using only d and q axes stator currents. The block diagram showing the stator flux estimation at standstill is given in Fig. 3.14.

**Fig. 3.14 Block diagram of stator flux estimation at standstill**

In the proposed method a HPF is utilized to remove the DC offset that is present in the signal being integrated. The output hence
obtained from the HPF will be sinusoidal but gets affected in magnitude and phase. For obtaining accurate flux estimate compensations in phase and magnitude are required. To this affect, a second HPF similar to the first one fed directly by the later is utilized. A two stage high pass filter having a transfer function as given in equation (3.37) is considered to over come the DC offset and initial condition problems associated with the integrator.

\[
HPF(s) = \frac{s}{s + a}
\]  

(3.37)

The above equation of HPF can be re-written as

\[
1 - \frac{a}{s + a} = 1 - LPF(s)
\]  

(3.38)

From equation (3.38) it can be understood that a HPF can be obtained by subtracting a low pass filter (LPF) from the original one therefore we can represent a HPF as shown in Fig. 3.15

**Fig. 3.15 High Pass filter cell using LPF**

Consider ing a sinusoidal signal \(x(t)\) and its transfer function given by equation (3.39), its response after passing through a LPF is given by equation (3.40) and its phase shift can be represented by equation (3.41)
\[ x(t) = \sin \omega_e t, \quad X(s) = \frac{\omega_e}{s^2 + \omega_e^2} \quad (3.39) \]

\[ y(t) = \frac{a}{\sqrt{a^2 + \omega_e^2}} \sin(\omega_e t - \phi) \quad (3.40) \]

\[ \cos \phi = \frac{a}{\sqrt{a^2 + \omega_e^2}} \quad (3.41) \]

The equation of LPF in (3.38) can be written with respect to transfer characteristics as

\[ \frac{Y}{X} = \frac{1}{1 + j\tau \omega_e} \quad (3.42) \]

Where \( \tau \) = filter time constant, \( \omega_e \) = frequency. The phase gain and phase shift of the filter at frequency \( \omega_e \) are given respectively as

\[ A = \left| \frac{Y}{X} \right| = \frac{1}{\sqrt{1 + (\tau \omega_e)^2}} \quad (3.43) \]

\[ \phi = \tan^{-1}(\tau \omega_e) \quad (3.44) \]

The above equations are valid if there is one filter but if we go for multiple numbers of programmable filters then the total phase gain will be the product of the individual phase gains of the filter and the phase shift will be the sum of the entire individual phase shift. If “n” numbers of filters are being used then the total phase gain and phase lag can be given respectively as:

\[ A_T = A_1 A_2 \ldots A_n = \frac{1}{\sqrt{(1 + (\tau_1 \omega_e)^2)(1 + (\tau_2 \omega_e)^2)\ldots(1 + (\tau_n \omega_e)^2)}} \quad (3.50) \]
\[
\phi_T = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n = \tan^{-1}(\tau_1 \omega_e) + \tan^{-1}(\tau_2 \omega_e) + \cdots + \tan^{-1}(\tau_n \omega_e)
\]  
(3.46)

If the filters that are being used are identical then the corresponding expressions for phase gain and shift can be given as

\[
A_T = nA = \frac{1}{\sqrt{1 + (\tau \omega_e)^2}}^n
\]  
(3.47)

\[
\phi_T = n\phi = n\tan^{-1}(\tau \omega_e)
\]  
(3.48)

For better performance the value of “n” should be chosen a higher one. Higher the value of “n” means, higher number of filter being used which not only increasing the complexity but also increases the burden on the software computation when the digital implementation is utilized. Hence, a choice is necessitated between the performance of the drive and the complexity of the system. In this work the value of “n” is taken as two, i.e. two number of filters, known as 2-stage cascaded filters are being employed in the proposed work. As discussed above let this filter is used to perform integration of a sinusoidal voltage at frequency \(\omega_e\), then \(\phi_T = \pi/2\) and \(G_{A_T} = 1/\omega_e\), where \(G\) = gain compensation needed for the integration. Substituting these conditions in equations (3.47) and (3.48) respectively, the equations for \(G\) and \(\tau\) are obtained as

\[
G = \left(\frac{1}{\omega_e}\right)\sqrt{1 + (\tau \omega_e)^2}^2
\]  
(3.49)
\[ \tau = \left( \frac{1}{\omega_c} \right) \tan \left( \frac{\pi}{2 \times 2} \right) \] (3.50)

As can be seen from the above equations the parameters \( G \) and \( \tau \) are a function of frequency. The cascaded filters used can be realized by using an op-amp equivalent circuit.

The two stage high pass filter utilized is shown in Fig. 3.16 where \( \phi_a = \left( \frac{\pi}{2} - \phi_f \right) \) per stage and \( \phi_f \) is the phase shift angle of the analog filter for reducing ripple in the stator voltage and current signals. The time constant of the filter and the compensation gain \( G \) are programmable as a function of frequency so as to get the ideal integration at any frequency.

![Fig. 3.16 Block diagram of two stage High Pass Filter based stator flux vector synthesis](image)
The frequency dependant phase lag $\phi_f$ with time constant $\tau_f$ for the analog filter is given as

$$\phi_f = Tan^{-1}(\tau_f \omega_e)$$  \hspace{1cm} (3.51)

The time constant $\tau$ and phase shift angle $\phi_a$ for each filter can be expressed as

$$\tau = \frac{1}{\omega_e} Tan \left[ \frac{1}{2} \left( \frac{\pi}{2} - \phi_f \right) \right]$$  \hspace{1cm} (3.52)

$$\phi_a = \frac{1}{2} \left[ \frac{\pi}{2} - Tan^{-1}(\tau_f \omega_e) \right]$$  \hspace{1cm} (3.53)

In the above equations the frequency $\omega$, can be calculated from stator voltage, stator current and stator flux [1]. The compensation gain for the overall filter can be given as

$$G = \frac{1}{\omega_e} \sqrt{1 + \left( \tau_f \omega_e \right)^2 \left[ 1 + \left( \tau \omega_e \right)^2 \right]^2}$$  \hspace{1cm} (3.54)

The transition from standstill to sensorless vector control mode is explained in Fig. 3.17. In the standstill mode of operation the stator flux ($\psi_s$) is established upto the rated value and with the torque command, the frequency $\omega_e$ reaches a value known as threshold value $\omega_e^t$. The threshold frequency is the minimum frequency required for transition from stand still to vector control mode. After the stator flux has reached the rated value the speed begins to develop and then the motor is switched into sensorless vector control mode.
where the two stage high pass filter is used for stator flux estimation. In Fig. 3.17 it can be seen that there is a time delay \( t_d \) present between the time when the rated stator flux has reached and in the time when the transition of motor takes place form stand still to vector control mode.

This time delay is caused due to the presence of programmable two stage high pass filter present for the estimation of stator flux. At the time of transition from standstill to vector control mode the filter also causes the time delay for stator flux build up and due to this the stator flux suddenly decreases. This sudden reduction of stator flux development generates a jerk of motor torque at starting. This torque jerk generated at the time of starting during transition from standstill to vector control mode should be avoided. To avoid the generation of torque jerk, the delay caused in the development of stator flux should be compensated. To compensate for the time delay produced in the development of stator flux the feed forward control of stator flux is proposed.

By utilizing the feed forward control of stator flux the time delay in the development of stator flux is avoided which means that the jerk produced in the motor torque during transition from standstill to vector control mode, essentially at starting, is prevented. The procedure for feed forward control of stator flux is explained below.

Initially the difference between the rated (reference) stator flux \( \psi^*_s \) and the actual stator flux \( \psi_s \) is calculated. The difference between the
rated and the actual stator flux is compared with a value known as tolerant limit \( \lambda \) at transient state. If the difference between the rated and actual stator flux is greater than the tolerant limit then the previous values of the two stage high pass filters are adjusted in such a way that the stator flux establishes the rated stator flux. In the feedforward control of stator flux, the two previous values of high pass filter in Fig. 3.16 are calculated in the reverse direction from the rated stator flux \( \psi_s^* \).

\[\text{Fig. 3.17 Transition from Stand still to vector control Mode}\]

The d and q axes stator flux components are given as:
$$\psi_{sd} = \psi_s^* \cos \theta e$$  \hspace{1cm} (3.55)

$$\psi_{sq} = \psi_s^* \sin \theta e$$  \hspace{1cm} (3.56)

As discussed above the output of each filter is derived in the reverse direction from the d and q axes stator fluxes given in equations (3.55) and (3.56), the input of each filter is obtained from the output of the previous filter. The output of the second filter will be stator flux divided by the compensation gain $G$ and the output of the first filter leads by phase angle $\phi_a$ and multiplied by $\sqrt{1 + (\tau_0 e)^2}$ with respect to the output of second filter. The two outputs of the filter are assigned to the previous values of each filter. The previous values of d and q axes stator flux of the two filters is given respectively as:

$$\psi_{sd}^2 = \frac{\psi_{sd}^*}{G} = \frac{\psi_s^*}{G} \cos \theta e$$  \hspace{1cm} (3.57)

$$\psi_{sd}^1 = \frac{\psi_s^*}{G} \sqrt{1 + (\tau_0 e)^2} \cos(\theta e + \phi_a)$$  \hspace{1cm} (3.58)

$$\psi_{sq}^2 = \frac{\psi_{sq}^*}{G} = \frac{\psi_s^*}{G} \sin \theta e$$  \hspace{1cm} (3.59)

$$\psi_{sq}^1 = \frac{\psi_s^*}{G} \sqrt{1 + (\tau_0 e)^2} \sin(\theta e + \phi_a)$$  \hspace{1cm} (3.60)

In the above equations

$\psi_{sd}^2$ is the previous d axis stator flux of second filter,

$\psi_{sd}^1$ is the previous d axis stator flux of first filter,
\(\psi_{sq}^{2}\) is the previous q axis stator flux of second filter,

\(\psi_{sq}^{1}\) is the previous q axis stator flux of first filter.

By utilizing the above equations the output of each filter is derived in the reverse direction. By calculating the previous values of the filter in the reverse direction the feed forward control of stator flux is obtained. The feed forward control of stator flux avoids the time delay in the development of stator flux thereby the reducing the generation of torque jerk during transition from standstill to vector control mode.

The cut-off frequency should be selected as low as possible since the purpose is just to remove the DC component and therefore a value of 1 Hz is chosen. For the considered HPF an equation that describes the excitation frequency must be added to eq. (3.13) [141]

\[
\Delta \omega_e = -K_p \Delta p_e
\]  \hspace{1cm} (3.61)

where input power perturbations are computed using a first order filter as

\[
\Delta p_e = \frac{s}{s + \frac{1}{\tau}} p_e
\]  \hspace{1cm} (3.62)

where \(\tau\) is HPF time constant. The fifth differential equation is

\[
\Delta \phi_e + \frac{\Delta \omega_e}{\tau} = -K_p \phi_e
\]  \hspace{1cm} (3.63)

The input power can be calculated using the Park's voltages and currents as

\[
p_e = \frac{2}{3} \left( v_{ds}^r i_{ds}^r + v_{qs}^r i_{qs}^r \right)
\]  \hspace{1cm} (3.64)
Supposing that the voltage is constant, i.e. no perturbation is applied, the time derivative of the power can be expressed as

$$\dot{p}_s = \frac{2}{3} V_s \left( -i_{qs}^d \sin \delta + i_{qs}^r \cos \delta - (i_{ds}^r \cos \delta + i_{qs}^r \sin \delta) \dot{\delta} \right)$$  \hspace{1cm} (3.65)

To obtain the fifth system equation, equation (3.65) must be substituted in (3.63). Substituting time derivative expressions of currents and load angle, the fifth differential equation is obtained as

$$\Delta \dot{\delta}_s = \frac{3}{2} K_p V_s \left( \frac{\omega_s L_d \cos \delta}{L_q} - \frac{R_s \sin \delta}{L_d} + (\omega_e - \omega_r) \cos \delta \right) i_{ds}^r$$

$$+ \frac{3}{2} K_p V_s \left( \frac{\omega_s L_q \sin \delta}{L_d} + \frac{R_s \cos \delta}{L_q} + (\omega_e - \omega_r) \sin \delta \right) i_{qs}^r$$

$$+ \frac{3}{2} K_p V_s \frac{\lambda_m \cos \delta}{L_q} \omega_r$$

$$- \frac{3}{2} K_p V_s \left( \frac{V_s \sin^2 \delta}{L_d} - \frac{V_s \cos^2 \delta}{L_q} \right) - \frac{1}{\tau} \Delta \omega_r$$  \hspace{1cm} (3.66)

By linearizing the system, the stability characteristics can be obtained with the modulation of the frequency using power perturbations. A high pass filter with low cut off frequency is required to remove the integrator drift and any initial condition problems.

**3.5.2 Results and discussions**

Simulations are carried out to validate the proposed MRAS scheme using high pass filter for control of induction motor drive. Simulations are carried out for MRAS scheme using HPF for both with and without feedforward control of stator flux. The simulation parameters and the specifications of induction motor used are given in Appendix – I.
In the standstill model the d-axis stator flux is made to build up keeping the q-axis stator flux to zero value. The stator flux is made to reach the rated value and then the torque command is applied. As can be seen from Fig. 3.18 there is a time delay in the buildup of stator flux after the torque command is given when a HPF is used without the feedforward control of stator flux. To avoid the torque jerk at starting the feed forward control of stator flux is proposed in this work. As can be seen from Fig. 3.18, after the torque command is applied the time delay in the stator flux is compensated and the stator flux is kept constant by using the feedforward control of stator flux approach. Due to the time delay in built of stator flux, a jerk of motor torque is developed at the starting as can be seen from Fig. 3.19, this is reduced with the proposed feedforward control. The variations of d and q axes flux for both feedforward and without feedforward control are shown in Fig. 3.20 and Fig. 3.21 respectively. The speed of the drive and angle $\theta_e$ are shown in Fig. 3.22 and Fig. 3.23 respectively.

![Fig. 3.18 Build up of Stator flux at starting with and without feedforward control](image)
Fig. 3.19 Torque at starting with and without feedforward control

Fig. 3.20 Flux ds at starting with and without feedforward control

Fig. 3.21 Flux qs at starting with and without feedforward control
3.5.3 Conclusions

Although different methods have been presented in literature to control the speed during low speed region, many of those methods have some draw backs or other as can be seen from the literature. In the proposed work, a high pass filter with low cut off frequency is proposed, which is cascaded to the reference model, to overcome the pure integration and DC drift problems. However, with the use of HPF to overcome the pure integration and DC drift a time delay is
produced in the development of stator flux when the motor is switched form standstill to vector control mode after the torque command is given. Due to this delay in the development of stator flux, a jerk of motor torque at the time of starting is produced. Hence, to eliminate jerk of motor torque, at starting, in this work feedforward control of stator flux is proposed which compensates the time delay in the development of the stator flux thereby eliminating the torque jerk at starting and the motor is switched from standstill to vector control mode without motor torque jerk.

3.6 MRAS based sensorless control of Induction Motor using SVPWM

3.6.1 Introduction

Although the MRAS based sensorless control of induction motor is one of the robust control schemes for speed estimation it has few drawbacks such as presence of ripples in torque and stator current. The conventional MRAS scheme for control of induction motor drive uses a voltage source inverter which produces the pulsating torque due to the application of non-sinusoidal voltages. Many researchers have focused on this for reducing the ripples present in torque and current. The pulsating torque is mostly influenced by the type of pulse width modulation (PWM) method employed. In the initial stage of PWM techniques the sine-triangle PWM (SPWM) was mostly utilized which reduces the ripples present in the torque. In due course of time space vector PWM (SVPWM) was developed and it was found to result
in fewer ripples in the torque than the SPWM. Hence, a lot of focus is being diverted to improve the SVPWM method.

The ripples present in the torque and stator current can be reduced by selecting a voltage vector not for the entire switching period, but for only a part of the switching period. In the remaining period zero voltage vectors are utilized. By inserting the zero voltage vectors the slip frequency can be controlled more accurately. By using SVPWM the ripples present in the torque and current can be reduced to a large extent when compared to SPWM method. Also the switching frequency of the inverter can be kept constant.

### 3.6.2 Space Vector Pulse width Modulation (SVPWM):

With the ever increasing demand for reduction of ripples present in torque and current, the PWM techniques have found a lot of focus for doing the same. A voltage source inverter is widely used to generate 3-phase ac with variable voltage and variable frequency that is required for drives used in variable speed applications. It is a well known fact that the ac voltage can be defined by its amplitude and frequency. Therefore, for variable speed drive applications the control of these two quantities is very much needed. In the conventional pulse width modulation (PWM) method, the production of pulses of variable duty cycle controls the average output voltage over a sufficiently small period known as sampling period.

In Fig. 3.24 a three phase voltage source inverter is shown where a, b, c are three phases and the voltages with respect to these three
phases is taken as $V_a, V_b, V_c$ that are applied to the induction motor.

Six power transistors i.e. 3 in the upper and 3 in the lower half forms a bridge inverter with the applied DC voltage $V_{dc}$ is split into two as

$$\frac{+V_{dc}}{2} \quad \text{and} \quad \frac{-V_{dc}}{2}$$

with 0 being the DC bus centre.

As can be seen from Fig. 3.24 every terminal of the induction motor will be connected to the pole of one of the inverter legs. It means that every terminal of the induction motor will be connected to either the upper transistor or the lower transistor. When the induction motor terminal is connected to the upper transistor it is taken as positive and when the terminal is connected to the lower transistor it is taken as negative.

![Fig. 3.24 Three phase VSI feeding an Induction Motor](image)

Therefore, it can be said that, at any instant, each one of the three pole voltages $[V_{a0}, V_{b0}, V_{c0}]$ of inverter connected to the induction motor, as measured with respect to the DC bus centre (0), will be
either $\frac{+V_{dc}}{2}$ or $\frac{-V_{dc}}{2}$. When ever the upper transistor is connected then for that phase the voltage is $\frac{+V_{dc}}{2}$ and this is taken as 1 for a or b or c phases. Similarly when the lower transistor gets connected then for that phase the voltage is $\frac{-V_{dc}}{2}$ which is taken as 0 for a or b or c phases. As an example, if upper transistors or the positive bus are connected to “a phase” and “b phase” and lower transistor or the negative bus is connected to “c phase” then this state is designated as “110” or alternately “++-”. By this it can be understood that, each phase of a three phase voltage source inverter can be connected either to the upper (positive) or the lower (negative) DC bus. For a three phase VSI, there exist two possibilities for each phases (either positive or negative) which in effect means that the total number of combinations possible for switching states will be $2^3$ or 8. These 8 combinations are designated by numbers usually known as switching codes or states. The possible 8 switching states are given as $V_0(000)$, $V_1(100)$, $V_2(110)$, $V_3(010)$, $V_4(011)$, $V_5(001)$, $V_6(101)$, $V_7(111)$ or these switching states can also be given in other form as $V_0(- - -)$, $V_1(+ - -)$, $V_2(+ + -)$, $V_3(- + -)$, $V_4(- + +)$, $V_5(- - +)$, $V_6(+ - +)$, $V_7(+ + +)$. Among the 8 possible switching states, 6 states i.e. states from $V_1$ to $V_6$ are known as active voltage vectors or active states in which one pole from the upper and two from the lower group or two poles from the upper and one from the lower group gets connected thereby the power gets transferred between DC bus and the induction motor.
As seen from Fig. 3.24 the poles can be connected to either upper or lower group. One such switching state $V_0$ is shown in Fig. 3.25. During the states $V_0$ and $V_7$ the power transfer between DC bus and the induction motor does not take place and hence these states are known as zero voltage vector or zero states.

**3.6.3 Voltage Space Vector:**

In the present work the PWM technique considered utilizes the space vector representation of the voltage in the stationary reference frame and is described in this section. From the inverter pole voltages $\left[ V_{a0}, V_{b0}, V_{c0} \right]$, the vector components, in $d$ and $q$ axes ($V_d, V_q$), in stationary reference frame can be derived by using the well known forward Clarke’s transformation. The vector components ($V_d, V_q$) in the stationary frame of reference is given as
where \( V_s \) is voltage space vector, \( x = e^{j\frac{2\pi}{3}} \) and \( x^2 = e^{-j\frac{2\pi}{3}} = e^{j\frac{4\pi}{3}} \).

The relationship between the three pole voltages \( \{V_{a0}, V_{b0}, V_{c0}\} \) and the three phase voltages \( \{V_{an}, V_{bn}, V_{cn}\} \) is given by the following equation.

\[
\begin{align*}
V_{a0} &= V_{an} + V_{n0} \\
V_{b0} &= V_{bn} + V_{n0} \\
V_{c0} &= V_{cn} + V_{n0}
\end{align*}
\]  

(3.68)

By using the relation \( V_{an} + V_{bn} + V_{cn} = 0 \) the equation for \( V_{n0} \), the common node voltage, can be given as

\[
V_{n0} = \frac{V_{a0} + V_{b0} + V_{c0}}{3}
\]  

(3.69)

With the above discussion the relationship between the switching variable vector \( [a, b, c]^T \) and the phase voltages \( \{V_{an}, V_{bn}, V_{cn}\}^T \) can be given in the matrix form as

\[
\begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]

(3.70)

It is evident from equations (3.67) and (3.68) that the phase voltages \( \{V_{an}, V_{bn}, V_{cn}\} \) also result in the same space vector \( V_s \). The space vector \( V_s \) can be resolved into two components \( V_d \) and \( V_q \) as shown in Fig. 3.26. The relationship between \( V_d, V_q \) and the phase voltages \( \{V_{an}, V_{bn}, V_{cn}\} \) can be obtained by conventional three phase to two
phase transformation utilizing the relationship between abc reference frame and stationary d-q reference frame shown in Fig. 3.26 and given by the equation (3.71)

\[
\begin{bmatrix}
V_q \\
V_d
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -1 \\
\frac{2}{\sqrt{3}} & -\frac{2}{\sqrt{3}}
\end{bmatrix} \begin{bmatrix}
V_{an} \\
V_{bn} \\
V_{cn}
\end{bmatrix}
\]  

(3.71)

The summary of the switching states, both active and zero, and corresponding phase voltages in abc reference and in stationary dq frame of reference are given in Table 3.1. Using the equation (3.67), the set of balanced 3-phase voltages can be represented by a space vector of constant magnitude, having a value equal to the amplitude of the voltages (Vm), and rotating with angular speed \( \omega = 2\pi f \) [1].

The possible switching states of an inverter can be represented as voltage space vectors as shown in Fig. 3.27. The location of the active vectors or states form an origin centered hexagon with 6 symmetrical sectors and the zero vectors or states at the origin.

![Fig. 3.26 Relationship between abc reference frame and stationary dq reference frame](image-url)
### Table 3.1 Switching vectors, phase voltages, d, q axes voltages.

<table>
<thead>
<tr>
<th>Voltage Space Vector</th>
<th>Switching Vectors</th>
<th>Phase Voltages</th>
<th>d, q axes voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>a b c</td>
<td>V&lt;sub&gt;an&lt;/sub&gt;</td>
<td>V&lt;sub&gt;bn&lt;/sub&gt;</td>
<td>V&lt;sub&gt;cn&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;0&lt;/sub&gt;(----)</td>
<td>0 0 0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>V&lt;sub&gt;1&lt;/sub&gt;(+-+)</td>
<td>1 0 0</td>
<td>2/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>-1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;2&lt;/sub&gt;(++--)</td>
<td>1 1 0</td>
<td>1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;3&lt;/sub&gt;(+-)</td>
<td>0 1 0</td>
<td>-1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>2/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;4&lt;/sub&gt;(+++)</td>
<td>0 1 1</td>
<td>-2/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;5&lt;/sub&gt;(--+)</td>
<td>0 0 1</td>
<td>-1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>-1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;6&lt;/sub&gt;(+-+)</td>
<td>1 0 1</td>
<td>1/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>-2/3 V&lt;sub&gt;dc&lt;/sub&gt;</td>
</tr>
<tr>
<td>V&lt;sub&gt;7&lt;/sub&gt;(+++)</td>
<td>1 1 1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The maximum boundary of the space vectors form the hexagon and the circle, shown in Fig. 3.27, is the maximum trajectory of the sinusoidal outputs in the linear modulation. The inverter PWM output patterns are shown in the figure as I – VI sectors. It can be seen from Fig. 3.27 that the angle between each voltage space vector is 60° or π/3, and the active voltage vectors are given by equation (3.72) or in general by (3.73)

\[
\begin{align*}
V_1 &= \frac{2}{3} V_{dc} \ e^{j \frac{\pi}{3}}, \\
V_2 &= \frac{2}{3} V_{dc} \ e^{j \frac{2\pi}{3}}, \\
V_3 &= \frac{2}{3} V_{dc} \ e^{j \frac{3\pi}{3}}, \\
V_4 &= \frac{2}{3} V_{dc} \ e^{j \frac{4\pi}{3}}, \\
V_5 &= \frac{2}{3} V_{dc} \ e^{j \frac{5\pi}{3}}
\end{align*}
\]

(3.72)
During a subcycle, a combination of switching states can be used, at the same time maintaining the volt-second balance for generating a given sample. This combination of switching states generating a sample is known as switching sequence. However, one point to be remembered is that for generating a PWM waveform any arbitrary set of active states (vectors) and zero states (vectors) cannot be applied maintaining the volt-second balance. There are few restrictions that are to be imposed so as to generate a PWM waveform resulting in harmonic distortion a minimum.

A PWM algorithm should be designed in way that there are no pulses of opposite polarity in the line to line voltage waveforms. This effectively means that the line to line voltage should be either 0 or $+V_{dc}$ but not $-V_{dc}$ at any instant during the positive half cycle.

\[
V_n = \frac{2}{3} V_{dc} e^{j(n-1)\frac{\pi}{3}} \quad \text{where} \quad n = 1, 2, 3, 4, 5, 6 \tag{3.73}
\]

**Fig. 3.27 Voltage space vectors as produced by Inverter**
If \(-V_{dc}\) is present then it will lead to large ripple current. Another important requirement is that, the switching of two simultaneous phases must not take place for efficient utilization of the available switching frequency of the inverter. Therefore, when the reference voltage vector is present with in a given sector, only two active states, x and y, whose vectors form the boundaries of that particular sector as shown in Fig. 3.28, can be applied.

![Fig. 3.28 Active and Zero states (vectors) in a sector](image)

If any other active states are applied then this results in a pulse of opposite polarity. The zero state or vector which is closer to x or which is just one switch away is referred as \(z_x\) and the other state which is closer to y is referred as \(z_y\). In each sector, the active and zero states that are applicable are summarized and given in Table 3.2.

The left hand side of equation (3.71) is also known as the voltage vector \(V_{ref}\) which represents the reference space vector or sample. The reference inputs are sampled in regular sampling time period or subcycle for determining the switching durations of effective vectors.
Table 3.2 Active and Zero states in each sector

<table>
<thead>
<tr>
<th>Sector</th>
<th>$\theta_h$</th>
<th>x</th>
<th>Y</th>
<th>$z_x$</th>
<th>$z_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0-60</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>II</td>
<td>60-120</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>120-180</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>IV</td>
<td>180-240</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>V</td>
<td>240-300</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>VI</td>
<td>300-360</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

Different voltage vectors which are produced by the inverter are applied over different time durations with in a sampling period such that the average vector produced over the subcycle is equal to the sampled value of the reference vector, both in magnitude and phase angle. The vectors required to generate any sample are the zero vectors and two active vectors forming the boundary of the sector in which the sample lies. The reference voltage vector $V_{ref}$ can be resolved into two components as shown in Fig. 3.29

![Fig. 3.29 Decomposition of $V_{ref}$ in sector–I](image)

In Fig. 3.29 $X_{V1}$ and $X_{V2}$ in sector – I can be given as
\[ X_{V1} = \frac{T_1}{T_s} V_1 \quad \text{and} \quad X_{V2} = \frac{T_2}{T_s} V_2 \quad (3.74) \]

where \( T_1 \) and \( T_2 \) are the durations for which the active states 1 and 2 respectively are to be applied in a given sampling time period \( T_s \).

In general the equation for \( V_{ref} \) is given as

\[ V_{ref} = \frac{T_n}{T_s} V_n + \frac{T_{n+1}}{T_s} V_{n+1} \quad \text{where} \ n=1, 2, 3, 4, 5, 6 \quad (3.75) \]

As all the six sectors are symmetrical the procedure for calculation of time period and the equation of \( V_{ref} \) will be identical with only the difference in suffix. Hence the discussion here is limited to sector – I only. Let \( z \) be the duration of time during which the zero states are to be applied then by utilizing the volt-time balance the time period of \( T_1, T_2 \) and \( z \) are calculated as follows. In sector – I the equation for \( V_{ref} \) can be given as

\[ V_{ref} = \frac{T_1}{T_s} V_1 + \frac{T_2}{T_s} V_2 \quad (3.76) \]

By using equation (3.72) \( V_{ref} \) at an angle \( \alpha^o \) can be given as

\[ V_{ref} \angle \alpha^o = \frac{1}{T_s} \left[ \frac{2}{3} V_{dc} \angle 0^o T_1 + \frac{2}{3} V_{dc} \angle 60^o T_2 \right] + 0 \angle 0^o \]
\[ \alpha = \tan^{-1} \left( \frac{V_d}{V_q} \right) \quad (3.77) \]

\[ V_{ref} (\cos \alpha + j \sin \alpha) = \frac{1}{T_s} \left[ \frac{2}{3} V_{dc} \angle 0^o T_1 + \frac{2}{3} V_{dc} \left( \cos 60^o + j \sin 60^o \right) \angle T_2 \right] \quad (3.78) \]

Equating the real and imaginary part in equation (3.78) we have
\[ V_{\text{ref}} \cos \alpha = \frac{1}{T_s} \left[ \frac{2}{3} V_{dc} \times T_1 + \frac{2}{3} V_{dc} \cos 60^\circ \times T_2 \right] \]  
\text{(3.79)}

\[ V_{\text{ref}} \sin \alpha = \frac{1}{T_s} \left[ \frac{2}{3} V_{dc} \sin 60^\circ \times T_2 \right] \]  
\text{(3.80)}

Rearranging the equations (3.79) and (3.80) and with necessary simplifications we get

\[ T_1 = T_s \times M_i \left[ \frac{\sin(60^\circ - \alpha)}{\sin 60^\circ} \right] \]  
\text{(3.81)}

and

\[ T_2 = T_s \times M_i \left[ \frac{\sin \alpha}{\sin 60^\circ} \right] \]  
\text{(3.82)}

where \( M_i \) is the modulation index and given is given as

\[ M_i = \frac{3 V_{\text{ref}}}{2 V_{dc}} \]  
\text{(3.83)}

Knowing the values of \( T_1 \), \( T_2 \) and \( T_s \), the time \( T_z \) spent on the zero vectors for keeping the switching frequency constant can be given as

\[ T_z = T_s - \left( T_1 + T_2 \right) \]  
\text{(3.84)}

In SVPWM approach, the total zero voltage vector time \( T_z \) is equally divided between the two zero states \( V_0 \) and \( V_7 \) i.e. \( T_z / 2 \) for each zero state and also the zero voltage vector time is symmetrically distributed at the start and at the end of the subcycle. Hence, the SVPWM make use of the sequences as 0127-7210 in sector – I, 0327-7230 in sector – II and so on. The switching sequences for all the six sectors are
shown in Table 3.3 and in Fig. 3.30 shows the switching sequence when the sample is situated in sector – I.

**Table 3.3 Switching sequence in all the six sectors**

<table>
<thead>
<tr>
<th>Sector number</th>
<th>On-sequence</th>
<th>Off-sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-1-2-7</td>
<td>7-2-1-0</td>
</tr>
<tr>
<td>2</td>
<td>0-3-2-7</td>
<td>7-2-3-0</td>
</tr>
<tr>
<td>3</td>
<td>0-3-4-7</td>
<td>7-4-3-0</td>
</tr>
<tr>
<td>4</td>
<td>0-5-4-7</td>
<td>7-4-5-0</td>
</tr>
<tr>
<td>5</td>
<td>0-5-6-7</td>
<td>7-6-5-0</td>
</tr>
<tr>
<td>6</td>
<td>0-1-6-7</td>
<td>7-6-1-0</td>
</tr>
</tbody>
</table>

As can be seen form Table 3.3 in each sector two active and two zero states are switched.

In Fig. 3.30 the space vector switching pattern in sector – I are shown from which it can be said that the total zero voltage vector time is divided between the zero states V0 and V7. The switching pattern is shown for both the upper and lower leg switching devices. With the above discussions it is clear that for proper implementation of SVPWM the following steps are to be followed:

1. Determination of \( V_d, V_q, V_{ref}, \alpha \)
2. Determination of time durations \( T_1, T_2, T_z \)
3. Determination of switching times of all the six transistors.
3.6.4 Proposed SVPWM based MRAS speed observer

One of the main requirements of space vector PWM (SVPWM) is the construction of reference voltage space vector $V_{ref}$ which in literature, can be achieved by different ways. To reduce the complexity of the SVPWM algorithm, in this work, the reference voltage space vector is constructed by using the errors between the reference d and q axes stator fluxes and d and q axes estimated stator fluxes sampled from the previous cycle.
Implementation of this SVPWM based MRAS speed observer for sensorless control of induction motor drive is shown in Fig. 3.31. Conventional MRAS utilizes a voltage source inverter and with the utilization of SVPWM, the advantages of the conventional MRAS, mainly its robustness, still remains but the complexity involved with the use of SVPWM for generation of pulses for the inverter increases.

In the proposed method of SVPWM based MRAS speed observer for sensorless control of induction motor, the position of the reference stator flux vector $\psi_s^*$ is derived by the addition of actual rotor speed and the slip speed. The actual synchronous speed of the stator flux vector $\psi_s$ is obtained from the adaptive motor model. After each sampling interval the actual stator flux vector is compared with the reference stator flux vector and the flux error hence obtained is
minimized in each sampling interval. By utilizing the following procedure the reference voltage vector \( V_{ref} \) is obtained in d and q axes components.

In the \( V_{ref} \) (reference voltage vector) calculator block shown in Fig. 3.31 the reference values of the d and q axes stator fluxes and actual values of the d and q axes fluxes are compared and the error hence between the two is given by the equations (3.85) and (3.86) respectively.

\[
\Delta \psi_{sd} = \psi_{sd}^* - \psi_{sd} \quad (3.85)
\]

\[
\Delta \psi_{sq} = \psi_{sq}^* - \psi_{sq} \quad (3.86)
\]

By knowing the values of stator flux error in d and q axes, from above equations, and the stator ohmic drop the appropriate reference voltage space vectors in terms of d and q axes can be calculated by utilizing the equations (3.87) and (3.88) respectively.

\[
V_{sd}^* = \frac{\Delta \psi_{sd}}{T_s} + R_i \frac{i_{sd}}{T_s} \quad (3.87)
\]

\[
V_{sq}^* = \frac{\Delta \psi_{sq}}{T_s} + R_i \frac{i_{sd}}{T_s} \quad (3.88)
\]

In the above equations, \( T_s \) is the duration of sampling period or the subcycle which is half of the period of the switching frequency. This in effect means that the flux is controlled twice per switching cycle. The d and q components of the reference voltage vector obtained in
equations (3.87) and (3.88) are fed to the SVPWM block from which the actual switching times for all the six transistors in inverter leg are obtained.

3.6.5 Results and Discussions:

Simulation studies are carried out to validate the proposed space vector PWM (SVPWM) based Model Reference Adaptive System (MRAS) speed observer for sensorless control of induction motor drive. The simulation parameters and the specifications of induction motor used are given in Appendix – I. Various conditions are considered in the proposed work and simulated. The performance of the drive on no-load, step change in load (addition and removal of load), step change in speed, external load torque disturbances have been considered. The simulations are also performed at different speeds ranging from low to near rated speed to show the effectiveness of the proposed scheme for the complete speed range. Simulations have been performed with SVPWM to show the effectiveness of the proposed SVPWM based MRAS speed observer for control of induction motor drive.

The response of the drive with proposed SVPWM based MRAS speed observer for no-load are shown in Fig. 3.32 a) and Fig. 3.32 b) shows the harmonic spectrum of the stator current. In Fig. 3.33 the response of the drive with proposed SVPWM on load with a load of 10 N-m applied at 1 sec are shown and it can be inferred from these figures that the ripples are very much reduced in torque and current by utilizing the proposed SVPWM. Moreover, the response of the
drive, with external load torque disturbances, for the proposed SVPWM based drive is shown in Fig. 3.34 where it can be inferred that as the load is varied the speed of the drive also gets varied in the same fashion. With increase in load the speed gets reduced and with decrease in load the speed of the motor drive gets increased. With the step change in speed from 800 rpm to 1300 rpm, known as acceleration of the drive, where the speed change command is given at 1 sec is shown in Fig. 3.35. All the above cases of no-load, on load, external load torque disturbances and step change in speed are also simulated for medium speed range. In this work 800 rpm is considered as medium range and speed around 100 rpm is considered as low range speeds.

In Fig. 3.36 the response of the proposed SVPWM based MRAS speed observer for sensorless control of induction motor drive for a speed of 800 rpm on no-load is shown. The response of the drive at a speed of 800 rpm with load of 10 N-m applied at 1 sec is shown in Fig. 3.37. With the external load torque variations the speed, torque and current responses at a speed of 800 rpm are shown in Fig. 3.38. The response of the drive when speed is changed from 1300 rpm to 800 rpm with speed change command given at 1 sec is shown in Fig. 3.39 for the proposed drive. The responses of the drive for low speed range i.e. 100 rpm is shown for the proposed SVPWM based drive in Fig. 3.40.
Fig. 3.32 a) Response of SVPWM based MRAS at 1300 rpm on no load
Fig. 3.32 b) Harmonic Spectrum of stator current by SVPWM based MRAS

THD=0.034228
Fig. 3.33 Response of SVPWM based MRAS at 1300 rpm with step change in load (Load of 10 N-m applied at 1 Sec)
Fig. 3.34 (a) Response of SVPWM based MRAS at 1300 rpm with external load torque disturbance
Fig. 3.34 (b) External load torque disturbance
Fig. 3.35 Response of SVPWM based MRAS with step change in speed from 800 rpm to 1300 rpm at 1 sec
Fig. 3.36 Response of SVPWM based MRAS at 800 rpm on no load
Fig. 3.37 Response of SVPWM based MRAS at 800 rpm with step change in load (Load of 10 N-m applied at 1 Sec)
Fig. 3.38 (a) Response of SVPWM based MRAS at 800 rpm with external load torque disturbance
Fig. 3.38 (b) External load torque disturbance
Fig. 3.39 Response of SVPWM based MRAS with step change in speed from 1300 rpm to 800 rpm
Fig. 3.40 Response of SVPWM based MRAS at 100 rpm on no load
3.6.6 Conclusions

In the conventional MRAS speed observer using a Voltage Source Inverter ripples are present both in current and torque as can be seen from the waveforms. Hence, to improve the performance of the drive in terms of ripples in this section, space vector PWM algorithm based on reference voltage vector is proposed and implemented. The proposed algorithm utilizes the stator flux components as control variables and enables the operation at constant switching frequency. The errors in d and q axes are obtained by comparing the reference values of d and q axes stator flux with estimated d and q axes stator flux. With the help of these errors the voltage space vectors that are to be applied to the induction motor in the next sampling interval by the VSI are determined. The required space voltage vector is synthesized by using SVPWM algorithm to generate the triggering signals for the inverter. Simulations are performed and analyzed for various conditions such as no load, on load, external load torque disturbance, step change in speed and for different speeds like 1300, 800, 100 rpm. The results presented show the effectiveness of the proposed SVPWM based MRAS drive which is used to reduce the ripples present in the torque and currents.