CHAPTER 1

GENERAL INTRODUCTION

1. INTRODUCTION

Since the early days of quantum mechanics, scientists have been trying to understand the many strange implications of the theory such as the superposition, wave – particle duality, observer’s role in measurements, etc. Now, the recent developments on the relationship between the geometry of physical reality, set theory and logic help us answer some of these questions. This work tries to understand the implications of quantum logic in the description of physical reality.

In physics the description of physical reality is usually based on definition and observation (measurement). Classical physics definitions and measurements are always considered to be complete. Therefore, the classical physics descriptions of physical reality are considered to be complete descriptions.

“……a definition and measurement of physical variables of S allows us (in classical physics) to determine univocally all future evolution and all past situations of S. For this we also speak of complete description.”

Unlike classical physics measurements, quantum measurements are always described in terms of probability distribution that gives the likelihoods of the possible results.

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1 Auletta Gennaro, Foundations and Interpretations of Quantum Mechanics, 21
2 Auletta 21
“Quantum mechanics furnishes only probability distributions of physical quantities. All this can be expressed by saying that quantum states, unlike classical ones, determine probability measures with dispersion.”

Therefore, the use of probabilities in quantum mechanics is not only a convenient mathematical tool, but also it is an intrinsic necessity to the description of quantum systems. Many theoretical physicists agree to the fact that there is no other way to deal with the quantum measurements but through the theory of probability.

The process of observation and measurements is associated with logic because logic provides us with basic rules governing the required correlations that we need in order to form propositions about the physical world. Classical logic as we shall see later handles the process of observation and measurements in classical physics while quantum logic is the study of how logic handles the process of observation and measurements of quantum systems.

Quantum measurements are associated with the non classical probability theory which is associated with set theory and propositional logic. In fact, the more recent developments in our understanding of quantum probability and the geometry of space are stimulated by the appreciation of the relationship between Boolean algebra, set algebra and logic. George Boole (1854) was the first to give an algebraic formulation to the theory of sets in logic. He established correlation between algebra and Aristotelian propositional logic by showing how logic can be expressed in algebraic terms.

In 1937 Birkhoff and von Neumann introduced the idea of quantum logic which not only, treats physical properties of quantum systems as logical propositions, but also studies the relationships and structures formed by these properties, with specific emphasis on quantum measurements.

The idea of quantum logic was later on developed, extended and reviewed by Mackey, Jauch,

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3 Robert Giuntini, Quantum Logic and Hidden Variables, p. 6
Putnam, Varadarajan, Piron, Beltrametti and Cassinelli, Bub and many others. The detailed investigation on the works of Birkhoff and von Neumann provides us with the new insights;

- It gives Boolean algebra a central role not only in logic, but also in the theory of probability.
- It leads us to the developments of the theory of lattices and the geometry of sets in quantum mechanics.
- The theory of lattices leads us naturally to the concept of reality in postulation.

A state of an entire system of the classical state is represented by a single point of phase space. The concept of a phase space was introduced for the description of classical systems. In quantum mechanics, the appropriate analogous concept is that of a Hilbert space. A single point of a Hilbert space represents the quantum state of an entire system. The dimensions in the Hilbert space (complex vector space) do not represent ordinary space dimensions. Instead, each Hilbert space dimension corresponds to one of the different independent physical states of a quantum system. The analysis of the works of Birkhoff and von Neumann in terms of phase space makes us realize that the algebraic description of phase space leads to the idea of the probability measure on the lattice of subspaces.

Orthogonality between rays is an important concept for quantum mechanics. The ray consists of all possible multiples of a particular state – vector. Orthogonal rays in quantum mechanics refer to states that are independent of one another such as; the different possible positions of a particle or different possible momentum states. Orthonormal are mutually orthogonal and normalized to be unit vectors. In the propositional logic terminology Yes/No propositions are orthogonal complements of one another.
According to von Neumann, the lattice of quantum mechanics proposition is given by lattice that is orthocomplemented and non distributive. However, in recent years many theoretical physicists prefer the non – Boolean orthomodular description because of the consequence that for a complex system the intersection between a set (properties, propositions) and its complement is not equal to the empty set, as it is in Boolean description, but it includes both in a fuzzy sense. Moreover, in this model the concepts of states, observables, symmetries, etc. can be developed in a more or less natural way.4

The non-Boolean orthomodular description leads us to a non – commutative characteristic of quantum propositions which are understood in terms of possible multiples of a particular state or as a distribution on the range of the spectral values. The representation of the possible multiples of a particular state (in terms of degrees, forms and sizes) not only, provides us with logical indication on probability usage in quantum mechanics, but also appears to be the most suitable and coherent way for the understanding the structure of quantum reality.

1.1. The Aim and Objectives of the Research

The more recent developments in our understanding of quantum probability and the geometry of space are stimulated by the appreciation of the relationship between set algebra and logic. This relationship proposes a new geometric framework for understanding the basic foundations of

4 Robert Piziak, “Orthomodular lattices and Quantum Physics”, in Mathematical Association of America, p. 299
quantum physics. The framework suggests the existence of a state space (the set of all possible states of the universe) within which as smaller subsets (fractional) of state space is embedded.

Based on the above developments, this research argues that the number of possible states of physical reality is limited only by our observations and measurements. We attempt to use the logical reformulation of quantum mechanics to show that “despite human experimental ingenuity, nature will undoubtedly stay at least one step ahead.” [Berthold-Georg] We are going to achieve it by starting from the very foundations upon which the probability theory is based; Aristotelian two – valued logic.

1.2. Reasons for the Choice of the Topic

There are three main reasons that led to the choice of the topic;

(i) The standard wave version of quantum mechanics does not automatically resolve the “measurement problem” implicit in the behavior of quantum variables. Therefore we need an alternative approach that would, not only, resolve the “measurement problem” but also, would provide us with probability interpretation that would lead us to the consistent description of quantum reality.

(ii) The developments of quantum mechanics have brought a radical shift in understanding of the geometry of physical reality. Quantum mechanics has made us realize that the geometry of physical reality is not a fixed Euclidean geometry as it was supposed classically, but rather a non – Euclidean geometry that changes dynamically.
Physicists are not clear when it comes to the issues of quantum reality. Most of them ‘prefer to pass over the uncomfortable reality issue in favor of questions more concrete.’

Moreover, the never ending wealth of publications on the subject of physical reality shows that the situation remains rather unsatisfactory.

1.3. Method

Our method is theoretical rather than experimental (empirical). That is, we are going to reinterpret present physical theories in order to develop new concepts of physical reality. In this thesis we are going to use von Neumann’s concept of a *quantum logic* understood as a possibility structure for events, and Bohm’s ideas on *hidden variables*, to arrive at the consistent description of quantum reality.

1.4. Scope of the Present Work

Quantum mechanics is the basis for our present understanding of physical reality not only on an atomic scale, but also at the macroscopic scale. The discovery that the properties of materials on appropriate time and length of scale does not behave according to our classical expectations has led researchers to exploit new fabrication possibilities at the NANO and atomic scale structures with highest performance.

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5 Nick Herbert, Quantum Reality, p.16
However, despite the success of quantum mechanics in giving engineers, the Biologists and the Chemists the tools with which to study and control objects on an atomic scale\textsuperscript{6}, yet there is still controversy with regard to the \textit{nature} of quantum reality, our \textit{perception} of it and the way we \textit{speak} about it.\textsuperscript{7} The divergence of opinion shows that physicists have lost their grip on the concept of physical reality; “most of them are distinctly uncomfortable about addressing the issue of reality at all;”\textsuperscript{8} some of them say “nature is just like that and apply the ideas of quantum physics to their study or research without worrying about their fundamental truth or falsity;”\textsuperscript{9} while others argue that mathematical formalism of quantum mechanics tells us nothing essentially about an actual quantum reality but it allows us to compute probabilities for alternative realities that may occur.

“For a long time, physicists gave up the idea of digging more deeply into such questions, finding that in their eyes they had taken too philosophical a turn. For them there was, and still is, a more urgent need: to make use of calculation rules of quantum physics in the very many domains where prodigious advances were and still are possible thanks to that wonderful tool.”\textsuperscript{10}

Though many physicists are reluctant in addressing the issue of quantum reality, Roger Penrose says that the question of reality must be addressed in quantum mechanics and by physicists, especially if one takes the view (as many physicists appear to) that the quantum formalism applies universally to the whole of physics. In fact avoiding discussing great issues such as the issue of reality has the unfortunate effects; “it remains difficult to see these issues in their true light.”\textsuperscript{11}

Moreover, quantum mechanics is the most accurate theory that describes nature at all possible scales. Quantum physics is now the basis for our understanding not only of the behavior of
matter and energy on the scale of atoms and subatomic particles, but also, it is believed to hold for all matter including the very large objects, such as stars and galaxies and cosmological events, such as the Big Bang. The universal characteristics of quantum mechanics imply that “if there is no quantum reality, there can be no reality at any level (all levels being quantum levels, on this view).”\(^{12}\)

In exploring the link between geometry of physical reality and the logical structure of the world, we are searching for the modern understanding of probabilities that fits in with the non–Boolean structures of the quantum world. The modern understanding of probabilities in turn provides us with the level of understanding and insights that will enable us make contribution to the consistent description of quantum reality.

### 1.5. Other works done in the same area

The issue of quantum reality started attracting physicists from the time of EPR thought Experiment which proposed the criterion of physical reality. EPR thought experiment was followed by the Bell’s theorem, which is a simple but powerful proof concerning the structure of physical reality. Bell’s theorem postulated that reality must be non–local.

“Bell’s theorem is one of the clearest windows that physicists posses into the nature of deep reality.”\(^{13}\)

The issue of quantum reality drew more attention of the physicists when the results of Aspect’s experiments which conclusively established that quantum reality is inherently non–local started coming out from the laboratory in 1982. Since then the issue of quantum reality has continued to

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12 Roger Penrose, pg. 508
13 Nick Herbert, Quantum Reality, p. 2
shape the debates on the conceptual foundations of quantum mechanics until today. Some of the publications on the debates on the issue of physical reality are as follows:

- Einstein, Albert, Boris Podolsky, and Nathan Rosen (1935), ‘Can Quantum – Mechanical Description of Physical Reality Be Considered Complete?’ Physics Review, 47, 777 – 780, Reprinted in Wheeler and Zurek (1983), pp. 138 -41. All started with EPR thought experiment. Einstein, Podolsky and Rosen were the first to propose the definition of ‘reality’ in their EPR thought experiment. They proposed an experiment in which a pair of particles is produced. When the particles are far apart, suitable measurements are made on particle 1. From these measured values, $p_2$ and $q_2$ are inferred. Using their proposed definition of reality, EPR then claimed simultaneous reality for $p_2$ and $q_2$, in contradiction to quantum mechanics. Therefore, Einstein, Podolsky and Rosen concluded that quantum mechanics must be incomplete since it does not represent reality.


- Aspect, Alain, Philippe Grangier and Gerard Roger (1982), ‘Experimental realization of Einstein – Podolsky – Rosen – Bohm Gedankenexperiment: A new violation of Bell’s inequalities’, Physics Review Letters, 49, 91-94. The results of Aspect’s experiments conclusively established that quantum reality is inherently non – local. The role of non – local hidden variables is paramount in the description of quantum reality. We come to
realize that the basic features of quantum mechanics are not merely simulated by Hilbert space mathematics but they are intrinsic structure of physical reality.

- Jeffrey Bub (1997) ‘Interpreting the Quantum World’, Cambridge University Press. In this book Bub argues that the possibility structure of a quantum world is represented by a dynamically evolving (non-Boolean) sublattice in $\mathcal{L}$, while the possibility structure of a classical world is fixed for all time as the Boolean algebra $\mathcal{B}$ of subsets of a phase space. The dynamic evolution of the quantum state tracks the evolution of the possibilities (and probabilities defined over these probabilities).

- Jean-Francois Froger & Robert Lutz, (2009) ‘The Hidden Structure within Reality’, Editions DesIris. In this book Jean-Francois and Robert argues that the understanding of reality involves a close examination of the logical bedrock of scientific theories and of their common presuppositions. The latter consists in admitting as ‘real’ what can be observed through the senses and be calculated from models obeying binary logic. The shift from Boolean logic to non-Boolean logic enables us to infer the existence of fundamental particles and to render their properties understandable.

- Lisa Zyga, “The Law of Physics Could Explain Quantum Mysteries”, Phys Org, August 17, 2009. In this work the invariant set postulate proposed by Tim Palmer is presented as a new geometric framework for understanding basic foundations of quantum physics. The invariant set postulate differentiates between reality and unreality, suggesting the existence of a state space, within which a smaller subset space (reality) is embedded.
1.6. Novelty in my Research

Description of quantum reality in terms of non–classical models and the theory of lattices lead us to the notion of physical reality that in general does not reveal itself totally in space and time or as an actual state of the system as it is supposed by classical mechanics but rather it has other aspects which are determined by the probability distribution over possible physical states. In other words, there is always something more to reality which completes the concrete state but is not reducible to it; this is what we call in this thesis reality in postulation.
I: CLASSICAL DESCRIPTION OF PHYSICAL REALITY

Non-Contextual Description of Reality

Classical Physics
- Observation
- Measurement

Boolean Algebra/Classical Logic

Aristotelian Propositional Logic (Yes/No Propositions)
II. QUANTUM MECHANICS

Contextual Description of Reality

Probability Measurement

Non-Locality (Measurement Problem)

Quantum Logic

Non-Separability

Non-Boolean Logic (Certain/Uncertain Propositions)
III. DESCRIPTION OF PHYSICAL REALITY

Clasical Reality [Non - Contextual]
- Locality and Separability
- Postulation of Reality

Quantum Reality [Contextual]
- Non - Locality and Inseparability
- Reality in Postulation