Chapter 7

Dynamics of Modal Interpretations
7.1. Introduction

The description of quantum reality based on quantum logic interpretation of quantum mechanics faces two main challenges; the first is the dynamics and the second one is the measurement problem. On the measurements problem, quantum logic has overcome it by asserting that the set of all properties detectable by us forms a Boolean algebra and in quantum mechanics, the properties that define the space of possibilities at a given time form a proper sublattice of the non-Boolean property structure.¹

On the challenge of dynamics, we need to find a way in which quantum logic could recapture the observed dynamics and their observed non-dynamical properties of the macroscopic objects from the dynamics of quantum objects. According to Michael Dickson, the dynamics of the modal interpretation provides us with the way forward of overcoming this challenge, because it

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¹ Bub, Interpretations of the Quantum World, p. 233
defines a probability measure over the possible physical states. In other words, the modal interpretation “chooses a subset of $\mathcal{L}$ to be the domain of the quantum probability measure.”

**7.2. Bohm’s Interpretation of Probability**

Quantum mechanics describes the dynamical evolution of probabilities, through the dynamical evolution of quantum states. Therefore, it appears that what is physically meaningful in a quantum world according to Jeffrey Bub is just:

- The lattice $\mathcal{L}$ of possibilities and
- The dynamically evolving probabilities defined on $\mathcal{L}$.

However, it is impossible to make sense of probabilities over a domain of possibilities without introducing the notion of *truth value*. This can be achieved if we consider that the *ray* representing the quantum state assigns probabilities to all the propositions represented by subspaces of the Hilbert space:

- Probability 1 to propositions in the property state defined by the ray,
- Probability 0 to propositions represented by subspaces orthogonal to the ray, and
- Non zero probabilities to all other propositions.

It follows that the probabilities represented by subspaces of Hilbert space cannot be represented on a standard (Kolmogorov) probability space, defined over the different possible property states.

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2 W. Michael Dickson, Quantum Chance and Non–locality, p. 75
3 Bub, Interpretations of the Quantum World, p. 236
4 As everybody knows traditional QM refers to the frequentistic probability worked out within the Copenhagen Interpretation (CIQM). It is essentially a sub specie probabilitatis Boolean logics extension. The values between [0,1] – i.e. between the completely and always true proposition I and the always false O – are meant as expectation values, or the probabilities associated to any measurable property. (Ignazio Licata, p. 8)
of ‘possible worlds’ specified by the orthodox interpretations, as measures over these property
states (as in the case of a classical probability theory)\(^5\)

> “Let us consider Boolean algebra with \(n\) atoms corresponding to the \(n\) commensurable
> observables. It is not difficult to check that any positive function

\[
P(p_i) \geq 0,
\]

\[
\sum_{i=1}^{n} P(p_i) = 1
\]

Satisfies the Kolmogorov axioms; in particular, the Kolmogorov axioms are satisfied by an
equidistribution;

\[
P(p_i) = \frac{1}{n} \text{ for all } i = 1, \ldots, n.
\]

The other extreme is a two valued probability measure

\[
P(p_i) \in \{1, 0\} \text{ for all } i = 1, \ldots, n.
\]

Sometimes two-valued probability measures are called 0–1 or dispersion free measures,
valuations or truth assignments.”\(^6\)

Since the Hilbert space cannot be represented on a standard (Kolmogorov) probability space,
Bub suggests that we look at the features of the projective structure itself. In this case, Bohm’s
interpretation appears to be a good possible candidate because it does not involves the
measurement problem which is implied by Von Neumann proposal and which identifies actuality
with unity probability. The identification of actuality with unity probability seems to lead to the
measurement problem because it chooses the preferred determinate observable as the unit
observable.

> “In standard presentation of quantum theory two types of evolution are recognized: unitary
> evolution, described by the Schrodinger equation or one of its relativistic generalizations – and
> non-unitary collapses. The latter processes are associated with measurements: in a measurement a
> superposition of states is assumed to be reduced to one of its components, corresponding to the

\(^5\) Bub, Interpretations of the Quantum World, p. 31

\(^6\) Svoil, Quantum Logic, p. 62
actual outcome. Mathematically, this is achieved by a projection of the state onto the component in question.  

Bohm’s interpretation solves the measurement problem because it is the non–collapse probabilistic interpretation. Non–collapse interpretation is preferred here because of the conviction that there is no fundamental difference between microscopic and macroscopic objects; both are subject to the same quantum mechanical principles. In fact in the collapse interpretation, the main problem arises in finding mathematics that would allow us to connect the deterministic quantum possibilities with the actuality of a single observed event. In Bohm’s interpretation, which is a non–collapse interpretation, such problem does not arise.

According to Bohm’s non–collapse probabilistic interpretation, the state $|\psi\rangle$ in Hilbert space has two aspects;

- The first is about possibilities; it is about what may be the case.
- The second is the theoretical quantity that occurs in the evolution equation, and its time development governs how the set of definite valued quantities changes.

The double role of $|\psi\rangle$, on the one hand probabilistic and on the other hand dynamical and deterministic is a well–known feature of the Bohm interpretation. Bohm’s interpretation may therefore be considered as a specific version of non–collapse interpretation, that is one in which

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7 D. Dieks, Probability in Non–Collapse Interpretation of Quantum Mechanics, in Beyond Quantum Mechanics, p. 345.
8 There is equivalence between probability interpretation and the many–Worlds interpretation. According to D. Dieks, the many–Worlds interpretation says that each element of the measure space corresponds to an actual state of affairs, whereas the probabilistic alternative tells us that each element may correspond to the one actual but unspecified state of affairs. There is consequently no difference in the symmetry properties or simplicity of the interpretations, but rather a difference in the nature of their ranges: in the one case this is a collection of many real worlds, in the other it is a collection of candidates for the one realworld. So, in the end the significant difference boils down to the difference between one and many real worlds – and it surely is not a principle of metaphysics that many is simpler than one. General considerations concerning symmetry and simplicity do therefore not favor many worlds over a probabilistic interpretation.
there is an a priori given definite – valued observable (position). This implies that this double *deterministic* and *probabilistic* aspects of \( |\psi\rangle \) is not only specific to the Bohm interpretation but also it is typical of all interpretation of quantum mechanics in which there are no - collapses and in which \( |\psi\rangle \) relates in a probabilistic way to the physical world.

Though probabilistic interpretation treats all elements of the probability space in exactly the same way, by mapping all of them to possibilities that may be realized, it does not tell us which possibility *is* realized. Each single element of the interpretation’s range may correspond to reality.

### 7.2.1. Bohm’s Interpretation of Probability in Non – Boolean Structures

In Bohm’s interpretation the relation of non – Boolean properties to the intrinsic indeterminism in a quantum world brings in two new insights which are associated with probabilities on the lattice structures;

- *Reducible* to measures over 2 – valued homomorphisms on the determinate sublattice of \( \mathcal{L} \)
- *Irreducible* on the lattice \( \mathcal{L} \)

In the quantum world, the properties that define the space of possibilities at a given time form a proper sublattice of the non – Boolean property structure. Although they do exist 2 –valued homomorphism that select property states on the sublattice, this determinate sublattice is itself

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9 D. Dieks, Probability and Non – Collapse Interpretation, p. 351.
non–Boolean and non–embeddable into a Boolean lattice because “it is easy to see that if a sublattice $T$ of $L(H)$ is Boolean, then all its elements commute…”\(^\text{10}\)

We can therefore speak of the determinate sublattices as maximal, in the sense that they contain the maximal subcollections of properties that can fit together in a quantum world, or the maximal subcollections of propositions that can be determinately *true or false*, given the constraints of the theorem.

“If $M$ is a subspace, its complement $M^\perp$ is the subspace containing all vectors orthogonal to all members of $M$. The subspaces are partially ordered by the subset relation. This ordering permits the following definition: If $\{M_i\}$ is a family of subspaces, its span $\cup_i M_i$ is the least subspaces containing all the $M_i$, $\cap_i M_i$ is the greatest subspace contained in each of the $M_i$, and $\bigcup\cap_i M_i$. It can be shown that any subspace and its orthocomplement span the entire space.”\(^\text{11}\)

In this sense each determinate sublattice is uniquely defined by a (pure) quantum state – an atom in the non–Boolean property structure – and some preferred observable that is stipulated as determinate.

“Among the observables of quantum mechanics the simplest are those which correspond to properties which the system may have (in which case the value of the observable is 1) or lack (in which case the value of the observable is zero). Such observables are represented by (orthogonal) projections on closed subspaces of $H$.\(^\text{12}\)

We already know that the dynamical change in a quantum world is tracked by the evolution of the quantum state over time, via Schrodinger’s equation of motion. So the quantum state functions as a dynamical state in tracking the temporal evolution of *what is possible* and what is *probable*, through the temporal evolution of the determinate sublattice associated with the quantum state, with respect to a preferred determinate observable. This is to say, “the actual

\(^\text{10}\) Itamar, Quantum Probability – Quantum Logic, p. 61
\(^\text{11}\) Michael R. Gardner, Is Quantum Logic Really Logic?, p. 511
\(^\text{12}\) Itamar, Quantum Probability – Quantum Logic, p. 57
properties in a classical world evolve in a fixed Boolean possibility space, while the possible properties in a quantum world evolve in a dynamically changing non-Boolean possibility space. Classically, only the actual properties are time-indexed; quantum mechanically, both the actual properties and the possible properties are time-indexed.”

From this perspective, the possibility structure of a quantum world is represented by a dynamically evolving (non-Boolean) sublattice in $\mathcal{L}$, while the possibility structure of a classical world is fixed for all time as the Boolean algebra $\mathcal{B}$ of subsets of a phase space.

“The set $\mathcal{L}$ of subspaces of $H$ forms a complete lattice; i.e. it has a maximal and a minimal element, and $\bigcup_j M_j$ and $\bigcap_j M_j$ exist for all subset $\{M_j\}$ of $\mathcal{L}$.”

In this sense quantum mechanics is therefore interpreted as a *dynamical theory of particle trajectories* rather than a statistical theory of observation. And the transition from classical mechanics to quantum mechanics involves the realization that a probability distribution of results [possibility] is *dynamical* rather than a classical fixed Boolean structure.

“The transition from classical mechanics to quantum mechanics involves the discovery that possibility is dynamical: the possibility structure of our universe is not a fixed, Boolean structure, as we supposed classically, but is in fact a non-Boolean structure that changes dynamically. The unitary Schrödinger evolution of the quantum state in time tracks the evolution of this possibility structure…”

### 7.2.2. Bohm’s Interpretation of Probability Versus Orthodox Interpretation

The slight difference between Bohm’s interpretation of probability and the orthodox interpretation, is in the fact that orthodox interpretation refers to the probability of a particle

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13 Bub, Interpretations of the Quantum World, p. 234
15 Bub, Interpretations of the Quantum World, p. 235.
being found somewhere in a suitable measurement, while Bohm refers to the probability that a particle “is” at some position. In this way interpretation of probability in quantum mechanics is not only epistemic and independent of our state of knowledge but also ontological.

Bohm’s interpretation of probability is shared by a number of physicists.

- Werner Heisenberg was influenced by the probability characteristics of quantum mechanics and said that it is possible to understand the probabilistic interpretation in the way that, before measurement, a physical observable has no determinate value but only possible values\textsuperscript{16}

- Dickson also argues, the fact that the state of a quantum system at any time generates a probability measure over all possible values of each observable\textsuperscript{17}, implies that the dynamical state provides us with possible value states for all physical systems.

The notion of possible value states has the advantage of removing any form of dualism and moreover, no contradiction can follow between the observables because a contradiction can only arise between existent realities. The notion of possible value state in this sense is a preferred candidate for the description of quantum reality.

\textsuperscript{16} De Broglie, Une Tentative d'interprétation causale et non - linéaire de la Mécanique Ondulatoire: la théorie de la double solution, Gauthier-Villars, Paris, 1956, p.48

\textsuperscript{17} W.M. Dickson, Quantum Chance and Non locality, p.8
7.3. Modal Description of Quantum Systems

Quantum logic arises essentially in a probabilistic framework and any probabilistic context gives rise in a natural way to some kind of modalities. Therefore, the dynamics of the modal interpretation provides us with the way forward of overcoming the challenges arising from quantum logical interpretation as a model for the description of quantum reality.

7.3.1. van Fraanssen Modal Interpretation

Following Bohm’s interpretation of probability, van Fraanssen in 1970es proposed what is known as modal interpretation. The aim of Fraanssen’s proposal like Bohm’s was to do away with projection postulate which is associated with the collapses of states. The orthodoxy interpretation had proposed the collapse of the wave function or projection postulate in order to account for the process of how the quantum state changes into classical state. However, the problems with the projection postulate are serious; “nobody has (yet) shown how to eliminate ‘measurement problem’ from the statement of the projection postulate, it may be that there are good reasons nonetheless to suppose that collapse does occur upon measurement.”\(^\text{18}\)

Modal interpretations have a straightforward way out of the measurement problem because it asserts that, “although only one state of affairs is actual, the total state describes all possibilities – it gives rise to a probability distribution that comprises both the actual and the possible.”\(^\text{19}\) All modal interpretations have one common characteristic; they draw a distinction between physical states and theoretical states, though different authors may use different terms for the same distinction.

\(^{18}\) W. Michael Dickson, Quantum Chance and Non–locality, p.28
\(^{19}\) D. Dieks, Probability in Non–Collapse Interpretations, p. 345
In every modal interpretation, this set depends at least in part on the theoretical state, and may be denoted $\mathcal{A}_w$. The set of possible physical states is a set of maps from $\mathcal{A}_w$ to the set $\{\text{'doesnotoccur'}, \text{'occurs'}\}$, or $\{0, 1\}$. Finally, modal interpretation defines a probability measure over the possible physical states, by adopting the quantum probability measure over $\mathcal{A}_w$.\(^{20}\)

- **Physical state**: A specification of all possessed properties, for a system (what occurs and what does not occur)
- **Theoretical state**: It determines all probability distributions over possible physical states.

The theoretical state yields probabilistic predictions about which events are now occurring, and which will occur in the future. In other words, if a quantum mechanical state is not equal to an eigenstate, then the value of a given parameter is in *superposition* of all eigenvalues that correspond to the eigenstates that make up the state. A measurement of that parameter will return any of the allowed eigenvalues with probability equal to the square absolute value of the coefficient in front of that eigenstate.

"In particular, the suggestion is that, granted the usual interpretational links between eigenstates of observables and values of physical quantities, a superposition of such eigenstates should be interpreted as representing the joint existence of the corresponding values."\(^{21}\)

According to all non–collapse interpretation, the state does not only represent what is actually the case in the world we observe, but also contains information about the possibilities that have not become *actual* in our world. Therefore there is a *modal* aspect to all non–collapse approach. Van Fraassen (1972, 1974, 1991.) modal interpretation proposes a distinction between what he calls:

- **Value state of the system**

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\(^{20}\)W.M. Dickson, Quantum Chance and Non–locality, p. 75

\(^{21}\)D. Dieks, Probability in Non–Collapse Interpretations, p. 351.
Dynamical state of a system

The value state is similar to what we call a physical state or actual state which is conceived as realization in reality. The dynamic state is what is considered as theoretical state or possible value properties which may be actualized due to some determinism or indeterminism change of the system.

Definition 7.1: The value state at any instant represents the system’s physical properties at that instant. It specifies the sharply defined values of all physical quantities for the system at the point in time in question.

Definition 7.2: The dynamical state determines the evolution of the system. It provides the distribution of properties that the system might have at later stage.

When the dynamical state of a system is mixed, it does not fix the value state, but it determines the set of possible values states.

The central idea in van Fraassen’s proposal and of modal interpretation in general is that the physical systems at all times possess a number of well-defined physical properties which change in time. This means that the change of a physical system is described also by a change of its properties. The dynamical state determines the set of possible value states and their possible evolution in time.

Moreover, according to van Fraanssen’s proposal, a system may have a sharp value of observable even if the dynamical state is not an eigenstate of that same observable. This is a bit different from the traditional eigenstate-eigen value link which says that a system can only have a sharp value of an observable if its quantum state is the corresponding eigenstate. This means the value state corresponds to a given eigenvalue if and only if its dynamical state is an
eigenstate of the observable corresponding to that eigenvalue. In quantum systems, not every subset of the state space will correspond to a physical property.

“It should be stressed that even in the strictly quantum case, most of the self – adjoint operators actually do not represent ‘interestingly’ physical quantities; only a few of them represent physical quantities that are useful and meaningful for the description of the physical system (e.g. energy, momentum, position, angular momentum).”

7.3.2. van Fraanssen Modal Interpretation in Non – Boolean Structures

From the point of view of the algebraic approach to quantum mechanics, van Fraansen’s interpretation is in resonance with non – Boolean algebra which is the logic structure of quantum propositions. According to Fraansen’s, propositions about a physical system cannot be jointly true unless they can be jointly certain according to the standard quantum rules. In other words, propositions about a physical system can be simultaneously true if they are represented by non - commuting observables. This is due to the fact that non – commuting observables impose limits on the possibilities of joint existence of properties themselves, independently of our knowledge. Non – commuting observables do not so much restrict our knowledge about the properties of a system; a physical property and its orthocomplement can both be possible in the same realization.

From the point of view of Einstein – Podolsky – Rosen experiment, the modality in van Fraansen’s interpretation, neither does it arise from an incompleteness of the description by the state vector nor from ignorance as proposed by EPR arguments. The dynamical state provides us with possible value states for all physical systems that are compatible with all the observable data.

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22 Beltrametti and Casinnelli, The Logic of Quantum Mechanics, p. 3
From the late 1980s, various researchers such as Kochen (1985), Bub and Clifton (1996), Dieks (1995, 2005, 2007), Healey and many others developed further the modal interpretation after realizing the possibilities that it offered to the solution of conceptual problems of quantum mechanics.