CHAPTER 5

THEORETICAL INTERPRETATION
AND DESIGN EQUATIONS

Development of closed form expressions for calculating the \( TM_{10} \) and \( TM_{01} \) mode frequencies of the arrow shaped microstrip antenna is presented in this chapter. The accuracy of the method is validated by experimental results. Characteristics of the new geometry are also analyzed by calculating the current and field distribution over the surface and edges of the patch using IE3D simulation software.
5.1 INTRODUCTION

The design equations of the arrow shaped patch antenna are developed by modifying the standard equations for a rectangular patch. These relations can predict both $f_{10}$ and $f_{01}$ mode frequencies very accurately. This provides a fast and simple way to predict the characteristics of the antenna. The design is also analyzed using experimental measurements and IE3D simulation package. The theoretical predictions are found to be very close to these results and thus establish the validity of design formulae. The design equations of the rectangular patch is discussed followed by the modifications incorporated for the new geometry. Finally the current distributions of the patch at the antenna surface are calculated using IE3D simulation software in order to verify the modes of the resonant frequencies determined from calculations.

5.2 RESONANT FREQUENCIES OF A RECTANGULAR MICROSTRIP PATCH

The design equations for a rectangular patch antenna based on cavity model proposed by Lo et al. [22, 23, 24] is described in this section. In this model microstrip antennas are considered to resemble dielectric loaded cavities and exhibit higher order resonances. The normalized fields within the dielectric substrate can be found more accurately by treating the region as a cavity bounded by electric conductors above and below the patch and by magnetic walls along the perimeter of the patch.

The field configurations can be found using the vector potential approach. Referring to Figure 5.1, the volume beneath the patch can be treated as a rectangular cavity loaded with a dielectric material having dielectric constant $\varepsilon_r$. The dielectric material
of the substrate is assumed to be truncated and not extended beyond the edges of the patch. The vector potential $A_x$ must satisfy the homogenous wave equation of

$$\nabla^2 A_x + k^2 A_x = 0 \quad (5.1)$$

whose solution can be written as,

$$A_x = [A_1 \cos(k_x x) + B_1 \sin(k_x x)] [A_2 \cos(k_y y) + B_2 \sin(k_y y)] [A_3 \cos(k_z z) + B_3 \sin(k_z z)] \quad (5.2)$$

where $k_x, k_y, k_z$ are the wave numbers along $x, y, z$ directions.

The electric and the magnetic fields are related to the vector potential $A_x$ by

$$E_x = -\frac{j}{\omega \mu \varepsilon} \left( \frac{\partial^2}{\partial x^2} + k^2 \right) A_x \quad H_x = 0 \quad (5.3)$$

$$E_y = -\frac{j}{\omega \mu \varepsilon} \left( \frac{\partial^2 A_x}{\partial x \partial y} \right) \quad H_y = \frac{1}{\mu} \frac{\partial A_x}{\partial z} \quad (5.4)$$

$$E_z = -\frac{j}{\omega \mu \varepsilon} \left( \frac{\partial^2 A_x}{\partial x \partial z} \right) \quad H_z = \frac{1}{\mu} \frac{\partial A_x}{\partial y} \quad (5.5)$$

Applying the boundary conditions it can be shown that

$$B_1, B_2, B_3 = 0, \text{ and}$$

$$k_x = \frac{m \pi}{h}, \quad m = 0, 1, 2, \ldots$$

$$k_y = \frac{n \pi}{L}, \quad n = 0, 1, 2, \ldots$$

$$k_z = \frac{p \pi}{W}, \quad p = 0, 1, 2, \ldots \quad (5.6)$$

Thus the final form for the vector potential $A_x$ within the cavity is
\[ A_x = A_{\text{np}} \cos(k_x x) \cos(k_y y) \cos(k_z z) \]  

(5.7)

Since the wave numbers are subject to constraint equation,

\[ k_r^2 = k_x^2 + k_y^2 + k_z^2 = \left( \frac{m\pi}{h} \right)^2 + \left( \frac{n\pi}{L} \right)^2 + \left( \frac{p\pi}{W} \right)^2 = \omega_r^2 \mu \varepsilon \]  

(5.8)

the resonant frequencies of the cavity are given by

\[ (f_r)_{\text{np}} = \frac{1}{2\pi \mu \varepsilon} \sqrt{\left( \frac{m\pi}{h} \right)^2 + \left( \frac{n\pi}{L} \right)^2 + \left( \frac{p\pi}{W} \right)^2} \]  

(5.9)

If \( L > W > h \), the mode with the lowest frequency is \( \text{TM}_{10} \) whose resonant frequency is given by

\[ (f_r)_0 = \frac{1}{2L\sqrt{\mu \varepsilon}} = \frac{c}{2L\sqrt{\varepsilon_r}} \]  

(5.10)

where \( c \) is the speed of light in free space.

The next higher order mode is \( \text{TM}_{01} \) whose resonant frequency is given by

\[ (f_r)_{01} = \frac{1}{2W\sqrt{\mu \varepsilon}} = \frac{c}{2W\sqrt{\varepsilon_r}} \]  

(5.11)

If \( W > L > h \), then the dominant mode is \( \text{TM}_{01} \) and the second order mode is \( \text{TM}_{10} \).

In all these discussions it was assumed that there are no fringing fields along the edges of the cavity. This is an assumption and is not totally valid.

Since the dimensions of the patch are finite along the length and width, the fields at the edges of the patch undergo fringing. This is illustrated along the length in Figures 5.2 for the two radiating slots of the microstrip antenna. The amount of
fringing is a function of dimensions of the patch and the height of the substrate. Since for microstrip antennas the thickness of the patch is much small compared to patch dimensions, fringing is less; however it must be taken into account because it influences the resonant frequency of the antenna.

Fringing makes the patch look wider electrically compared to its physical dimensions. This is illustrated along the length in Figure 5.2. Since some of the waves travel in the substrate and some in air, an effective dielectric constant \( \varepsilon_{\text{reff}} \) is introduced to account for fringing and the wave propagation in the line.

For TM\(_{10} \) mode,

\[
\varepsilon_{\text{reff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + \frac{12}{W} \frac{h}{W}\right)^{1/2} \tag{5.12}
\]

For TM\(_{01} \) mode,

\[
\varepsilon_{\text{reff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + \frac{12}{L} \frac{h}{L}\right)^{1/2} \tag{5.13}
\]

For calculating the TM\(_{10} \) mode frequency effective resonating length is considered to take into account the fringing field,

\[
L_{\text{eff}} = L + 2\Delta L_1 \tag{5.14}
\]

where \( \Delta L_1 \) is the line extension along the length.

\[
\Delta L_1 = 0.412h \frac{(\varepsilon_{\text{reff}} + 0.3)\left(\frac{W}{h} + 0.258\right)}{(\varepsilon_{\text{reff}} - 0.258)\left(\frac{W}{h} + 0.8\right)} \tag{5.15}
\]
Similarly for TM\textsubscript{01} mode,

\[ W_{\text{eff}} = W + 2\Delta L_2 \]  

(5.16)

where

\[ \Delta L_2 = 0.412h \frac{(\varepsilon_{\text{refl}} + 0.3) \left( \frac{L}{h} + 0.258 \right)}{(\varepsilon_{\text{refl}} - 0.258) \left( \frac{L}{h} + 0.8 \right)} \]  

(5.17)

Since (5.10) and (5.11) do not account for fringing, must be modified to include the edge effects and can be computed using

\[ (f_r)_0 = \frac{1}{2L_{\text{eff}} \sqrt{\varepsilon_{\text{refl}}} \sqrt{\mu_0 {\varepsilon_0}}} = \frac{c}{2L_{\text{eff}} \sqrt{\varepsilon_{\text{refl}}}} \]  

(5.18)

\[ (f_r)_0 = \frac{1}{2W_{\text{eff}} \sqrt{\varepsilon_{\text{refl}}} \sqrt{\mu_0 {\varepsilon_0}}} = \frac{c}{2W_{\text{eff}} \sqrt{\varepsilon_{\text{refl}}}} \]  

(5.19)

These equations are modified to calculate the TM\textsubscript{10} and TM\textsubscript{01} mode frequencies of the new arrow shaped geometry which is explained in the next section.
Chapter 5  

Theoretical Interpretation And Design Equations

Figure 5.1  Rectangular Microstrip Patch Geometry

Figure 5.2  Physical and effective lengths of Rectangular Microstrip Patch
5.3 RESONANT FREQUENCIES OF ARROW SHAPED MICROSTRIP PATCH

The resonance frequencies of the arrow shapes microstrip patch antenna are obtained by suitably modifying equations 5.18 and 5.19. Here the effective resonating lengths are calculated by taking into account the height of intruding and protruding triangles.

5.3.1 Coaxially Fed Arrow Shaped Microstrip Antenna

The geometry of the dual frequency arrow shaped microstrip antenna is shown in Figure 4.1. The antenna is coaxially fed at $f_p(x_0, y_0)$ to excite $TM_{10}$ and $TM_{01}$ mode frequencies.

The frequencies $f_{10}$ and $f_{01}$ can be calculated as follows

$$f_{10} = \frac{c}{2(S_{eff} + 2\Delta l_1)\sqrt{\varepsilon_1}} \quad (5.20)$$

$$f_{01} = \frac{c}{2(W_{eff} + 2\Delta l_2)\sqrt{\varepsilon_2}} \quad (5.21)$$

In these equations the effective resonating lengths, effective permittivity and line extension factors are modified, which can be calculated using the following empirical relations.

For $TM_{10}$ mode the line extension factor and effective permittivity expressions are obtained from 5.12 and 5.15 of the rectangular patch antenna.
\[ \varepsilon_1 = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + 12 \frac{h}{W}\right)^{-1/2} \quad (5.22) \]

\[ \Delta l_1 = 0.412h \frac{(\varepsilon_1 + 0.3) \left(\frac{W}{h} + 0.258\right)}{(\varepsilon_1 - 0.258) \left(\frac{W}{h} + 0.8\right)} \quad (5.23) \]

For TM\(_{01}\) mode, these expressions are obtained from equation 5.13 and 5.17, replacing the length \('L'\) of the rectangular patch by \('S'\). \('S'\) is the mean length calculated from the intruding and protruding triangle lengths \(S_1\) and \(S_2\) as shown in Figure 4.1.

\[ \varepsilon_2 = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left(1 + 12 \frac{h}{S}\right)^{-1/2} \quad (5.24) \]

\[ \Delta l_2 = 0.412h \frac{(\varepsilon_2 + 0.3) \left(\frac{S}{h} + 0.258\right)}{(\varepsilon_2 - 0.258) \left(\frac{S}{h} + 0.8\right)} \quad (5.25) \]

where \[ S = \frac{S_1 + S_2}{2} \quad (5.26) \]

The calculation of effective length and width are as follows. Two different cases are discussed. The case in which \((L \geq W)\), TM\(_{10}\) is the dominant mode excited and TM\(_{01}\) is the second order mode. The case \((L < W)\), the dominant mode cannot be predicted unless the effective resonating lengths are known.
For these two cases effective length and width are calculated by defining a checking factor.

\[
\text{Checking Factor (C.F.) = } \frac{W_{cd}}{W},
\]

For C. F. > 1/2 and C. F. \(\leq 1/2\) two sets of equations are defined for both the cases.

**CASE \((L<W)\)**

\[
\begin{align*}
S_{eff} &= S_i - (.0001/L) + .01W - .68(W_{cd} - .01) - .03(W_{cp} - .01) \quad \text{C.F.} \leq 1/2 \\
W_{eff} &= W + .58W_{cp} - .43W_{cd}
\end{align*}
\]

\[
\begin{align*}
S_{eff} &= 0.5(S_i + L) + .4W_{cd} - .175W - .03(W_{cp} - .01) \quad \text{C.F.} > 1/2 \\
W_{eff} &= .78W + .025W_{cd} + .49W_{cp}
\end{align*}
\]

**CASE \((L\geq W)\)**

No C.F for the calculation of \(S_{eff}\).

\[
\begin{align*}
W_{eff} &= W + .58W_{cp} - .43W_{cd} + .0023(L - W)/W \quad \text{C.F.} \leq 1/2 \\
W_{eff} &= .78W + .025W_{cd} + .49W_{cp} + .0025W_{cd}/W + .17(L - W - .01) \quad \text{C.F.} > 1/2
\end{align*}
\]

\[
S_{eff} = S_i + 2.3(L - 2W - 0.0046/L)W_{cd} + 0.00006/L - .1(W_{cp} - .01)
\]
5.3.1.1 Comparison between Theoretical and Experimental Results

A comparison between the theoretical and experimental results is made by calculating the percentage errors.

\[
\% \text{ error} = \frac{\text{measured} - \text{calculated}}{\text{measured}} \times 100\%
\]

The theoretical and experimental results for various lengths of arrow shaped antenna for different combinations of \( W_{cd} \) and \( W_{cp} \) are shown in Figure 5.3. From the graph it can be noted that \( f_{10} \) mode frequency varies rapidly and \( f_{01} \) frequency remains almost constant for particular \( W_{cd} \) and \( W_{cp} \). The theoretical and experimental results are in very good agreement.

Figure 5.4 shows the variation of both calculated and measured frequencies with \( W \). The width variations effect mainly \( f_{01} \) mode frequency keeping the other almost constant. Table 5.1 shows the variation of calculated and measured values with intruding triangle height \( W_{cd} \) of the patch and Table 5.2 shows similar variation with \( W_{cp} \). The results of variation of the resonant frequencies with different ‘h’ and ‘\( \varepsilon_r \)’ combinations are shown in Figure 5.5 and Table 5.3. All the above tables and graphs prove that theoretical results almost follow the experimental values in all cases.

By comparing the experimental and calculated data, it is found that in all cases the percentage error is less than 2. These equations provide a fast and simple method for the design of compact dual frequency microstrip antenna. For calculating the resonant frequencies, the present design equations are less time consuming than general e.m. simulation packages.

Using the above resonant frequency calculation it is possible to design and optimize the various antenna dimensions for desired operating frequency or frequency ratio.
Figure 5.3 Graph showing the variation of calculated and measured $\text{TM}_{10}$ and $\text{TM}_{01}$ mode frequencies with Length L.

($W=5$ cm, $c_r=4.28$, $h=0.16$ cm)
Table 5.1 Measured and calculated TM\(_{10}\) and TM\(_{01}\) mode frequencies

(Variation with \(W_{cd}\))

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<th>(W_{cd}) (cm)</th>
<th>(W_{ee}) (cm)</th>
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<th>% error</th>
<th>TM(<em>{01}) mode frequency ((f</em>{01})) (GHz)</th>
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Figure 5.4 Graph showing the variation of calculated and measured TM_{10} and TM_{01} mode frequencies with width ‘W’

(L=5 cm, \(c_r=4.28\), h=0.16 cm)
Figure 5.5 Graph showing the variation of calculated and measured $\text{TM}_{10}$ and $\text{TM}_{01}$ mode frequencies with height $'h'$ and permittivity $'\varepsilon_r'$.

($W = 5 \text{ cm}, W_{cp} = 1 \text{ cm}, W_{cd} = 1 \text{ cm}$)
Table 5.2 Measured and calculated TM<sub>10</sub> and TM<sub>01</sub> mode frequencies with % error
(Variation with W<sub>cp</sub>)

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<th>% error</th>
<th>TM&lt;sub&gt;01&lt;/sub&gt; mode frequency (f&lt;sub&gt;01&lt;/sub&gt;) (GHz)</th>
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## Table 5.3 Measured and calculated TM_{10} and TM_{01} mode frequencies with % error
(Variation with height and permittivity)

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<th>L, W, W_{cp}, W_{ed} (cm)</th>
<th>h (cm)</th>
<th>( \varepsilon_r )</th>
<th>TM_{10} mode frequency (( f_{10} )) (GHz)</th>
<th>% error</th>
<th>TM_{01} mode frequency (( f_{01} )) (GHz)</th>
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5.3.2 Electromagnetically Coupled Dual Port Arrow Shaped Microstrip Antenna

The arrow shaped antenna is reconfigured using two perpendicular microstrip feed lines to eliminate cross talk between the two polarizations and to achieve excellent isolation between the ports as explained in Section (4.4). The geometry of the proposed antenna is shown in Figure 4.31. The antenna is etched on a dielectric substrate of thickness \( h_2 \) and dielectric constant \( \varepsilon_r_2 \) and fed by proximity coupling using two 50\( \Omega \) perpendicular microstrip lines etched on a substrate of thickness \( h_1 \) and dielectric constant \( \varepsilon_r_1 \).

The equations given above for the co-axial fed arrow shaped microstrip antenna is modified to obtain the frequencies for the dual ports. Here the thickness of the substrate is modified due to the effect of another substrate with microstrip feedline. Hence \( 'h' \) used in the equations 5.22 through 5.25 should be replaced by effective thickness \( h_{eff}=h_1+h_2 \). Here only the special case where both the substrates are of the same permittivity are studied and hence the dielectric constant in equations 5.22 and 5.24 are replaced by \( \varepsilon_r=\varepsilon_r_1=\varepsilon_r_2 \).

So, the effective dielectric constant for \( TM_{10} \)

\[
\varepsilon_1 = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 12 \frac{h_{eff}}{W} \right)^{-1/2}
\]  \hspace{1cm} (5.27)

\[
\Delta f_1 = 0.412 \frac{W}{h_{eff}} \frac{(\varepsilon_1 + 0.3)\left( \frac{W}{h_{eff}} + 0.258 \right)}{(\varepsilon_1 - 0.258)\left( \frac{W}{h_{eff}} + 0.8 \right)}
\]  \hspace{1cm} (5.28)
For TM_{01},

\[
\varepsilon_2 = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 12 \frac{h_{\text{eff}}}{S} \right)^{-1/2}
\]  

(5.29)

\[
\Delta l_2 = 0.412 h_{\text{eff}} \frac{(\varepsilon_2 + 0.3) \left( \frac{S}{h_{\text{eff}}} + 0.258 \right)}{(\varepsilon_2 - 0.258) \left( \frac{S}{h_{\text{eff}}} + 0.8 \right)}
\]  

(5.30)

where \( h_{\text{eff}} = h_1 + h_2 \), \( \varepsilon_r = \varepsilon_r 1 = \varepsilon_r 2 \)  

(5.31)

For Port1,

\[
f_{10} = \frac{c}{2(W_{\text{eff}} + 2 \Delta l_2) \sqrt{\varepsilon_2}}
\]

For Port2,

\[
f_{10} = \frac{c}{2(S_{\text{eff}} + 2 \Delta l) \sqrt{\varepsilon_1}}
\]

\( S_{\text{eff}} \) and \( W_{\text{eff}} \) are calculated as explained in the coaxial feeding technique.

The theoretical variation of the two resonant frequencies with \( L \) for different values of \( h_{\text{eff}} \) and \( \varepsilon_r \) are shown in Figure 5.6. The measured curves are given in the same figure to validate the computed results. Here the theoretical results are in good agreement with measured values with maximum percentage error less than 2 as shown in Table 5.4.
Figure 5.6 Graph showing the variation of calculated and measured \( \text{TM}_{10} \) and \( \text{TM}_{01} \) mode frequencies of the dual port antenna with Length \( L \)

\((W=5\text{cm}, W_{cp}=1\text{cm}, W_{cd}=1\text{cm})\)
### Table 5.4 Measured and calculated TM\(_{10}\) and TM\(_{01}\) mode frequencies for the dual port antenna

<table>
<thead>
<tr>
<th>L,W,Wcp, Wcd (cm)</th>
<th>(h) (cm)</th>
<th>(\varepsilon_r)</th>
<th>TM(<em>{10}) mode frequency ((f</em>{10})) (GHZ)</th>
<th>TM(<em>{01}) mode frequency ((f</em>{01})) (GHZ)</th>
<th>Measured</th>
<th>Calculated</th>
<th>% error</th>
<th>Measured</th>
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<th>% error</th>
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5.4 Verification of the resonating modes using IE3D

Mode verification can be done by calculating the 3D average current density on the patch surfaces. Mode identification is possible by seeing the current distribution along the edges of the patch.

For verification, a rectangular patch having dimension $L=6 \text{ cm}$, $W=5 \text{ cm}$ is simulated and the current distributions on the surface of the patch is calculated. The fundamental modes are given in Figure 5.7(a, b). From the figure it is clear that the first frequency (Figure 5.7(a)) is having a mode $TM_{10}$ as there is one half wave of the field variation along length, second frequency (Figure 5.7(b)) is $TM_{01}$ as there is one half wave variation along width.

In arrow shaped antenna modes determined by this technique found to agree with the mode predicted using theoretical calculations. For verification, current distributions are calculated for the patches by trimming $W_{cd}$ and compared it with experimental results as shown in Table 5.1. Patch having dimensions $L=4 \text{ cm}$, $W=5 \text{ cm}$, $W_{cd}=1 \text{ cm}$, $W_{cp}=1 \text{ cm}$ is simulated using IE3D and axial currents are determined as shown in Figure 5.8(a, b). Here the dominant mode frequency $f_1=1.423 \text{ GHz}$ is of $TM_{01}$ mode as there is one half wave variation of current along the width (Figure 5.8(a)). The second frequency $f_2=1.695 \text{ GHz}$ is of $TM_{10}$ mode as there is one half wave of current variation along the length (Figure 5.8(b)). Comparing it with modes shown in Table 5.1 it is clear that the modes predicted using theoretical calculations are in good agreement. Now $W_{cd}$ is varied and the same procedure is repeated. For $W_{cd}=2 \text{ cm}$, we can notice that there is a change in dominant mode. Here the dominant mode is $f_{10}$ as clear from the Figure 5.8(c, d). Checking Table 5.1 the same change can be noticed for theoretical calculations.

From the above results we can conclude that modes predicted through theoretical calculations are established from the current calculations using IE3D.
Chapter 5

Theoretical Interpretation And Design Equations

\[ f_1 = 1.189 \text{ GHz} \]

\[ f_2 = 1.418 \text{ GHz} \]

**Figure 5.7** 3D average current density on the surface of the rectangular patch (L=6 cm, W=5 cm)
Figure 5.8 3D average current density on the surface of the patch

(L=4 cm, W=5 cm, W_{cp}=1 cm)
5.5 Mode verification for slotted geometries

Mode verifications can be done for slotted geometries also. For the unslotted geometry \((L=6\text{cm}, W=5\text{cm}, W_{cd}=1\text{cm}, W_{cp}=1\text{cm})\) the current distributions for the first three modes are shown in the Figure 5.9 (a, b, c, d). The first frequency is \(TM_{10}\) as there is one variation of current along length (Figure 5.9 (a)), and second \(TM_{01}\) as there is variation along width (Figure 5.9 (b)), and the third \(TM_{11}\) as there is one variation along both length and width (Figure 5.9 (c)). In Figure 5.9 (d) there is a double variation with almost null current at the centre and edges and identified this mode as \(TM_{20}\).

Current is calculated for the above arrow shaped geometry with an embedded rectangular slot as shown in Figure 4.19. The slot dimensions are \(l_s=5\text{cm}, w_s=0.2\text{cm}, s=0.5\text{cm}\). The current distributions for the fundamental modes are shown in Figure 5.10 (a, b). From the graph it is clear a new mode is generated which is less than \(TM_{10}\) and \(TM_{01}\) mode frequencies of the unslotted patch. The new frequency has current distributions at the circumference of the slot. This new mode have same polarization as \(TM_{01}\) as presented in Chapter 4. Hence this new frequency is regarded as the \(TM_{01}\) mode \((0<\delta<1)\) as it is a new mode generated by the embedded slot.

Current distribution is also calculated for the arrow shaped geometry with two slots embedded close to the non radiating edges of the patch as shown in Figure 4.22. The slot is having dimension \(l_s=5\text{cm}, w_s=0.2\text{cm}, s=0.2\text{cm}\). Current distributions for the fundamental mode are shown in Figure 5.11 (a, b). The first frequency is the fundamental mode generated as in the case of unslotted geometry. For the second frequency we can see the current variation along the circumference of the slot. This frequency has the same polarization as the fundamental mode. This new frequency generated can be said to have a mode \(TM_{80}\) \((1<\delta<2)\). It is clearly cited in the figure.
Figure 5.9 Current distribution on the surface of the unslotted patch (L=6cm, W=5cm, W_{cp}=1cm, W_{cd}=1cm)
Figure 5.10 Current distribution on the surface of the arrow shaped patch with a rectangular slot ($l_y=5$cm, $w_y=0.2$cm, $s=0.5$cm)