PREFACE

Nature abounds with the periodic phenomena; from the motion of a swing to the oscillations of atoms, from sunrise to the sunset. But in nature there are numerous other phenomena in which linearity breaks down and instead of periodicity, we get aperiodic or chaotic motion. For example, the smooth waves on a well behaved lake turn to violent turbulence in the mountain brook, and the daily sunrise is overshadowed by cloud formation.

A fractal is a nonlinear geometric object which is rough or irregular on all scales of length, so it appears to be broken up in a radical way. Fractals are said to possess infinite details. In many cases, a fractal can be generated by a repeating pattern, in a typically recursive or iterative process. Objects that are now called fractals were discovered and explored long before when the word was coined in 1975 by B. B. Mandelbrot.

Verhulst logistic map $f(x) = r x (1 - x)$ is a widely studied and applicable model in discrete dynamical system. The discrete time variable version of Verhulst’s growth law, the logistic map, is the foundation stone for the theory of nonlinear dynamics, and basis of modern chaos theory. In English language chaos means state of total disorder or mismanagement. Although there is no universal definition of chaos, this is the general acceptance that the breakdown of predictability is called chaos. The term chaos was proposed by Yorke and Li.

It is a general belief that once chaos occurs in the system, the system becomes unpredictable, and it is necessary to handle this type of situation. Parrondo’s paradox was introduced in dynamical system to show that combination of two unstable systems can become stable. Although superior iterates itself increase the convergence of range of the
logistic map, but still chaos occurs. Parrondo’s paradox further controls some of the chaotic situations. In Chapter II, we have analyzed the stability of superior logistic map for period-1 and period-2. Further, we applied Parrondo’s paradox to superior logistic map and found various combinations of “chaos$_1$ + chaos$_2$ = order” for $r > 4$, i.e, the combination of two chaotic systems can become stable.

The logistic map is used to study the population growth of asexual reproduction. In Chapter III, we have analyzed the stability of modified and extended logistic maps for describing multi-scaled population (i.e., sexual reproduction) in superior orbit. In this chapter, it has been shown that the stability of above maps is extended and also found various examples of “undesirable$_1$ + undesirable$_2$ = desirable” system.

In Chapters II and III, it has been shown that the two unstable systems can become ordered jointly. But in Chapter V, we apply PS algorithm to the combination of $N$ chaotic systems in random and deterministic manner to get the stable system. PS algorithm has been applied on logistic map, modified logistic map and extended logistic map in the superior orbit to control the chaos (i.e., chaos$_1$ + chaos$_2$ + ... + chaos$_N$ = order), and to perform anti-control of chaos (i.e., order$_1$ + order$_2$ + ... + order$_N$ = chaos) as well.

Today Julia sets have been focused subject for the investigation in nonlinear fields. Till now, Julia and Mandelbrot sets have been intensively studied, but most of the studies are limited to their properties and drawing of graphics for different types of functions. So, control over the Julia and Mandelbrot sets have become an active research area. In Chapter V, we have given a review on control of Julia and Mandelbrot sets by controlling parameters, changing iterative procedures, noise method, and applying alternation method. We used alternation method, one of the controlling methods, in superior Julia sets in Chapter VI.
In Chapter VI, we have introduced the alternation in superior quadratic and cubic Julia sets, and studied the connectedness to show that superior Julia sets are connected or disconnected or totally disconnected as well.

Finally, Chapter VII presents the some of the applications of the work done in Chapter II-VI.