CHAPTER 2
THEORETICAL BACKGROUND
2.1 INTRODUCTION

This chapter provides a theoretical aspects of risk management, conceptual clarity of various Value at Risk models and basic statistical tools employed in the study. Over a period of time with the advancement of the study in risk management in the financial literature, many modifications has been done to the Value at Risk Models. Given the growth of literature on Value at Risk models it is highly impossible to cover the whole gamut of VaR models. Again, targeting the users of risk management techniques in the Indian context based on the available literature, the study limits to the application of select VaR models only.

This chapter explains financial risk management, types of risks, meaning of Value at Risk, features and process of various Value at Risk model like Historical Simulation, Historical Simulation-GARCH, Historical Simulation asymmetric GARCH, Monte Carlo Simulation, Exponentially Weighted Moving Average, Conditional VaR models. This chapter also explain the meaning of Backtesting, and various Backtesting models.

The following abbreviations are used for the above models in the entire study:

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<td>VaR</td>
<td>Value at Risk</td>
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<td>Historical Simulation Model</td>
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2.2 THEORETICAL UNDERSTANDING

2.2.1 Meaning of Risk
Risk can be defined as a situation where the expected results deviate from the realised results. The term deviation here indicates negative deviation which amount to losses. The positive deviation would mean profit for a rational investor purely from financial or economic point of view.

2.2.2 Meaning of Financial Risk Management:
‘Financial Risk Management refers to the design and implementation of procedures for controlling financial risks’ (Jorion, 2002). Risk management is the process of identifying various risks exposed by the firms, quantifying those risks and using appropriate methods to control those risks.

2.2.3 Types of Financial Risks

2.2.3.1 Meaning of Financial Risk:
Financial risks are the risks of losses due to the financial transactions. Financial Risks are classified into six categories. They are market risk, credit risk, liquidity risk, operational risk, legal and regulatory risk, and human factor risk.

Figure 2(A): Types of Financial Risks
2.2.3.2 Meaning of Market Risk

Market risk is defined as the risk of loss or gain which occurs due to the unanticipated changes in the market prices of the financial markets. In other words the unexpected changes in the security prices, interest rates, foreign exchange rates.\(^2\) The Bank for International Settlements (BIS) defines market risk as “the risk that the value of ‘on’ or ‘off’ balance sheet positions will be adversely affected by movements in equity and interest rate markets, currency exchange rates and commodity prices”\(^4\).

Classification of Market Risk:

**Figure 2(B): Types of Market Risks**

![Diagram of Market Risks]

- **Interest Rate Risk** arises due to the change in the absolute level of interest rates. This change causes the value of the investment to increase or decrease. This risk also arises in case of differences between two rates. The interest rate changes adversely affect the securities value inversely. Such risks can be reduced through diversification of investments in fixed income securities with different maturity periods or through hedging.

- **Equity Price Risk** arises due to the fluctuations in the security prices of the companies. These risk are influenced by purchase of stocks, dividend policy, company’s financial position, market conditions etc.

- **Foreign Exchange Risk** arises due to the drastic changes in the exchange rates of one currency to another currency. In order to cover the losses. The investor either
takes a long or short position in a foreign currency. Commodity Risk arises due to the uncertainties in the future values and fluctuation in prices of commodities. The commodities can be precious metals, grains, crude oil prices, electricity etc.

2.2.3.3 Meaning of Credit Risk

Credit Risk arises when the counterparties fail to fulfil their contractual obligations. The companies dealing with bonds, loans and derivative instruments are exposed to credit risk. The changes in the market prices of debt due to changes in the credit ratings also give rise to credit risk. The decrease in the credit rating indicates the decline of the borrowing capacity of a company. Credit Risk includes Sovereign Risk and Settlement Risk. Sovereign Risk arises when a country imposes restrictions on foreign exchange transaction which in turn makes it impossible for the traders to honour the international obligation. Further, suppose if two parties are supposed to make settlements on the same day, wherein one party defaults to make payment, Settlement Risk occurs. This risk most often is visible in foreign exchange transactions.

2.2.3.4 Meaning of Liquidity Risk

Liquidity Risk are of two types namely, asset liquidity risk and funding liquidity risk. Failure to carry out the transaction at the present market prices due to the size of the position to normal trading lots, give rise to asset liquidity risk. Major currencies or government securities which have deep markets face the asset liquidity risk. Funding liquidity risk arises due to the inability to meet payment obligations. This risk may result in early liquidation of the companies by transforming the paper losses into real loss, leading to the closure of the companies.
2.2.3.5 Meaning of Operational Risk

Operational Risk are the results of the technical or administrative failures (inadequate or improper procedures and controls) and frauds. These risk includes human and technical errors. Operational risks can also give rise to market risk or credit risk.

2.2.3.5 Meaning of Legal Risk

Legal Risk arises when there is default in the compliance of rules and regulations, when there is impact of changes in the tax laws on the market value of a position, when a third party file a legal suit for financial losses etc.

2.2.3.6 Meaning of Human Factor Risk

Human Factor Risk arises due to the intentional or unintentional errors committed by the office bearers in the administrative works. For ex, entering the wrong figures, pressing a wrong button on the computers, destruction of a file etc.

2.2.4 Statistical Concepts

The various statistical concepts used in the study for the purpose of measuring Value at Risk are summarised below:

2.2.4.1 Mean

Mean or simply called as average is one of the popular measures of central tendency. Mean is arrived at by dividing the sum of all the observations in a sample or population by the number of observations. Symbolically it is given as follows:

\[
\bar{X} = \frac{\sum X}{N}
\]
Where:

\( \overline{X} \) (Sometimes called the X-bar) is the symbol for the mean.

\( \Sigma \) (The Greek letter sigma) is the symbol for summation.

\( X \) is the symbol for the scores.

\( N \) is the symbol for the number of scores.

### 2.2.4.2 Variance and Standard Deviation

Variance and standard deviations are based on squared deviations from the mean.

‘The arithmetic mean of the squared deviation from the mean’. The variance is non-negative and is zero only if all the observations are the same. The square root of the variance is standard deviation.

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}
\]

Standard deviation and variance are the standard and widely used measures of dispersion. It indicates to what extent the observations are deviated from the average. The standard deviation is considered to be the basic measure for calculating risk.

### 2.2.4.3 Skewness

Two important elements of skewness are

- To compare the length of two tails of the distribution
- To measure the symmetry of the asset returns around the mean.
The distribution is asymmetrical if it is impacted more by negative outliers than positive outliers. As such the skewness indicate how outliers events influence the shape of the distribution. The distribution can be negatively skewed or positively skewed. If the tails tail falls to the far left of the distribution, it is termed as negatively skewed distribution. If the tails tail falls to the far right of the distribution, it is termed as positively skewed distribution.

### 2.2.4.4 Kurtosis

The fatness of a tail of distribution is defined as kurtosis. Kurtosis describes the risk of the distribution using outlier events. Outlier events also called as ‘black swans’, indicate they are far away from average returns.

![Figure 2(C): Skewness](image)

![Figure 2(D): Kurtosis](image)
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Leptokurtic are fat tails distribution, which implies highly peaked (returns are high) and risks are reflected in tails. In case of platykurtic, there are less number of observations in the tail. Mesokurtic indicate perfectly normal shape curve.

2.2.4.5 Jarque Bera Test

The Jarque Bera test is used to check the null hypothesis whether the financial-asset returns are normally distributed or not. The test is based on the assumption that skewness and kurtosis is equal to zero. Following is the formula for calculating JB test:

\[ JB = \frac{n}{6} \left[ (Skew x)^2 + \left( \frac{Kurt x}{4} \right)^2 \right] \]

2.2.4.6 Covariance & Correlation

Correlation and covariance helps to determine whether there exists some sort of association between the variables. It is demoted by ‘r’. The value of r ranges between -1 to +1. If the value or r is near to -1, it indicates the two variables are inversely related. If the value of r is near to +1, it indicates high perfect relationship between the variables.

2.2.4.7 Unit root Testing

This is also called as stationary test. Unit root in a data series is a problem which indicates that data is non-stationary. It is a necessary condition that financial series to be stationary before applying any time series model. The existence of unit root in the data can be tested through popular Augmented Dickey Fuller test or Philips Pherron Test. If the data are not stationary the series should be converted into first difference or lognormal returns of the series has to calculate before applying any models.
2.2.4.8 Normal Q-Q Plots

Q-Q plots are the quantile-quantile plots. It is a graphical method to check whether a data set follows a theoretical distribution. The straight line in the Q-Q plot is considered as a standard line. If the data points follow the straight line it is said the data is normally distributed. If the data points consistently appear above-below-above the line pattern, then the data is said to be left skewed. If the data points show ‘below-above-below’ the line pattern then the data is right skewed. If the pattern follows ‘above-below-above-below’ the line pattern than the data is leptokurtic in nature. On the other hand, if the data points are consistently ‘below-above-below-above’ the line pattern, then they are platykurtic in nature.

2.2.5 Meaning of Value at Risk

Value-at-Risk can be either expressed in number or a percentage. It depends basically on two important components namely the holding period and the confident interval. It states the maximum expected loss of an investor for an investment in a portfolio or an instrument, at a certain confidence interval and for a holding period.
2.2.6 Historical Simulation (HS)

HS VaR was introduced by Boudoukh et al. (1998)\(^5\) and Barone-Adesi et al (1998, 1999)\(^6,7\). This model is based on the assumption that ‘tomorrow’s portfolio returns, \(R_{PF,t+1}\), is well estimated by the empirical distribution of the past \(m\) observations, \(\{R_{PF,t+1-r}\}_{t=1}^{m}\)’ (Carol, 2008)\(^8\). As such the HS approach assumes that past is the good indicator of future (Das, 1998)\(^9\) and therefore the model values the current prices of the individual assets or the portfolio based on the historical prices. Perignon and Smith (2006)\(^10\) find through their results that ‘64.9% of firms that disclosed their methodology, 73% reported the use of Historical Simulation method for calculating VaR’ (Carol, 2008)\(^11\).

The VaR with coverage rate is calculated as follows:

\[
VaR_{t+1}^p = -\text{percentile}\left(\{R_{PF,t+1-r}\}_{t=1}^{m}, 100p\right)
\]

Methodology of Historical Simulation:

1. Select a sample of actual daily risk factor changes over a given period of time, say 250 days (i.e one year’s worth of trading days).
2. Apply those daily changes to the current value of the risk factors and revalue the current portfolio as many times as the number of days in the historical sample.
3. Construct the histogram of portfolio values and identity the VaR that isolates the first percentile of the distribution in the left-hand tail, assuming VaR is derived at the 99% confidence level.
The main advantages of popularity of the Historical Simulations VaR are as follows:

1. This method is very simple and easy to implement.

2. Under this method it is not necessary to make any assumption of distributions’ form of the risk factor returns.

3. This method is applicable to both linear and non-linear portfolios.

4. By relying on actual prices, the method allows non-linearities and non-normal distributions. Full valuation is obtained in the simplest fashion: from historical data. The method captures gamma, vega risk and correlations. It does not rely on specific assumptions about valuation models or the underlying stochastic structure of the market.

5. The method also directly deals with the choice of horizon for measuring VaR. Returns are simply measured over intervals that correspond to the length of the horizon.

6. This method also short-circuits the need to estimate a covariance matrix. This simplifies the computations in cases of portfolios with a large number of assets and short sample periods. All that is needed is the time series of the aggregate portfolio return.

7. This method can account for the fat tails and because it does not rely on valuation models, is prone to model risk.

8. The model is robust and intuitive and as such, is perhaps the most widely used method to compute VaR.
Historical Simulation suffers from the following setbacks:

Firstly, it is difficult to apply historical VaR to risk assessments with a horizon longer than a few days due to data limitations.

Secondly, Historical Simulation VaR estimation is based on the distribution of daily portfolio returns (or P&L) and then scale the 1-day VaR estimate to an h-day horizon. But finding an appropriate scaling rule for historical VaR is not easy.

Thirdly, the main drawback of historical simulation is its complete independence on a particular set of historical data – and thus on the idiosyncrasies of this data set. The underlying assumption is that the past, as captured in the historical data set, is a reliable representation of the future. This implicitly presumes that the market events embedded in the data set will be reproduced in the months to come.

Fourthly, Historical Simulation may lead to a distorted assessment of the risk if we employ the technique regardless of any structural changes anticipated in the market – such as the introduction of the Euro at the beginning of 1999.

Fifthly, another practical limitation of Historical Simulation is data availability. One year of data corresponds to only 250 data points (trading days) on average, i.e 250 scenarios. By contrast Monte Carlo Simulation usually involves at least 10,000 simulations (i.e. scenarios). Employing small samples of historical data inevitably leaves gap in the distributions of the risk factors and tends to under-represent the tails of the distributions, i.e., the occurrences of unlikely but extreme events.

Sixth, the serious drawback of HS is seen when it comes to the calculation of VaR at 10-day horizon rather than the next day. Ideally 10 day VaR should be calculated from 10 day non-overlapping past returns, which would entail coming up with 10 times as many past daily returns, which is often not feasible. As such HS method is not considered as a dynamic
No doubt, we multiply 1-day VaR calculated by HS method by square root of 10 to get 10-day VaR. This is possible under the assumption of normality, which the HS method does not take into account.

2.2.7 Historical Simulation – Volatility Adjustment

Historical Simulation requires large historical data. But one need to be extra cautious while using the data of a longer duration since the market conditions changes over a period of time. Particularly, equity markets and currency markets tend to display the period of stability, large swings and dynamic behaviour with frequent changes in the prices etc, making the market participants difficult in predicting the movements and taking appropriate investment decisions. Hence arises the necessity to incorporate the volatility element in the Historical Simulation method using the GARCH parameters.

*Duffie and Pan (1997)* and *Hull and White (1998)* suggested a volatility weighting method for historical VaR using asymmetric GARCH model. The methodology is designed to weight returns in such a way that we adjust their volatility to the current volatility. To do this we must obtain a time series of volatility estimates for the historical sample of portfolio returns.

Following is the step for calculating the volatility adjusted Historical Simulation VaR using GARCH estimates.

- Denote the time series of unadjusted historical portfolio returns by \( \{ r_t \}_{t=1}^T \)
- Denote the time series of the statistical (e.g. GARCH or EWMA) volatility of the returns by \( \{ \tilde{\sigma}_t \}_{t=1}^T \), where T is the time at the end of the sample, when the VaR is estimated.
- The return at every time \( t < T \) is multiplied by the volatility estimated at time \( T \) and divided by the volatility estimated at time \( t \). That is, the volatility adjusted returns series is

\[
\hat{r}_{t,T} = \left( \frac{\hat{\sigma}_T}{\hat{\sigma}_t} \right) r_t
\]

where \( T \) is fixed but \( t \) varies over the sample, i.e. \( \{t=1,...,T\} \). A time-varying estimate of the volatility of the series, based on the same model that was used to obtain \( \hat{\sigma}_t \), should be constant and equal to \( \hat{\sigma}_T \) i.e. the conditional volatility at the time the VaR is estimated.

2.2.8 Generalised Auto Regressive Conditional Heteroscedasticity

GARCH model was developed by Bollerslev in 1986. GARCH models comprise of three main components: (a) long run mean of Variance, (b) previous market returns and (c) previous day volatility. Long run mean of variance is the mean reversion purpose which imply that both high and low prices in the long run tend to revert back or converge to long run mean. As per the symmetric normal GARCH the conditional variance are assumed to be dynamic in behaviour. Following is the conditional variance equation:

\[
\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad \varepsilon_t \mid I_{t-1} \sim N(0, \sigma_t^2)
\]

The GARCH conditional volatility is the annualized square root of conditional variance. The conditional variance and volatility are conditional on the information set. \( \varepsilon_t \) denotes the market shock or unexpected return. (Alexander, 2008)$^{14}$

The long term volatility

The long term or unconditional variance (also called long term volatility) found by substituting \( \sigma_t^2 = \sigma_{t-1}^2 = \bar{\sigma}^2 \) into the GARCH conditional variance equation. For instance, for the symmetric normal GARCH we use the fact that \( E(\varepsilon_{t-1}^2) = \sigma_{t-1}^2 \) and then put \( \sigma_t^2 = \sigma_{t-1}^2 = \bar{\sigma}^2 \) to obtain
\[ \hat{\sigma}^2 = \frac{\omega}{1 - (\alpha + \beta)} \]

\(\alpha\) measures the reaction of conditional volatility to market shocks. Low \(\alpha\) indicate that the volatility is not sensitive to market events. \(\beta\) measures the persistence in conditional volatility irrespective of changes in the market. The sum of \(\alpha\) & \(\beta\) determines the rate of convergence of conditional volatility to the long term average level. (Alexander, 2008)

2.2.9 Asymmetric GARCH

The A-GARCH model is the extension of GARCH with an additional term added to account for possible asymmetries. There is a single extra ‘leverage’ parameter denoted by \(\lambda\). The asymmetric response is rewritten to specifically augment the volatility response from only the negative market shocks:

\[ \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \lambda \mathbf{1}_{(\varepsilon_{t-1} < 0)} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]

where the indicator function \(\mathbf{1}_{(\varepsilon < 0)} = 1\) if \(\varepsilon < 0\), and 0 otherwise. Parameter estimation is based on the usual normal GARCH likelihood function, where again \(\sigma^2\) depends on the extra parameter \(\lambda\). (Alexander, 2008)

2.2.10 Monte Carlo Simulation VaR (MCS)

Monte Carlo Simulation is one of the popular and widely used VaR model in the financial markets. It has abundant applications in the area of finance. ‘It is often used as a method of last resort when analytic solutions do not exist, or when other numerical methods fail’ (Carol, 2008).

Monte Carlo Simulations cover a wide range of possible values in financial variables and fully account for correlations. MC Simulation method proceed follows two steps:

**First**, a stochastic or random process and parameters (risk and correlation obtained from historical / option data) are specified for financial variables by the risk managers.
Second, fictitious price paths are developed for a time horizon considered and portfolio is marked-to-market. These randomly generated price paths are then used to compile a distribution of returns from which VaR is estimated.

The method is summarized in the following figure.

**Figure 2(E): Monte Carlo Simulation Methodology**

(Source: Jorion, 2002)

The calculation of VaR under MCS is similar to the Historical Simulation method. The only difference that can be noted is that in case of MCS the hypothetical changes in prices for asset are created by random draws from a pre-specified stochastic process instead of sampled from historical data.

**Advantages:**

- Monte Carlo Analysis is by far the most powerful and flexible technique to compute VaR.
- MCS method take into account for a wide range of exposures and risks, including nonlinear price risk, volatility risk and even model risk.
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- MCS method is flexible enough to incorporate time variation in volatility, fat tails and extreme scenarios.
- Simulations generate the entire pdf, not just one quantile, and can be used to examine, for instances, the expected loss beyond a particular VaR.
- MCS applications can also be used for a longer holding periods which helps in calculating credit risk.
- Monte Carlo Simulation also can incorporate the passage of time, which will create structural changes in the portfolio. This includes the time decay of options; the daily settlement of fixed, floating, or contractually specified cash flows; or the effect of pre-specified trading or hedging strategies. These effects are especially important as the time horizon lengthens, which is the case for the measurement of credit risk.

Problems:

The biggest drawback of this method is its computational time. If 1000 sample paths are generated with a portfolio of 1000 assets, the total number of valuations amounts to 1 million. In addition, if the valuation of assets on the target date involves itself a simulation, the method requires a simulation within a simulation. This quickly becomes too onerous to implement on a frequent basis.

This method is the most expensive to implement in terms of systems infrastructure and intellectual development. The Monte Carlo simulation method is relatively onerous to develop from scratch, despite rapidly falling prices for hardware.

Another potential weakness of the method is model risk. Monte Carlo simulation relies on specific stochastic processes for the underlying risk factors as well as pricing models for securities such as options or mortgages. Therefore, it is subject to the risk that the models
are wrong. To check if the results are robust to changes in the model, simulation results should be complemented by some sensitivity analysis.

Finally, VaR estimates from Monte Carlo Simulation are subject to sampling variation, which is due to the limited number of replications. Consider, for instance, a case where risk factors are jointly normal and all payoffs linear. The delta-normal method will then provide the correct measure of VaR, in one easy step. Monte Carlo Simulations based on the same covariance matrix will give only an approximation, albeit increasingly good as the number of replications increases.

Overall, this method is probably the most comprehensive approach to measuring market risk if modelling is done correctly. To some extent, the method can even handle credit risks.

2.2.11 Exponentially Weighted Moving Average VaR
Historical Simulation VaR estimation is based on the equally weighted unconditional variance estimate. The VaR based on equally weighted averages produce unconditional parameter. It is the average value of the past conditional parameter over the past period data. Hence they are most suitable for calculating VaR for longer time horizon. What is to be noted is that, average volatility in the market are reflected for a longer period. The VaR estimates based on the equally weighted moving average is able to forecast only the volatility for a longer period of time. While the short period volatility estimates are not possible through equally weighted moving average. For predicting the short term volatility, exponentially weighted moving average (EWMA) method is used. It provides a time varying volatility estimate for short term VaR calculations. EWMA method assumes that the risk factors are i.i.d. They use the square-root-of-time rule to scale VaR over different risk horizons.
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EWMA method is very responsive to the market crashes. This is because, the model gives more weightage to the recent events and less weightage to the past distant events in calculating the conditional variances.

EWMA method is based on the assumption that the prices of the assets do not remain the same and changes over a period of time. And hence the model assumes that volatility in the financial-asset returns fluctuates very often. Thus EWMA is considered superior relative to historical simulation model in estimating market risk for a VaR model. JP Morgan uses EWMA model for estimating VaR.

The volatility is modelled using EWMA through the following equation:

\[
\sigma = \sqrt{(1 - \lambda) \sum_{t=1}^{n} \lambda^t (X_t - \mu)^2}
\]

Where \( \lambda \) is the exponential factor

\( n \) is the number of days.

\( \mu \) is the mean value of the distribution, which is normally assumed to be zero for daily VaR.

The equation can be stated for exponential weighted volatility:

\[
\sigma = \sqrt{\lambda \sigma_{t-1}^2 + (1 - \lambda)X_t^2}
\]

This form of the equation directly compares with GARCH model. The very decisive element for assessing the validity of the model in case of EWMA is the chosen value factor or the decay factor. JP Morgan`s RiskMetrics model uses 0.94 and 0.97 as the decay factor for daily and monthly volatility estimations respectively.
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The necessary number of days for calculating EWMA VaR can be calculated using the following formula: (Best, 1999:70)\(^{19}\).

\[
\text{Necessary data number} = \frac{\log(\text{required accuracy})}{\log(\text{factor value})}
\]

For asset \(i\) at time \(t\), exponential weighted volatility can be written as follows:

\[
\sigma_{i,t} = \sqrt{(1 - \lambda) \sum_{j=0}^{\infty} \lambda^j r_{t-j}^2}
\]

In equation \(\lambda\) = exponential factor, 

\[i\ t\ r, = \text{represent logarithmic return of asset } i \text{ at time } t\]

\[i\ t\ r, = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \text{ formula.}\]

If there are loads of data for past years, the data chosen for the model should be selective. The criteria given by RiskMetrics is 99% of the all available data. This can be formulated as stated \(1/(1 - \lambda)\). Here \(n\) number of return data’s serial weight is equal to \((1 - \lambda)/(1 - \lambda) n\). Thus if 99% of the weight wants to be included, the number of data should be calculated as

\[n = \frac{\ln(0.01)}{\ln(\lambda)} \text{ formula. Effective data number for forecasting volatility is based on exponential factor numbers. As seen on the formula, high exponential factor number means more data requirements.}\]

2.2.12 Conditional VaR

Conditional VaR or CVaR is also known as Expected Tail Loss (ETL), Expected Shortfall (ES), Average Value at Risk (AVaR). It is used to assess the market risk and credit risk. The ETL is the expected value of our losses, \(L\), if we get a loss in excess of VaR:\(^{18}\)

\[
\text{ETL} = E [L \mid L > \text{VaR}]
\]
The VaR expresses the maximum loss one can expect, provided no bad (i.e tail) event occurs. ETL tells us the amount of loss one would incur if the VaR estimated is exceeded.

‘The underlying parameters and distributional assumptions of both Value at Risk and ETL are the same. Both are usually calculated at 95% confidence level and 1-day holding period. P/L s assumed to follow standard normal distribution with mean ‘0’ and standard deviation ‘1’ (Dowd, 2002).’

Dowd (2002) gives the following reason as to why ETL can be a better measure than VaR:

- VaR just tells the expected loss. While the ETL tells us what can be the expected loss in bad (i.e tail) states or if the VaR is exceeded. ‘It gives an idea of how bad bad might be’.
- The risk-expected return decision is applicable under all circumstances under ETL. Whilst this is not the case with VaR. Further, if risks are ranked through second-order stochastic dominance rule the ETL based risk-expected return decision rule is more consistent with expected utility maximisation. In case of VaR if risks are ranked through first-order stochastic dominance rule the VaR based risk-expected return decision rule is more consistent with expected utility maximisation (Yoshiba and Yamai, 2001).
- VaR does not satisfy sub-additivity, while ETL does satisfy.
- VaR now and then discourages risk diversification, while ETL encourages risk diversification.
- ETL being sub-additivity suggests that risk of a portfolio will be convex to its origin. ‘Convexity ensures that portfolio optimisation problems using ETL measures, unlike ones that use VaR measures, will always have a unique well-behaved optimum’ (Uryasev, 2000, Pflug, 2000, Acerbi and Tasche, 2001). This is
reiterated by Rockafeller and Uryasev (2000) who validate that ‘convexity guarantees that portfolio optimisation problems with ETL risk measures can be handled very efficiently using linear programming techniques.’

2.2.13 Variance Covariance Approach:
JP Morgan’s ‘RiskMetrics’ system developed the Variance CoVariance (CVC) approach. This model is an extension of Markowitz concept of portfolio risk (Jorion, 2001). The model consider the moving average concepts in calculating the VaR estimates at a certain confidence interval. This model assumes that the financial-asset returns are normally distributed and hence follow Gaussian probability density function. Hence the returns are described by mean, standard deviation or the variance and the correlation between various market returns.

Correlation between various market returns is given by variance-covariance matrix. ‘The CVC method assumes that correlation between risk factor remains same’ (Learning Curve, 2003). Correlation measures the degree to which the variables are related. For a given portfolio, risk can be reduced provided the assets are positively correlated due to diversification. Diversification in portfolio helps in reducing the total risk which will be less than the sum of the individual assets risks.

The assumption of normal distribution makes VCV approach to estimate VaR easily as skewness and kurtosis are not accounted for. This assumption is a serious limitation of the model as the empirical financial-asset returns exhibit excess kurtosis.

2.2.14 Backtesting
Various Value at Risk models are used by financial markets for forecasting market risks. The VaR estimates help the risk managers to monitor and reduce risks faced by the markets. But each VaR model generates varying figures of VaR. This poses difficulty for the risk managers in deciding the reliability and accuracy of the VaR estimates. As such there arises
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the need for checking the accuracy of the VaR estimates. The actual profits or loss are compared with the estimated profits or losses given by VaR models. If there exists no difference between the estimated and realised profits or losses, the model is said to be useful in predicting market risk. This procedure of testing the accuracy of VaR estimates is called as Backtesting. This sections provides different backtesting methods for validating VaR model. The study employs Kupiec’s (1995) proportion of failures-test, Christoffersen’s (1998) tests of independence of exceedances and the conditional coverage test (Haas, 2001) which is a combination of unconditional coverage and test of independence of exceedances.

2.2.14.1 Unconditional Coverage Test

Unconditional coverage test, introduced by Kupiec (1995), is a very common and simple method of backtesting. This test assumes that indicator functions follow an i.i.d Bernoulli process has a constant ‘success’ probability equal to the significance level of VaR, $\alpha$. Following are the steps of this method:

- Calculate VaR estimate at a chosen confidence interval and holding period
- Compare the VaR estimate with the daily market returns to arrive at profit or losses
- If the market returns exceeds the VaR estimate, it is considered as the exceedance or violation
- Count the number of exceptions for the entire data
- If the number of exceptions are more than the chosen significance level, we conclude that model under estimates VaR

The following null hypothesis is of the unconditional coverage test:

$H_0$: VaR model is accurate or the number of exceptions does not exceed the significance level.
The test statistic is a likelihood ratio statistic given by the following formula:

\[ LR_{uc} = \frac{\pi_{\text{exp}}^{n_1} (1 - \pi_{\text{exp}})^{n_0}}{\pi_{\text{obs}}^{n_1} (1 - \pi_{\text{obs}})^{n_0}} \]

Where,

\( \pi_{\text{exp}} \) is the expected proportion of exceedances = \( \alpha \)

\( \pi_{\text{obs}} \) is the observed proportion of exceedances = \( \frac{n_1}{n} \)

\( n_1 \) is the observed number of exceedances

\( n_0 = n - n_1 \), where \( n \) is the same sample size of the backtest.

\( n_0 \) is the number of returns with indicator 0 (we call these returns the good returns.)

The asymptotic distribution of \(-2 \ln LR_{uc}\) is chi-squared with one degree of freedom.

\[ \ln (LR_{uc}) = n_1 \ln(\pi_{\text{exp}}) + n_0 \ln(1 - \pi_{\text{exp}}) - n_1 \ln(\pi_{\text{obs}}) - n_0 \ln(1 - \pi_{\text{obs}}) \]

However, Jorion (2001) \(^{32}\) says that there are possibility of committing Type I error and Type II errors. Type I errors refers to rejecting the correct model and Type II error refers to accepting the incorrect model. Dowd (2006) \(^{33}\) say that the confidence level for any VaR model should be chosen to balance between Type I error and Type II errors.

### 2.2.14.2 Test of Independence of Exceedances:

It is enough to know whether the VaR model is accurate or not. The risk managers focus on the clustering of exceptions or exceedances. A good VaR model is assumed to spread the occurrences of exception evenly throughout the period. A good VaR model is capable of responding to volatility changes and indicate that the exceedances are independent of each other. In other words, a sequence of exceptions indicate that VaR model fails to respond to
changing market conditions. Even if the VaR model passes the unconditional coverage test, it is not necessary for the model to pass the test of independence.

‘A test for independence of exceedances is based on the formalisations of the notion that when exceedances are not independent the probability of an exceedance tomorrow, given there has been an exceedance today, is no longer equal to \( \alpha \).’ (Carol, 2008)  

The null hypothesis of test of independence is as follows:

\( H_0 \): The exceedances are identical and independent of each other.

Let \( n_i \) be the observed number of exceedances and number \( n_0 = n - n_1 \) be the number of good returns. Further, define \( n_{ij} \) to be the number of returns with indicator value \( i \) followed by indicator value \( j \), i.e. \( n_{00} \) is the number of times a good return is followed by another good return, \( n_{01} \) the number of times a good return is followed by an exceedance, \( n_{10} \) the number of times an exceedance is followed by a good return and \( n_{11} \) the number of times an exceedance is followed by another exceedance. So \( n_1 = n_{11} + n_{01} \) and \( n_0 = n_{10} + n_{00} \)

\[
\pi_{01} = \frac{n_{01}}{n_{00} + n_{01}} \quad \text{and} \quad \pi_{11} = \frac{n_{11}}{n_{10} + n_{11}}
\]

\( \pi_{01} \) is the proportion of exceedances, given that the last return was a good return and \( \pi_{11} \) is the proportion of exceedances, given that the last return was an exceedance. Now we can state the independence test statistic, derived by Christofferson (1998) as

\[
LR_{ind} = \frac{\pi_{obs}^{n_{11}} (1 - \pi_{obs})^{n_0}}{\pi_{01}^{n_{01}} (1 - \pi_{01})^{n_{00}} \pi_{11}^{n_{11}} (1 - \pi_{11})^{n_{10}}}
\]
2.2.14.3 Conditional Coverage Test

This test is a combination of both unconditional coverage test and test of independence. It is given by the following formula:

\[ LR_{cc} = \frac{\pi_{exp}^{n_1}(1 - \pi_{exp})^{n_0}}{\pi_{01}^{n_{01}}(1 - \pi_{01})^{n_{00}}\pi_{11}^{n_{11}}(1 - \pi_{11})^{n_{10}}} \]

The asymptotic distribution of \(-2 \ln LR_{cc}\) is chi squared with two degrees of freedom. On comparing the three test statistics it is clear that

\[ LR_{cc} = LR_{uc} \times LR_{ind} \]

\[-2 \ln LR_{cc} = -2 \ln LR_{uc} - 2 \ln LR_{ind} \]

Besides the above, there are various other backtesting models available for testing the accuracy of the VaR estimates. *Christoffersen’s Interval Forecast Test (1998)* \(^{36}\), Mixed Kupiec-Test, the Kupiec’s TUFF-test, Backtesting Based on Loss Function (*Lopez - 1998, 1999*) \(^{37,38}\), Backtests on Multiple VaR Levels by *Crnkovic and Drachman (1997)* \(^{39}\). However the choice of the backtesting model depends on the tradeoff approach to balance between Type I and Type II errors.
Chapter 2

Theoretical Background

2.3 REFERENCES


4. [www.bis.org](http://www.bis.org)


