2.1 Previous and Related Studies

Over the last forty years, a significant body of research has led to the development of numerical cable models which can assist in the accurate positioning and navigation of long (Kilometer scale), submerged towed systems [Eric, 1987]. The first such model was based on lumped mass spring system (LMSM). Paul and Soler [Paul, et.al, 1972] presented a two dimensional formulation in which, the inertial forces were considered insignificant. In earlier studies, the LMSM was devised to simulate the motion of towed systems during relatively steady tow-ship manoeuvres in which inertial effects are negligible. This quasi-static implementation was used to solve for the motion of a towed system during straight tows at a steady speed. Both Chapman [Chapman, 1984] and Sanders [Sanders, 1982] presented three-dimensional quasi-static lumped parameter models. Chapman calculated the steady state profile of a tow-cable during straight tows and turns of varying diameters. This work showed that a towed system undergoes large transient motions as it enters and exits the turns. This emphasised the need for cable models to include inertial effects in order to accurately capture these transient motions.

Shan Haung [Shan, 1994] presented a LMSM which considers the inertial effects. In his method, the forces as well as the masses are lumped at node points and equation of motion
is formed based on the Newton-Euler theorem. The resulting equations are solved using finite difference method.

The main advantages of LMSM are the relative ease of dealing with the strong nonlinearities associated with the hydrodynamic loads, and the ability to assemble a compact numerical model from the linear elements. As reported by Shan Huang [Shan, 1994] through his research in single point towing systems, representation of cable as discrete mass and linear spring elements, results in reduced numerical oscillations in the solution space because the LMSM work as a low pass filter which damp out spurious high frequency numerical oscillations. For these reasons the LMSM formed the basis for a significant body of towed systems research. Such lumped mass implementations have been presented by Vaz and Patel [Vaz, et.al, 1995] and Driscoll et al. [Driscoll et.al, 1996]. Both Driscoll and Huang [Shan, 1994] accounted for the elasticity of the tow- cable by using linear spring elements. In both cases, the spring stiffness was calculated from the material properties of the tow-cable.

An alternative approach that has been developed in parallel was segmental one. It was developed by Ablow and Schechter [Ablow, et.al, 1983]. In this model, cable is treated as a long thin flexible circular cylinder in arbitrary motion. It was assumed that, the dynamic motion of the cable is determined by gravity, hydrodynamic loading and inertial forces. Then the governing equations are formulated based on a local coordinate system, which moves with the cable. The equations are discretized using finite difference method, subsequently solved using an implicit scheme, which is centered at time and space. This formulation worked well for the simulation of the conventional towing system but numerical instabilities are found when it tries to apply for the two part towing system.

Most of the approaches described above, are mainly devised to analyze the single point towing system for predicting real-time estimation of depth and trail of the towed body. Very few studies have been reported for analysing the ship induced disturbances on the dynamics of the tow-fish. An exception to this is the work done by Chapman [Chapman, 1984]. He has devised a two-dimensional numeral model to study the response of a neutrally buoyant fish, which is towed using a fared cable and is subjected to ship induced disturbances. He identified two dominant modes of transmission of disturbances down to the cable, namely longitudinal as well
as transverse oscillations. It has reported that transverse modes of the cable are considerably damped, while the longitudinal mode prevails [Chapman, 1984]. This may be due to larger normal drag exerted by the fluid (second order damping force) compared to tangential one for the case of slender bodies like marine cables. Also, it has found that overall amplitude of the motion of the tow-fish is proportional to the sine of the angle made by top of the cable with the horizontal, provided that combined natural frequencies of the ship and cable system does not coincide with that of external disturbances. However, the study was limited to single point towing. Further, the tow cable not only transmits but also generates disturbances [Samuel, et.al 1993]. These disturbances depend on the cable used, mass and drag of the depressor and mass and drag of the tow-fish. For example, the cable may shed vortices causing it to strum. This strumming can be removed with the use of fairing. Another effect is kiting which can be either ship-induced or tow-fish-induced. Ship-induced kiting causes sway in the tow-fish whereas tow-fish-induced kiting causes sway and roll in the tow-fish.

Recently, Wu [Wu, et.al, 2000] developed a three dimensional numerical model to simulate dynamic performance of the two-part towing system considering the ship disturbance as the input. It was found that the degrees of freedom which lie on the central vertical plane of the towed body such as pitch and heave were reduced considerably rather than the out-of-plane responses like yaw and roll. Even though, the model was successful in simulating the dynamic performance of the two-part towing system, numerical oscillations were found in the solution space. Further, mathematical modeling of tow system and solution methodology devised in this particular study was much complicated compared to LMSM approach.

Considering all these factors it was decided to investigate the dynamic performance of the two-part towing system with ship induced motion as the input disturbance. Also, Lumped mass formulation was selected considering simplicity of the formulation and added numerical stability compared to other schemes like Ablow and Schechter model [Ablow, 1983] and Finite element model. A two dimensional model was attempted considering the fact that only in-plane (vertical) responses are benefited maximum by two-part towing system. Only longitudinal mode of cable oscillation, for the transfer of periodic disturbances from the tow-
point to the towed body, was considered for the study. The formulation of the towed vehicle was based on the Kirchoff’s equation of motion for rigid bodies [Thor, 1994].

2.2. Time/Space Discretisation of Equation of Motion

For all spatial discretization methods, the resulting equations are typically written as a non-linear matrix equation known as the semi-discrete equation of motion, because the time derivatives of the vector of dependent variables are left as continuous functions. The equations of motion for LMSM are most often presented in matrix form as a system of second-order ordinary differential equations – the semi-discrete equation of motion. Most temporal integration schemes in use today have their roots in the method developed by Buckham [Buckham, *et al.*, 1999]. Newmark [Hughes, 2004]. Hughes and Belytschko [Ted Belytschko, *et al.*, 1983] provide a summary of the development of these types of methods in the context of finite element solution of structural dynamic problems. The methods typically employ temporal finite differences, with a variety of different schemes used to interpolate the solution over the time step. Thomas [Thomas, 1994] studied the three “classic” methods (Newmark, Houbolt, and Wilson-θ) and their applicability to the mooring dynamics problem. He concluded that Houbolt was the best choice.

In addition to Newmark and its variants which are popularly employed with finite element based models, researchers in the cable dynamics field have devised a variety of different schemes for the temporal integration problem. Chiou and Leonard [Chiou, 1979] used simple backward finite differences. Sun et al. [Sun Y, 1998] used the generalized trapezoidal rule which is a first-order variant of the Newmark method. Sanders [Sanders V, 1988] used a computationally expensive but fourth-order accurate Runge-Kutta procedure.

In spite of all these developments, none of the previous studies related to two-part towing, considered the implementation of time integration procedures other than, the classical ‘BOX’ type scheme [Ablow, 1983]. In the BOX scheme, the governing equations are discretised on the half-grid point in both space and time. This method was first employed for the solution of tow cable dynamics by Ablow and Schechter [Ablow, 1983]. The BOX method has got
unconditional stability for the case of linear problems, but was subjected to phenomenon known as Crank-Nicolson noise [Gobat, 2000] in the solution space. Therefore, the present study considers the use of advanced time integration procedures such as HHT-α scheme and other classical variants like Newmark, Houbolt and Euler.

2.3 Parallel Computations

Due to the nonlinear characteristics of mathematical model (due to the presence of nonlinear drag forces) of underwater towing problems (see equation 3.3), the solver is heavily loaded, and it takes very large computational time even, with very few numbers of discretised nodes of the cables. As the serial programming has limitations in terms of computational speed, parallel computational approaches are adopted in this particular study. While, application of parallel computing is a common sight in numerical codes for the solution of fluid dynamic equations, most of the codes for the towing simulation are serially programmed. The present study employs shared memory based parallel computing techniques to solve the model equations of towing problems.

2.4 Verification of Numerical Model

Finally the developed numerical model needs to be verified by experimental methods. Early efforts of cable model validation involved experiments with very short cables (less than 5 m) with one end fixed, and either an anchor or a buoy attached to the other end. Chiou and Leonard [Chiou, 1979] used data from a submerged buoy given an initial disturbance to validate their cable model. Huston and Kamman [Kamman, 1999] validated their finite segment cable model using experimental data from a hanging chain and a submerged buoy. Delmer and Stephens [Delmer, 1983] and Palo et al. [Palo, 1995] used experimental ‘anchor last’ data to verify their respective lumped-mass cable models. Yamamoto et al. [Yamamoto, 1996] validated their lumped-mass cable model using experimental data in the open ocean to depths of over 5000 m. Chapman [Chapman, 1984] take the validation of the cable model one step further by validating a complete towed system. This system includes a tow point with prescribed motion and a passive towed body attached to the lower end of the cable. The dynamics model of the system showed good agreement with experimental tow tank data. Seto
et al. [Seto, 1999] went beyond this by including an actively controlled tow-fish, as well as the dynamics of the towing vehicle into their simulation. The simulation was then validated against experimental data.

While, a lot of previous studies, put efforts on the experimental verification of single point towing process, very few such initiatives are reported in the case of two-part towing. For the verification of the numerical model for the two-part towing system, Wu [Wu, 2001] devised an experiment which simulates the motion of the tow ship in a sinusoidal wave field and the corresponding heave and pitch responses of the towed vehicle were measured. The same experimental approach has been used in this particular study to validate the developed numerical model. The experiments were performed at offshore laboratory NIT, Kozhikode which is equipped with a model ship towing tank facility.