CHAPTER-I
GENERAL INTRODUCTION AND REVIEW OF LITERATURE

1.1 Introduction to the Classical Risk Theory and Review of Literature

The entire thesis is concerned with Ruin theory and its applications in the domain of insurance. The basic foundation enabling the propagation of the application of Ruin theory to the domain of Insurance is the Classical Risk model and this chapter gives a brief review of literature and related concepts for the classical Risk theory and the Probability of Ruin.

Classical Risk model constitute the base of the Ruin Theory as applied to Actuarial Science. The main essence for the Classical Risk Model is a Cash flow which is a realization of a Surplus process in which the Insurer starts with an initial surplus $u$ and collects premium continuously at a constant rate of $c$ while the aggregate claims process follows a Compound Poisson Process. We shall be dealing with what is known as “insurance Risk” which means the risk for a non-life insurance such as motor-vehicle insurance, home and content insurance and travel insurance.

As we will be dealing with risk modeling, it needs to be noted that in the Actuarial domain, there are two uncertainties from the viewpoint of the Insurer reflected in terms of the number of claims at any particular instant of time and claim severity. They jointly constitute what is known as the Aggregate Claim model. When the claim arrival process is a homogeneous Poisson Process, it leads to a compound Poisson Process which was introduced by Lundberg (1909), Cramer (1930) and Cramer (1955). Some
of the good references for this topic includes Bulmann (1970), Gerber (1979), Bowers et al (1998) and Dickson (2005)

Classical Risk model is basically the Cramer –Lundberg model and various extensions of this model have been proposed. For instance, Dufresne and Gerber (1991) added a diffusion process to the original model and Huzak and Perman (2004) included a Levy Process with mean 0 and no positive jumps. Andersen (1957) generalized the claim number process to a renewal process that includes a Poisson process. Albrecher and Teugels (2006) considers the possible dependence between the interclaim time and claim size.

Main Research goals in the Classical Risk Model have been the evaluation of finite and infinite time ruin probabilities. Then emerged another themes of research associated with the classical risk theory in the form of the distribution of the time to ruin, the surplus before ruin and the deficit at ruin.

The Cramer Lundberg model is given by

$$U_t = u + ct - S_t, t \geq 0$$

where

$$S_t = \sum_{j=1}^{N_t} X_j \text{ if } N_t > 0$$

where \( \{N_t; t \geq 0\} \) is a homogenous Poisson Process with intensity parameter \( \lambda \) and \( \{X_j\}, j = 1,2,3 \ldots \) are positive random variables representing the claim severities and \( u \) is the initial surplus whereas \( c \) is the premium income per unit time.

Gerber and Shiu (1998, a) introduced a function called the discounted penalty function which is a function of the time to ruin, deficit at ruin and surplus prior to ruin and which unifies all the classical results as its particular cases. For example, the results of Gerber, Goovaerts and Kaas (1987), Dufresne and Gerber (1988a, b), Dickson (1992) and
Dickson and Dos Reis (1996) are obtained as its particular cases when the discounting factor is zero.

Lin and Willmot (1999) and Willmot and Lin (2000) have proposed a solution to the defective renewal equation satisfied by the discounted penalty function in terms of a compound Geometric tail. The exact and the approximate results which have been developed for the tails of the compound Geometric distributions can thus be explored to find analytical expressions and construct numerical algorithms for determining the distribution of the time to ruin, the surplus just prior to ruin and the deficit at the time of ruin. In Lin and Willmot (1999), the approach proposed allows one to express the joint and the marginal moments in terms of the compound geometric tails and to reveal recursive relationship between these moments.

Malinovskii (1998) derived the Laplace transform of non ruin (survival) probability as a function of finite time \( t \) when the claim severity distribution is exponential and the inter arrival time of claims has a general distribution \( K \). This case was further extended to the claim severity being distributed as Mixture of Exponential by Wang and Liu (2002). In both the cases, even under the consideration that the inter arrival time of the claim has special forms, the Laplace Transforms were difficult to be inverted. Dufresne (2001) derived the Laplace transform of the integral equation satisfied by the non-ruin probability under the Spare Andersen model thereby giving the Laplace transform of the probability of non ruin under this model. Lima et al (2002) illustrates the computation of the probability of ruin by the inversion of the Fourier and Laplace transform of the probability of ruin both in the classical model and when the inter arrival claim time is Erlang (2).
Dickson (1998) and Dickson and Hipp (1998, 2001) considered the case where the inter arrival time follows a Gamma ($2, \beta$) distribution and this was further complemented by Cheng and Tang (2003) who derived expressions for the moments of the surplus before ruin and deficit at ruin when the inter arrival time of claims follows a Erlang($2$) distribution. The above model with the extension of Erlang ($2$) to Erlang ($n$) was studied by Li and Garrido (2004) who also derived the $n^{th}$ order integro-differential equation satisfied by the expected discounted penalty function under this model.

Erlang($n$) was further extended to consider the case of inter arrival time of claim being distributed as the sum of $n$ independent exponential random variables with possible different means by Gerber and Shiu (2003 a,b) and Gerber and Shiu (2004).

We shall be primarily concerned with the continuous time ruin models. As the name suggests, the continuous time models examine the surplus continuously over time. In some of the continuous time models, explicit expressions for the various actuarial quantities of interest are available whereas in cases, where they are not available, one has to resort to some numerical algorithms yielding approximate solutions to them or be content with upper bounds to the quantities of interest. Since, the foundation of this work is the classical Risk model, we give a brief introduction to some of its main ingredients and how it leads to the formulation of the basic framework for the determination of the Probability of ruin.

1.2 Claim arrival process as a Poisson Process

A Poisson process $\{N_t: t \geq 0\}$ denotes the number of claims arriving on a portfolio of business in an interval of length $t$.

The number of claims process $\{N_t: t \geq 0\}$ is a Poisson process with rate $\lambda > 0$ if the following three conditions hold
(1) \( N_0 = 0 \)

(2) The process has stationary and independent increments.

(3) The number of claims in an interval of length \( t \) is Poisson distributed with mean \( \lambda t \). That is for all \( s, t > 0 \), we have

\[
p(N_{t+s} - N_s = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0,1,2,...
\]  

(1.1)

The implication of Stationary increment is that the number of claims in any interval of time depends just on the length of that interval irrespective of where the interval is located. Independent increment on the other hand implies that the number of claims arriving in an interval of time is independent of the number of claims in any other non-overlapping interval. Stationary and independent increments together imply that the process can be thought of intuitively as starting over at any point of time.

An associated property of the Poisson process is that the inter arrival time between two successive claims is exponentially distributed with mean \( 1/\lambda \). Because of the memoryless property of the exponential distribution, from a fixed point in time \( t_0 \geq 0 \), the time until the next claim occurs is also exponentially distributed with mean \( 1/\lambda \).

(1.3) The Continuous Time Problem:

As mentioned earlier, the Poisson process \( \{N_t: t \geq 0\} \) model the claim arrival pattern whereas the individual losses or the claim severities \( \{X_1, X_2, ..., X_n\} \) are considered to be independent and identically distributed as positive random variables independent of \( N_t \) each with probability density function \( f(x) \) and continuous distribution function \( F(x) \) and mean \( p_\lambda < \infty \). Also we consider \( E(X^k) = p_k, k = 1,2,3,... \).

If \( S_t \) denotes the aggregate loss in \( (0,t] \), the process \( \{S_t: t \geq 0\} \) is said to be a compound poisson process given by

\[ S_t = 0, if \ N_t = 0 \]
and

\[ S_t = \sum_{j=1}^{N_t} X_j \text{ if } N_t > 0 \]  

(1.2)

Because \( \{N_t: t \geq 0\} \) has stationary and independent increments and so does \( \{S_t: t \geq 0\} \)

and we have \( E(S_t) = E(N_t)E(X) = \lambda p_1 t \).

We assume that the premiums are paid continuously over time at the rate of “c” per unit time i.e. the premium paid up to time "t" is "ct". For mathematical simplicity, we assume that surplus earn no interest. Furthermore, we assume that the premiums are positively loaded i.e.

\[ ct > E(S_t), \text{ i.e. } c = (1 + \theta)\lambda p_1 \]

Where \( \theta > 0 \) is called the relative security loading or the premium loading factor.

Now for this continuous ruin model, the surplus process is defined as

\[ U_t = u + ct - S_t, t \geq 0 \]  

(1.3)

Where \( u = U_0 \) is the initial surplus.

The time point at which the surplus becomes negative for the first time is the time at which ruin is said to occur i.e., if the surplus process ever becomes negative, we say that the ruin has occurred or otherwise, the company survives.

Hence the infinite time survival probability is defined as

\[ \phi(u) = p(U_t \geq 0 \text{ for all } t \geq 0 | U_0 = u) \]  

(1.4)

And the infinite time ruin probability (or the probability of ultimate ruin) is defined as

\[ \psi(u) = 1 - \phi(u) \]  

(1.5)

Similarly, the finite time ruin probability is defined.

Let \( T_u \) denote the time to ruin from initial surplus \( u \) so that

\[ T_u = \inf\{t; U_T < 0\} \]  

(1.6)
and
\[ \psi(u) = P(T_u < \infty) = 1 - \phi(u) \]  
(1.7)

and
\[ \psi(u, t) = P(T_u \leq t). \]  
(1.8)

\( \psi(u) \) is known as the ultimate ruin probability whereas \( \psi(u, t) \) is the finite time ruin probability. For a detailed discussion on the classical Risk model and the probability of ruin see Grandell (1991), Panjer and Willmot (1992) Klugman et al (1998) and Asmussen (2000).

(1.4) The Adjustment coefficient and the Lundberg’s inequality:

Next, we will be concerned with a very important quantity in actuarial literature namely the adjustment coefficient. It is important because it is vital in the deduction of a very important inequality called the Lundberg’s inequality which gives an upper bound to \( \psi(u) \).

Let \( X \) be an arbitrary claim size random variable and \( t = k \) be the smallest positive solution to the equation
\[ 1 + (1 + \theta)p_t t = M_X(t) \]  
(1.9)

Where \( M_X(t) = E(e^{tX}) \) is the moment generating function (mgf) of the claim severity distribution. If such a value exists, it is called the adjustment coefficient. It needs to be noted that adjustment coefficient is defined only for those distributions which have a closed form expression for its moment generating function (m.g.f.)

For example, for the exponential distribution with parameter \( \beta \), the adjustment coefficient is the smallest positive solution of the equation
\[ 1 + (1 + \theta)\beta t = (1 - \beta t)^{-1} \]
For all the light tailed distributions, where the (m.g.f.) exists in closed form, it is possible to find an explicit expression for the adjustment coefficient. But for a general claim amount distribution, it is not possible to explicitly find an expression for the adjustment coefficient and hence, one has to use numerical methods and as shown in Klugman et al 1998, an initial guess for which can be taken as \( \frac{2\theta p_1}{E(X^2)} \)

An alternative definition for the adjustment coefficient is given as the smallest positive solution of the equation

\[
1 + \theta = \int_0^\infty e^{kx} f_{e,x}(x) dx
\]

Where,

\[
f_{e,x}(x) = \frac{1 - F(x)}{p_1}, x > 0
\]  \( (1.10) \)

is the Equilibrium probability density function.

The first main use of the adjustment coefficient lies in the following result

**Theorem: 1** Suppose \( k \) is the adjustment coefficient, then the probability of ruin \( \psi(u) \) satisfies

\[ \psi(u) \leq e^{-ku}, u \geq 0 \]

Proof: For the proof see Klugman et al (1998)

It can be noted that because the ruin probability is non-negative, we have

\[ 0 \leq \psi(u) \leq e^{-ku} \]  \( (1.11) \)

\[
=> L_{t \rightarrow x} \psi(u) \leq 0
\]

\[
=> \psi(x) = 0
\]

We now state the integro differential equation satisfied by the probability of ruin which can be solved to get an explicit formula for the probability of ruin.
But fundamental to its derivation are some of the quantities and results stated below.

Let $G(u, y) = \Pr(\text{ruin occurs with initial reserve } u \text{ and the deficit immediately after ruin is at most } y)$

**Theorem 2**: The function $G(u, y)$ satisfies the following integro–differential equation

$$
\frac{\delta}{\delta u} G(u, y) = \frac{\lambda}{c} G(u, y) - \frac{\lambda}{c} \int_0^u G(u - x, y) dF(x) - \frac{\lambda}{c} \left[F(u + y) - F(u)\right], u \geq 0 (1.12)
$$

Proof: (See Klugman et al 1998)

**Theorem 3**: The function $G(0, y)$ is given by

$$
G(0, y) = \frac{\lambda}{c} \int_0^y \{1 - F(x)\}dx, y \geq 0 \quad (1.13)
$$

For proof refer to Klugman et al 1998

**Theorem 4**: The survival probability with no initial reserve satisfies

$$
\phi(0) = \frac{\theta}{1 + \theta} \quad (1.14)
$$

Proof: We have

$$
p_1 = \int_0^\infty \{1 - F(x)\}dx
$$

$$
\psi(0) = \lim_{y \to \infty} G(0, y) = \frac{\lambda}{c} \int_0^\infty \{1 - F(x)\}dx = \frac{\lambda p_1}{c} = \frac{1}{1 + \theta}
$$

From which it follows that

$$
\phi(0) = 1 - \psi(0) = \frac{\theta}{1 + \theta}
$$

We now state the integral equation satisfied by the probability of ruin

**(1.5) Integro-differential equation satisfied by the Probability of Ultimate Ruin**

**Theorem 5**: The probability of ultimate ruin $\psi(u)$ satisfies the following integral equation
\[ \psi'(u) = \frac{\lambda}{c} \psi(u) - \frac{\lambda}{c} \int_0^u \psi(u-x) dF(x) - \frac{\lambda}{c} [1 - F(u)], \ u \geq 0, \]  
(1.15)

(Klugman et al 1998)

Proof: from (1.12) with \( y \to \infty \) and using \( \psi(u) = \lim_{y \to \infty} G(u, y), u \geq 0 \), theorem 5 can be established.

The same equation in terms of the Survival probability can be casted as

\[ \phi'(u) = \frac{\lambda}{c} \phi(u) - \frac{\lambda}{c} \int_0^u \phi(u-x) dF(x), \ u \geq 0 \]  
(1.16)

The integral equation in (1.15) can be solved only when \( F(x) \) has a special form viz when \( F(x) \) is exponential or Mixture of Exponential.

The following is a simple illustration how this equation can be solved in case of mixture of 3 exponentials, the choice of the distribution being justified from the fact that it constitute one of the probability models which have been considered to describe the claim severity in our work.

The pdf of the mixture of three exponentials is given by

\[ f(x) = w_1 \lambda_1 e^{-\lambda_1 x} + w_2 \lambda_2 e^{-\lambda_2 x} + w_3 \lambda_3 e^{-\lambda_3 x}, w_i > 0, \lambda_i > 0, x > 0, \sum_{i=1}^n w_i = 1 \]

Now taking Laplace transform on both sides of equation (1.15), and rearranging, we have

\[ \tilde{\psi}(s) = \frac{-c\psi(0) + \lambda \left( \frac{1 - \tilde{f}(s)}{s} \right)}{-cs + \lambda \left( 1 - \tilde{f}(s) \right)} \]  
(1.17)

Where \( \tilde{\psi}(s) \) and \( \tilde{f}(s) \) denote the Laplace transform of \( \psi(.) \) and \( f(.) \) respectively.

For the mixture of three exponentials,

\[ \tilde{f}(s) = \sum_{i=1}^3 w_i \frac{\lambda_i}{(\lambda_i + s)} \]  
(1.18)
Substituting this in (1.17), we have

\[
\psi(s) = \frac{-c\psi(0) + \lambda \left( 1 - \sum_{i=1}^{3} w_i \frac{\lambda_i}{(\lambda_i + s)} \right)}{-cs + \lambda \left( 1 - \sum_{i=1}^{3} w_i \frac{\lambda_i}{(\lambda_i + s)} \right)}
\]

(1.19)

The roots of the denominator are the roots of the Lundberg equation given by

\[
\sum_{i=1}^{3} w_i \frac{\lambda_i}{(\lambda_i + s)} \left( \frac{\lambda}{-cs + \lambda} \right) = 1
\]

(1.20)

Assuming that the three roots \( r_i (i = 1,2,3) \) of (1.20) are distinct and observing that \( \psi(s) \) is a rational function and that the denominator is zero whenever \( s \) is some \( r_i \), we have using the theory of partial fractions,

\[
\psi(s) = \frac{C_0}{s} + \sum_{i=1}^{3} \frac{C_i}{r_i + s}
\]

(1.21)

Where \( C_i (i = 1,2,3) \) are some constants determined from the theory of partial fractions

(For details refer to Chapter(7))

Taking Laplace Inverse of (1.21), we have

\[
\psi(u) = C_0 + C_1 e^{-r_1 u} + C_2 e^{-r_2 u} + C_3 e^{-r_3 u}
\]

Putting \( u = \infty \), we have

\[
\psi(\infty) = C_0 = 0
\]

Since, \( \psi(\infty) = 0 \)

Hence

\[
\psi(u) = C_1 e^{-r_1 u} + C_2 e^{-r_2 u} + C_3 e^{-r_3 u}
\]

(1.22)

This is the solution of the integro–differential equation when the distribution of claim severity is the mixture of 3 exponentials
(1.6) Maximal Aggregate Loss Random Variable: We now define a quantity called the maximal Aggregate Loss random variable which is very related to the general solution of the integro-differential equation.

Because, the surplus process has stationary and independent increments, the probability of dropping below the initial level \( u \) is the same for all \( u \) and so when \( u = 0 \), then this probability is \( \psi(0) \) i.e. the probability of dropping below the initial level \( u \) is \( \psi(0) \) for all \( u \). We now state a very important result in the form of the following theorem.

Theorem 6: Given that there is a drop below the initial level \( u \), the random variable \( Y \) which represents the amount of this drop has the pdf

\[
f_e(y) = \frac{1}{\psi_1} \{1 - F(y)\} \tag{1.23}
\]

We highlight its proof for better understanding of the behavior of the Surplus process.

Proof can be found in Klugman et al 1998

Proof: The property of the stationary and the independent increment of the surplus process lead us to conclude that the probability that the surplus drops below the initial level and the amount of this drop is at most \( y \) is given by \( G(0, y) \). Also by the theorem as represented by equation (1.13), we have

\[
G(0, y) = \frac{\lambda}{\xi} \int_0^y \{1 - F(x)\} dx, y \geq 0
\]

Therefore, the amount of this drop given that there is a drop has the cumulative distribution function (c.d.f.) as

\[
p(Y \leq y) = \frac{G(0, y)}{\psi(0)} = \frac{\lambda}{\xi \psi(0)} \int_0^y \{1 - F(u)\} du, y \geq 0
\]

Therefore, its probability density function (p.d.f.) is given by
If there is a drop of $y$, then the surplus immediately after drop is $u - y$ and because the surplus process has stationary and independent increments, the ruin thereafter would occur with probability $\psi(u - y)$ provided $u - y$ is non negative otherwise the ruin would have already occurred.

Now consider the following: Suppose that the first drop occurred whose probability is $\psi(0)$ and it makes the initial surplus $u - y$ and thereafter no drop occurred whose probability is $1 - \psi(0)$. Hence during the entire infinite time horizon, the probability of just one drop is $\psi(0)(1 - \psi(0))$

After the first drop has occurred, the initial surplus got reduced to $u - y$ and the probability that a second drop would occur henceforth is also $\psi(0)$. The amount of this second drop is independent of the first drop and has the pdf $f_e(y) = \frac{1}{p_1} \{1 - F(y)\}$. Due to the memory less property of the Poisson process, the number of drops, say, $K$ is geometrically distributed i.e. the probability of "$k" drops in the entire infinite time horizon is given by

$$P(K = k) = \psi(0)^k(1 - \psi(0)) = \frac{\theta}{1 + \theta} \left(\frac{1}{1 + \theta}\right)^k$$

(1.24)

Since $\psi(0) = \frac{1}{1 + \theta}$

After a drop, the surplus immediately begins to increase and the lowest level of the surplus that can be ever reached is $u - L$, where $L$ is called the maximal aggregate loss random variable and is the sum of all drop amounts.
Let $Y_j, j = 1, 2, ..., K$ be the amount of the $j^{th}$ drop. Because the process has stationary and independent increments, $Y_j$'s are i.i.d. each with pdf $f_y(y) = \frac{1}{\theta_1} [1 - F(y)]$. Again, since the number of drops $K$ is geometrically distributed, we have

$$L = Y_1 + Y_2 + \ldots + Y_K$$  \hspace{1cm} (1.25)

With $L = 0$ if $K = 0$. Thus $L$ is a compound geometric random variable with claim size density $f_y(y)$.

Also, it can be realized that if $L$ never falls below $u$, then the company survives forever. Thus the event $(L \leq u)$ constitute the event of ultimate survival for the insurance company thereby giving

$$\phi(u) = p(L \leq u), u \geq 0$$

$$= p(Y_1 + Y_2 + \ldots + Y_K \leq u)$$

$$= \sum_{k=0}^{\infty} p(Y_1 + Y_2 + \ldots + Y_K \leq u|K = k) p(K = k)$$

$$= \sum_{k=0}^{\infty} \frac{\theta}{1 + \theta} \left( \frac{1}{1 + \theta} \right)^k F_y^*(u), u \geq 0$$  \hspace{1cm} (1.27)

Where

$F_y^*(y) = 0$ if $y < 0$

and

$F_y^*(y) = p(Y_1 + Y_2 + \ldots + Y_K \leq y), y \geq 0$

denote the $k$-fold convolution of the distribution of $Y$ with itself.

(1.27) represents the general solution to the integro-differential equation and this general solution in terms of the ultimate ruin probability can be casted as

$$\psi(u) = \sum_{k=0}^{\infty} \frac{\theta}{1 + \theta} \left( \frac{1}{1 + \theta} \right)^k S_y^*(u), u \geq 0,$$  \hspace{1cm} (1.28)
where
\[ S_n^k(u) = 1 - F_n^k(u) \]

The above is known as the Pollaczek Khinchin formula for the probability of ultimate ruin.

The following is a systematic review on the extensions of the Classical Risk Model listed for the sake of completeness and they have no direct relevance to the content of the work for the work is bounded within the premises of the Classical Risk Model and it stresses on the computation aspects associated with the evaluation of the actuarial quantities which are obtained using some of the results of the Classical Risk theory and the chapter wise review of literature is placed at appropriate places.

(1.7) Some Extensions of the Classical Risk Model

De Finetti’s Model

In this model, the surplus at time \( t \) is

\[ U_t = u + ct - \sum_{i=1}^{t} X_i, t = 1,2,3 \ldots \]

Where \( X_1, X_2, \ldots \) are i.i.d. random variables with

\[ P(X_i = 0) = p, P(X_i = 2) = q = 1 - p, i = 1,2,3 \ldots \text{ and } p > 1/2 \]

This model proposed by De Finetti (1957) was later reviewed by Seal (1969), Gerber and Shiu (2004) and Auanzi (2009)

Approximations of the Cramér-Lundberg Model

The discretized version of the Compound Poisson Process was discussed by Dickson and Waters (1991) and it is given by

\[ U_t = u + ct - \sum_{i=1}^{t} X_i, t = 1,2,3 \ldots \]

The compound binomial process arises when the binomial process is used to model the claim number process $N_t$ and it is given by

$$U_t = u + ct - \sum_{i=1}^{N_t} X_i$$

Where $N_t$ is no longer a Poisson Process but a Binomial process, that is the increment $N_{t+h} - N_t$ for $h = 1, 2, 3, ...$ is a binomial random variable with parameters $h$ (number of trials) and $p$ (probability of success).


**Dividend Problem:**

According to De-Finetti (1957), it is unrealistic to assume that ruin probabilities can be minimized over an infinite time horizon for the companies can grow their surpluses without any limit. He proposed the dividend which restricts the infinite growth of the surplus for a payment of a part of the surplus is to be given to the shareholder as a dividend. However, it raises questions as to when dividend is to be given and how much of the surplus be declared as dividend. Avanzi (2009) reviewed a variety of dividend strategies such as the constant barrier strategy, linear and non-linear barrier strategies.

Dickson and Drekic (2006) also discusses this problem.

Relevant to the context of taxes being imposed on the Surplus process, is the concept of the Loss-carry forward system discussed by Albrecher and Hipp (2007) and Albrecher, Badescu and Landriault (2008).
The Wiener Process:
The continuous counterpart of the De-Finetti’s model is the Wiener Process. In this model, the surplus at time $t$ is

$$U_t = u + \mu t + \sigma W_t$$

where $\{W_t\}$ is a standard Weiner process. As a consequence, $U_{t+h} - U_t, f o r t > 0$ $h > 0$ is a wiener process with mean $h\mu$ and variance $h\sigma^2$. Gerber and Shiu (2003,2004) and Leung et al (2008) deal with this model.

Levy Process:
The approach to include a Levy process in risk theory was first introduced by Gerber (1970), where a diffusion component $\sigma W_t$ is added to the Classical Risk Process yielding

$$U_t = u + \mu t - \sum_{i=1}^{N_t} X_i + \sigma W_t, t \geq 0$$

where $\{W_t\}$ is a standard Weiner process. This new component is an extra uncertainty added to the classical Risk model that might account for the fluctuations in the number of customers, in the premium or return on the investment of the reserve. Rolski, et al (1999), Li (2006), Jang (2007) and Frostig (2008) are good references for this model.

The Sparre Anderson Models
The inter claim dependent claim size model

Albrecher and Teugels (2006), Boudreault et al (2006) Landriault (2008 a) and Huang and Li(2011) considered an extension to the classical Cramer Lundberg model in which a particular dependence structure is assumed to exist among the inter claim time and subsequent claim size. In the classical risk model it is assumed that the inter arrival time between any two claims say, $i^{th}$ claim and $(i - 1)^{th}$ claim denoted by $\Delta T_i$ follows an independent Exponential distribution with intensity parameter $\lambda$. In this model, we assume that the bivariate vector $(X_i, \Delta T_i)$ for each $i \in N$ are mutually independent whereas $X_i$ and $\Delta T_i$ are dependent.

(1.8) Objectives of the thesis:

We are concerned with the computation of the Probability of ultimate ruin in the classical risk model and some of other actuarial quantities related to the assessment of the solvency of an insurance company. Considering this, the following constitute the objectives of the thesis.

(1) To model the uncertainty in claim severity by six probability models namely the Pareto distribution, Log Normal distribution, Gamma distribution, Weibull distribution, Burr XII distribution and Mixture of Exponentials distribution and test for their goodness of fit.

(2) To evaluate the Probability of ultimate ruin under these models by a number of numerical algorithms like the Stable Recursive algorithm, method of Product Integration, the method of Upper and Lower bounds, Fast Fourier Transform and to the extent possible, carry out a comparative analysis between them.

(3) To derive the exact expressions for the Probability of ultimate ruin for the Mixture of 2 Exponentials and the Mixture of 3 Exponentials distribution.
(4) To compute the first two moments of the Time to Ruin, deficit at the time of Ruin and the Surplus just prior to ruin for these Probability models either numerically or explicitly, wherever possible.

(5) To compute the Quantiles of the Aggregate Loss distribution under these claim severity distributions by Panjer Recursive Algorithm and The Fast Fourier Transform Algorithm.

(6) To compute numerically the first four convolutions of Pareto, Log Normal, Gamma, Weibull and Burr XII distributions and to derive exact expressions for the first four convolutions of the Mixture of 3 Exponentials distribution. Additionally, to illustrate the application of these convolutions in computing the Probability function of the number of claims until ruin with zero initial surplus for these distributions.

(7) To compute the Probability of ultimate ruin under interest earnings and tax payments for Pareto, Log Normal, Weibull and Burr XII distributions and to compute the Upper and lower bounds to the probability of ultimate ruin under interest rate for the Mixture of 3 exponentials distribution.

(8) To derive methodology to simulate from the first order Equilibrium distribution of Burr XII and Weibull and illustrate its application in computing the Probability of ultimate ruin through Pollaczek Khinchin formula via Monte Carlo Simulation for these distributions.