ABSTRACT

The problems and the methodologies stated in this thesis are directly or indirectly related to uncertainty modeling of the environment in which actuaries must solve problems, thereby facilitating improved decision making by incorporating the information generated by these quantities. This thesis apart from incorporating some of the theoretical aspects of the Risk theory in Actuarial science renders vital contribution in applied field by means of using numerical techniques for the computation of some of these actuarial quantities for which explicit expression don’t exist. Probability models for modeling the claim severity, at times suffer from restricted utility due to the lack of explicit expressions for some of the important actuarial quantities associated with it. Overcoming this restriction has essentially been a motivation for this work.

In this thesis, the main problem we are concerned with is the evaluation of the probability of ultimate ruin for the claim severity distributions under consideration and we have tried solving with a view of obtaining an optimal approximation to it through a number of methods, each being efficient in one sense or the other.

Much of the literature on ruin theory is concerned with the classical risk model in which the insurer starts with an initial surplus "u" and collects premium continuously at a constant rate of “c” while the aggregate claim process follows a compound Poisson process. For the surplus process so generated, the evaluation of the finite time and the infinite ruin probabilities constitute the primary research goals. However, with the progress of time, some extra components got added to the Classical Risk Model. Some recent developments
includes the cases when dividends are allowed to the insured (Albrecher and Thonhoures, 2009), Spare Anderson Models, Multivariate ruin function (Rolski et al, 2009), Probability of Ruin under stochastic interest rates (Sundt and Teugels, 1995), Levy Insurance Risk Models (Kyprianou, 2006) etc. However, we have restricted ourselves within the premises of the classical Risk Model.

The thesis consists of twelve chapters. Starting with a survey of the results on the classical Risk theory in chapter 1, the next chapter deals with Loss modeling. The next five chapters are unified in the sense that each of them aims at the evaluation of the ultimate Ruin probability. In the last five chapters of the thesis, we deal with the computation of some of the other important actuarial quantities like the moments of the time to ruin, moments of the surplus just prior to ruin and deficit at the time of ruin, aggregate claim models, probability function for the number of claims until ruin and the influence of interest rate and tax payments on the probability of ultimate ruin. Each of these quantities render valuable insight to the risk associated with the process. We give a brief introduction to each of the 12 chapters of the thesis as follows:

In chapter 1, we have provided a survey of some of the fundamental results of the classical risk theory along with their proofs. Here, our primary interest is to introduce the basic concepts and terminologies related to the classical risk model and the ruin theory, which provide the basic framework for the entire work covered in the thesis. For example, we have given a brief introduction to the claim arrival process, the compound Poisson process, the integro-differential equation satisfied by the probability of ultimate ruin and some of the
related concepts like the maximal aggregate loss random variable. Some general procedures for obtaining the solution of the integro differential equation are also indicated.

Chapter 2 deals with loss modeling which constitute one of the main ingredients of risk management in the Actuarial domain. We have considered six probability models as potential models for modeling the claim severity and have used, each of them for modeling a set of data on insurance claims. We have estimated the parameters of these distributions through maximum likelihood estimation (MLE) technique, which is being followed by assessing the goodness of fit of these distributions. Irrespective of the fact whether the concerned probability model provides good fit to the data or not, we have used the parameter estimates of the model for the computational algorithms which are being dealt in the subsequent chapters.

In chapter 3, we are dealing with a recursive algorithm for the evaluation of the ultimate ruin probabilities which is stable in the sense that it doesn’t leads to the propagation of errors in the subsequent iterations and leads to the bounds of the ultimate ruin probability within a prescribed tolerance level. Associated with the computation arising out of this algorithm, is the determination of the equilibrium distribution, corresponding to each of the claim severity distribution. Lack of closed form expressions for them makes their computations numerically challenging.

Chapter 4 on the other hand, deals with an algorithm which is more suited for the numerical evaluation of the ultimate ruin probability for heavy tailed distributions like Burr XII, Pareto, Log Normal and Weibull. This method called the method of product integration is
faster than the stable recursive algorithm. It aims at the numerical solution of the integro-differential equation of the probability of ruin by treating it as a Volterra Integral equation of the second kind and makes use of a quadrature rule which exploits some of the features of the kernel, thereby reducing the number of recursions. It requires the computation of weights based on the kernel function specific to the underlying claim severity distribution.

Chapter 5 deals with the method of upper and lower bounds for the evaluation of the probability of ultimate ruin. It uses the connection between the probability of ultimate ruin and the maximal aggregate loss variable and the fact that the latter has a compound geometric distribution. It is one of the most original methods for approximating the probability of ruin and is to some extent, connected to the Stable Recursive algorithm discussed in chapter 1, in the sense that both require some quantities derived out of the equilibrium distribution corresponding to the claim severity distribution.

In Chapter 6, we have used the Fast Fourier Transform (FFT) algorithm to compute the probability of ultimate ruin for the claim severity distributions under consideration. The basic idea underlying this method is that the characteristic function of the maximal loss random variable is obtained in its discretized version and then inverted to get the discretized version (p.m.f) of the original maximal aggregate loss random variable from which, an estimate of the probability of ultimate ruin is obtained. However, it needs mentioning that we are just dealing with the naïve application of FFT without incorporating in it, some of the methods for reducing error due to discretization and truncation.
Chapter 7 deals with the determination of the explicit expression for the probability of ultimate ruin for the mixture of exponentials distribution. The method for deriving the exact expression for the probability of ultimate ruin for the mixture of exponentials is based on the moment generating function (m.g.f.) of the maximal aggregate loss random variable. This m.g.f., when the claim severity is mixture of exponentials give rise to an expression, which when split, by the use of partial fractions and the use of the roots of the Lundberg’s equation leads to an explicit expression for the probability of ruin.

In Chapter 8, we have computed the first two moments of the time to ruin, deficit at the time of ruin and the surplus just prior to ruin, explicitly for the mixture of three exponentials and for the remaining distributions, they are computed numerically through the use of numerical integration. The multiple integrals were actually handled by numerical integrals carried out in a nested manner. These numerical integrations were complex and the chapter significantly deals with them, pointing out the difficulties and the probable source of numerical error. The results which are used for the computation of these quantities are based on the so called discounted penalty function and the defective renewal equation satisfied by it.

In chapter 9, we are dealing with one of the classical problems in risk theory, which is the computation of the aggregate loss distribution. After giving a brief review on the various methods for the computation of the aggregate loss distributions, we have used two of the most efficient and widely used methods, namely the Panjer Recursion and the Fast Fourier transform to compute the quantiles of the aggregate loss distribution, which in our case, is a compound Poisson distribution. Fast Fourier Transform (FFT) is computationally faster
than the Panjer Recursion and is based on the characteristic function of the underlying compound distribution. However, FFT is not as accurate as the Panjer Recursion and is comparable to it only when it is subjected to exponential tilting. Moreover, it needs mentioning that determination of the higher order quantiles of the aggregate distribution, especially, when the underlying claim severity distribution is heavy tailed is computationally difficult and indicate the existence of many pitfalls. Hence, we remain restricted to the evaluation of the lower order quantiles.

In Chapter 10, we have tried calculating the probability function of the number of claims until ruin with zero initial surplus, based on the results derived in Dickson (2012). The computational requirement posed by it is rather complex, for it necessitates the evaluation of a double integral, with the outer integral extending from 0 to \( \infty \). Evaluation of the inner integral is also complex because it involves convolution of the claim severity distribution under consideration. Hence we have obtained the lower order convolutions of these distributions either analytically or numerically and illustrated their applications in evaluating the probability function of the number of claims until ruin. It needs stressing on the fact that ultimate evaluation of this function is rather complex for it amounts to handling a number of numerical integrals simultaneously, ordered sequentially in a nested pattern and consequently, we were successful in determining this function just for a very few number of claims, which may not have much value in practice.

In Chapter 11, we have added two more components to the Classical Risk model in the sense that now, we are considering the computation of the probability of ultimate ruin in the presence of interest earning and tax payments. In this direction, on the basis of a result
applicable for sub-exponential distributions, we have evaluated this probability for Burr XII, Weibull, Pareto and Log Normal distribution using an illustrative interest rate and tax structure. For the Mixture of Exponential distribution, we have just evaluated the upper and lower bounds to the probability of ultimate ruin under the influence of an interest rate.

Chapter 12 deals with the computation of the probability of ultimate ruin through simulation via the Pollaczek Khinchin formula. This requires random observation to be simulated from the equilibrium distribution of the corresponding claim severity distribution and hence, we have derived methodologies to simulate from the Equilibrium distribution of Burr XII and Weibull and thereafter, have used the simulated observations to obtain an approximation to the probability of ultimate ruin for these two distributions through simulation.