APPENDIX

1. The Newton Raphson Method: The Multiparameter Situation

One of the most used methods for optimization in the Multi Parameter situation in Statistics is the Newton–Raphson method which is described briefly as given below:

Assume \( \theta = \theta_1, \theta_2, \ldots, \theta_p \) is a vector of \( p \) (say) unknown parameters and the log likelihood of the distribution involving \( \theta \) is given by \( l(\theta, \bar{x}) \). Then the MLE for \( \theta \) are obtained by solving the equations

\[
\frac{\partial l}{\partial \theta_i} = 0,
\]

Let us now define what is known as the gradient matrix and the Hessian matrix given by

The gradient matrix is given by

\[
S(\theta) = \begin{pmatrix}
\frac{\partial l}{\partial \theta_1} \\
\frac{\partial l}{\partial \theta_2} \\
\vdots \\
\frac{\partial l}{\partial \theta_p}
\end{pmatrix}
\]

And the Hessian matrix is given by

\[
J(\theta) = \sum_{i,j=1,2,\ldots,p} J_{i,j}
\]

Where

\[
J_{i,j} = -\frac{\partial^2 l(\theta)}{\partial \theta_i \partial \theta_j}
\]
Then the iterative relationship for the multi parameter Newton Raphson method is given by

$$\theta^{(s+1)} = \theta^{(s)} + \left[ J(\theta^{(s)}) \right]^{-1} S(\theta^{(s)})$$

Where $\theta^{(s)}$ is the estimated value of $\theta$ at the $s^{th}$ iteration. The iteration is carried out until there is no significant difference between $\theta^{(s)}$ and $\theta^{(s+1)}$.

2. Simpson’s 1/3rd Formula for Numerical Integration:

Consider the set $f(x_n) = f_n$ of the values of the function $f(x)$ at equally spaced values of $x = x_0, x_1, x_2, ..., x_n$ constituting the set $x = \{x_n\}$. Let the common spacing between any two consecutive values of $x$ be

$x_{n+1} - x_n = h$ and let $n = 2q, q$ is a positive integer $>0$. Then the Simpson’s 1/3rd rule is given by

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left\{f_0 + f_{2q} + 4(f_1 + f_3 + \cdots + f_{2q-1}) + 2(f_2 + f_4 + \cdots + f_{2q-2})\right\}$$

3. Light and Heavy tailed Distributions

A distribution $F_X(x)$ is said to be light tailed if there exists constants $a > 0, b > 0$ such that $F_X(x) = 1 - F_X(x) \leq ae^{-bx}$ or equivalently there exist $z > 0$ such that $M_X(z) < \infty$ where $M_X(z) < \infty$ where

$M_X(z)$ is the m.g.f. of $z$. Distribution $F_X(x)$ is said to be heavy tailed if for all $a > 0, b > 0$, $F_X(x) > ae^{-bx}$ or equivalently for every $z > 0$, $M_X(z) = \infty$