CHAPTER - 2

LITERATURE REVIEW

2.1 INTRODUCTION

The behavioural analysis of composite laminates is an important field of research owing to the wide spread use of these composite materials in defense, aerospace, automobile, ship building, chemical industries and other commercial applications. It is their resistance to corrosion high strength to weight ratio and durability properties make them find potential applications.

The transverse delamination behaviour has been studied extensively because this type of failure leads to other damage modes. The analysis has to be such that it can account for both mechanical loading and temperature gradients as many applications like the heat shield of a re-entry vehicle or the wall of nuclear reactor, involve combined mechanical and thermal loads. Analysis of laminates can be based on a variety of theories from the rigorous 3-D theory of elasticity to the simple classical plate theories based on the direct extension of the corresponding theories for metallic structures. A meaningful compromise between these two extremes is achieved by employing a so-called shear deformation theory, which is actually a refinement of the simple classical laminated plate theory.

The following sections deal with an extensive review of the available literature on analytical and finite element methodologies for the analysis of transverse behaviour of different composite laminates.

2.2 STUDIES ON STATIC BENDING BEHAVIOUR OF COMPOSITE LAMINATED PLATES:

It has been recognized that classical plate theory is to be modified to include certain higher-order effects like warping of cross-sections. The first generalization of classical plate theory (CPT) is given by Reissner E. (1945) and Mindlin R.D. (1951).
Reissner E (1944), is the first to provide consistent theory which incorporates the effect of shear deformation. The basic assumption used by Reissner explained various forms of stress distribution across the thickness. A special variational theorem is used to determine both the equations of equilibrium in terms of stress-strain relations and in terms of resultants and assumed displacements functions.

Reissner, E (1947), has refined his own theories with the introduction of correction factors similar to Timoshenko's beam theory which are determined by matching result of the approximation theory with the exact solution to static problems.

Stavsky Y. (1959), has introduced first the shear deflection into laminated plate theory and evaluated mechanical behaviour for various laminated stacking sequences.

Reissner E, Stavsky Y (1961), improved the CLPT by including the influence of bending–extensional coupling in un-symmetrical laminates. Using the CLPT they discussed the effect of the coupling phenomenon between in-plane stretching and transverse bending which does not occur in the theory of homogeneous plates and it was not considered in earlier work for the plate consisting of two orthotropic sheets of equal thickness, which are laminated in such a way that the axis of elastic symmetry enclose an angle $\theta$ with the $(x, y)$ axes in one sheet and an angle $\theta'$ in the other sheet. Using this theory Whitney and Liessa 1969 solved a number of problems relating to the bending vibration and stability of coupled laminates.

Yang P.C, Norris C.H et al (1966), extended Mindlin's theory for homogeneous plates to laminates consisting of an arbitrary number of bonded anisotropic layers. Although this approach is much less cumbersome than that of Ambartsumyan, its consequences have not been explored because of lack of solutions to specific boundary value problems and he only considered the frequency equations for the propagation of harmonic waves, in a two layer isotropic plate.

Pagano N.J. (1968), studied an isotropic laminated composite where the ratio of 'E' to shear modulus (G) can be very large, however a theory analogous to those presented by Reissner (1945) and Mindlin (1951) for isotropic plates assumes greater importance.
Whitney J.M. (1969), has presented a theory for laminated plates which includes the effect of transverse shear deformation. Solutions have been obtained for bending, fundamental vibration frequencies and buckling of simply supported plates having special construction of symmetric and non-symmetric laminates. Also closed form solutions are obtained for bending, flexural vibrations and buckling loads of simply supported rectangular plates of special construction and he showed that shear deformation can significantly affect gross plate response for highly anisotropic laminate.

Ambartsumyan S.A (1969), developed cumbersome approach to define transverse shear stress that satisfies the required continuity conditions at the layer interfaces. This bending theory allows for the specification of three boundary conditions per edge but the analysis is restricted to laminates consisting of orthotropic layer stacked systematically with respect to midplane of the plate and having the axes of material symmetry coincides with the plate co-ordinate axes.

Pagano N.J (1969), studied the mechanical response of composite laminates by considering the problem of cylindrical bending of bi-directional \(^{0^\circ} - 90^\circ\) laminates. Exact solutions within the framework of the linear theory of elasticity were developed and compared with the respective solutions of CLPT.

Whitney J.M (1969), has extended Ambartsumyan approach to solve certain specific boundary value problems of more general material and geometric properties considered in Ambartsumyan (1969).


Pagano N.J. (1970), studied the 3-D elasticity solutions constructed for rectangular laminates with pinned edges. Also presented a sufficiently general exact elastic response of rectangular pinned edge laminates consisting of any number of orthotropic or isotropic layers more general loadings are treated in straight fashion by use of the Fourier series expansion of the load function. It is also capable of extension to describe the stability and vibration response of the types of the laminates considered.
Whitney J.M and Pagano N.J (1970) investigated the first order shear deformation theory for bending of anisotropic laminated plates developed by Yang, Norris C.H et al (1966). The theory includes shear deformation and rotary inertia in the same manner as Mindlin's theory for isotropic homogeneous plates. The governing equations reveal that un-symmetrically laminated plates display the same bending extensional coupling phenomenon found in classical laminated plate theory based on Kirchoff's assumptions. To test the generality of this theory, solutions are presented for a number of specific boundary value problems relating to the static bending and natural vibration frequencies of symmetric and non-symmetric laminates, for certain fiber reinforced composite materials.

Pagano N.J. and Hatfield S.J (1972a, 1972b), assumed a uniform shear strain through the thickness of the plate and neglected local effects for the finite element formulation based on the non confirming rectangular plate bending elements. The state of stress and deformation at the layer interfaces which may be important in situations where the stiffness properties vary drastically from layer to layer and they also examined a conservative estimate of the magnitude of the error reflected in the simplifying assumptions of CPT for multi layered systems by comparison of exact and approximate solutions for laminate consisting of only several layers.

Dong S.B. and Tso F.K.W. (1972), presented a constitutive relation for laminated orthotropic shells which include transverse shear deformation. This relation involves composite correction factors which are determined from an analysis of plane waves in a plate with the same layered construction. And the applicability of the present theory and the effect of transverse shear deformation are evinced in a problem concerned with the natural oscillations of a three layered freely supported cylinder. In conclusions the authors have emphasized that the present theory is applicable for materials where the transverse normal stiffness is sufficiently great.

Whitney J.M, Sun. C.T (1973), developed a refined laminated plate theory which is applicable to fiber reinforced composite materials under impact loading. In addition to the usual bending and extensional motions, the theory also includes the first symmetric thickness shear and thickness stretch motions. The governing equations reveal that un-symmetrically laminated plates display a coupling phenomenon between all of the deformations modes present in the theory.
Nelson R.B., Lorch D.R. (1974), presented a refined structural theory that accurately models the static and dynamic behaviour of laminated orthotropic plates. This is one of the extensions of classical theory to include transverse shear, transverse normal and quadratic displacement terms in the kinematic assumptions. Hamilton’s principle is used to formulate displacement equations of motion with appropriate boundary and initial conditions. The composite correction factors $k, i, j$ are introduced in a manner consistent with their indifference to choice of reference surface and are determined by procedure in which plane wave solutions for the plate are adjusted to match corresponding exact solutions. Examples of homogenous isotropic, orthotropic and laminated orthotropic plates are presented to show the capability of the theory.

Mawenya A.S. & Davies J.D. (1974), developed a general quadratic multilayer plate element for the analysis of arbitrarily layered curved plates. In the formulation each layer of the multilayer plate considered has different orthotropic properties and can deform locally. The formulation incorporates the effects of transverse shear deformation in each layer and is applicable to any arbitrarily layered curved plates.

Reissner E (1975), developed a plate theory which gives very accurate results compared with the elasticity solution for the bending of a plate with a circular hole. It is noted that the theory developed represents the lowest order correction for out-of-plane deformation effects to the classical theory. It neglects the contribution of in-plane modes of deformation by considering only out-of-plane effects. But, such in-plane effects may be of important in certain other plate problems.

Essenburg F (1975), considered a general problem of a beam of rectangular section with displacement components prescribed over portions of top and bottom surfaces. He had developed a beam theory including the effect of transverse normal strain. The present formulation has a heuristic appeal in that it satisfies equilibrium requirements locally and in a simple manner accommodates the prescription of surface displacement components at two surfaces.

Thomas J.R. Hughes, Taylor R.L et al (1977), have introduced a simple and efficient finite element for plate bending applications. They have employed a bilinear displacement and rotation functions in conjunction with selective reduced integration. The
element that has been introduced for the bending of thin and moderately thick plates involves minimal programming and is highly efficient and competitively accurate. Due to these attributes, the element offers an attractive basis for the non-linear developments.

Lo K.H and Christensen R.N. et al (1977a, 1977b), derived a theory of plate deformation which accounts for the effects of transverse shear deformation, transverse normal strain and a non-linear distribution of the in-plane displacements with respect to the thickness coordinate. The theory is compared with lower order plate theories through application to a particular problem involving a plate acted upon by a sinusoidal surface pressure. It is found that when the ratio of the characteristic length of load pattern to the plate thickness is of the order of unity, lower order theories are inadequate and they also extended the theory developed in (1977a) to model the behaviour of laminated plates. Through comparison with elastic solutions they have shown that the present theory correctly models the effects not attainable from the classical theory.

Thomas J.R. Hughes, Martin Cohen et al (1978), described the development of effective plate bending finite elements by reduced integration techniques. The basis for the development is thick plate theory. The variables in the theory are displacements, and independent rotations. For $C^0$ continuity, iso-parametric elements are considered that result with several advantages over thin plate elements. It is shown that the avoidance of shear locking may be facilitated by reduced integration techniques. They have considered both selective and uniform schemes. Singular problems, which have proved insolvable by thin plate elements, are easily handled. Lumped, positive definite mass matrices which yield large critical time steps for explicit transient calculations are simply derived.

Pugh E.D.L., Hinton E. and Zienkiewicz O.C. (1978), the accuracy of certain thick plate elements when used in the context of thin plate problems is improved by adopting reduced integration of the stiffness matrices. They have demonstrated a series of numerical experiments on five different quadrilateral thick plate elements over the use of reduced integration. Finally they have shown that the performance of nine noded Lagrangian element is near optimal.

Thomas J.R. Hughes and Martin Cohen (1978) developed a new highly accurate, finite element for thick and thin plate bending based upon Mindlin plate theory. The
element chosen is a nine noded quadrilateral, which exhibits improved characteristics in comparison with 8-node serendipity or 9-node Lagrange elements. In particular, the element stiffness possesses correct rank, and high accuracy is attained for extremely thin plates.

Kant T (1981), presented a method for the numerical analysis of elastic plates with two opposite simply supported ends. A variety of boundary conditions including the mixed and the non-homogenous types can be prescribed along either of the remaining two opposite edges. A less known formulation for economical numerical analysis of elastic plates is presented.

Kant T, Owen D.R.J and O.C. Zienkiewicz (1982), attempted a general finite element formulations for plate bending problem based on a higher-order displacement model and a three dimensional state of stress and strain. The theory incorporates linear and quadratic variations of transverse normal strain, transverse shearing strain and stresses respectively through the thickness of the plate. The quadrilateral element from the family of two dimensional continuous isoparametric elements are introduced and it’s performance is evaluated for a wide range of plates under uniformly distributed load and with different support conditions and ranging from very thick to extremely thin situations. The effects of full, reduced and selective integration scheme on final numerical results are examined.

Kant T and Hinton E (1983), presented a method for the numerical analysis of rectangular plates based on Mindlin’s theory. A variety of boundary conditions including the mixed and the non-homogenous type can be specified along either of the remaining two opposite edges. A new formulation for the computer analysis of rectangular plates based on Mindlin’s theory is presented.

Lawrence W. Rehfield and Rao Valisetty R (1983), developed a comprehensive theory for planar bending of laminates. This theory influences, laminated resin matrix composite structures to exhibit more pronounced transverse shear deformation. Research shows that there are additional non-classical influences that also affect bending related behaviour of composites. They include the consequences of section warping and its non-classical axial stress contribution and transverse normal strain
Reddy J.N. (1984), developed a higher order shear deformation theory of laminated composite plates. The theory contains the same dependent unknowns as in the first order shear deformation theory of Whitney and Pagano (1970), but accounts for parabolic distribution of the transverse shear strains through the thickness of the plate. Exact closed form solutions of symmetric cross-ply laminates are obtained and the results are compared with three dimensional elastic solutions and first order shear deformation solutions.

Phan N.D. and Reddy J.N. (1985), used a higher order shear deformation theory to analyze laminated anisotropic composite plates for deflections, stresses, natural frequencies and buckling loads. The theory accounts for parabolic distribution of the transverse shear stresses and requires no shear correction coefficients. For plates with special lamination schemes and simply supported boundary conditions, exact solutions are obtained and compared with the classical plate theory and 3-dimensional elasticity solutions. A displacement finite element model of the theory is also developed. The effects of shear deformation, coupling between bending and extension and material anisotropy on the response of laminated composite plates are also investigated.

Simitses G.J., Sallam S et al (1985), analyzed a simple one-dimensional model for predicting delamination buckling loads. This model is employed to predict critical loads for delaminated homogenous plates with both simply supported and clamped ends. And for both the sets of boundary conditions, the effect of delamination position, size and thickness are studied extensively. In the results they have revealed that for certain geometries the buckling load can serve as a measure of the load carrying capacity of the delaminated configuration. And in other cases, the buckling load is very small and delamination growth is a strong possibility depending on the toughness of the material.

Kant T and Pradeep B. Kulkarni (1986), attempted to design a simple continuous beam/flexural element based on a shear deformable theory. They have emphasized on the development of low order linear/bilinear element. In this formulation total strain energy is split into bending and shear energies and an anti-parameter in the shear energy terms is introduced to avoid locking.

Murakami H. (1986), developed a new laminated plate theory in order to improve the accuracy of in-plane response of the shear deformable laminated composite plate theory
based upon a variational principle proposed by Reissner (1984). The improvement is achieved by including a zig-zag shaped $C^0$ function to approximate the thickness variation of in-plane displacements. The advantage of using the new Reissner variational principle is that it automatically yields the transverse shear constitutive relations with appropriate shear correction factors.

Pandya B.N. and Kant. T (1987), presented a novel approach to satisfy the zero transverse shear stress conditions on the top and bottom of the plate. This theory includes the effect of transverse normal stress, doesn't require arbitrary shear correction factors for transverse shear stiffness and results in parabolic variation of shear strain/stresses through the plate thickness. The numerical results obtained using finite element technique proves the validity of the theory for the flexure of laminated plates.

Owen D.R.J and Li Z.H. (1987), formulated a local finite element model for thick and Mindlin anisotropic laminates, based on a refined approximate theory. They have reduced the three dimensional problem into two dimensional case by assuming piecewise linear variation of the displacements $u$ and $v$ and a constant value of the displacement $w$ across the thickness. It is shown that by using a sub-structuring technique the present local model is both practical and economical. They have obtained the numerical results for the flexure of simply supported square laminates.

Kant T and Pandya B.N. (1988a, 1988b), developed a higher order theory which satisfies zero transverse shear stress conditions on the bounding planes of a generally laminated fiber reinforced composite plate, subjected to transverse loads. The displacement model accounts for non-linear distribution of in-plane displacement components through the plate thickness and the theory requires no shear correction coefficients. A $C^0$ continuous displacement finite element formulation is presented and the coupled membrane flexure behaviour of laminated plates is investigated. The simple iso-parametric formulation is developed which is capable of evaluating transverse shears and transverse normal stress by using the equilibrium equations. They have also presented a $C^0$ continuous displacement finite element formulation of a higher-order theory for flexure of thick arbitrary laminated composite plates under transverse loads. The displacement model accounts for non-linear and constant variation of in-plane and transverse displacements respectively through the plate thickness. The assumed displacement model eliminates the use of shear correction factors.
coefficients. This displacement model is chosen to bring out the effects of neglected transverse normal stress/strain but at the same time retaining higher-order in-plane degrees of freedom in the formulation.

Kant T and Gupta A (1988), developed a higher order shear deformable beam theory. It is based on a higher order displacement model and incorporates linear and quadratic variation of transverse normal strain and transverse shearing strain through the beam thickness. The warping of the transverse normal cross-section of the beam is automatically incorporated in the mathematical model.

Pandya B.N. and Kant T (1988), presented a finite element formulation for a flexure of generally orthotropic plate based on a higher-order displacement model and a 3-dimensional state of stress and strain. This higher-order theory incorporates linear variation of transverse normal strain/stress and parabolic variation of transverse shear strains through the thickness of the plate. The quadrilateral element from the family of 2-dimensional $C^0$ continuous isoparametric Lagrangian element is developed as general orthotropic higher-order elements. The performance of this element is evaluated on square plates with different support conditions under uniform distributed and center point loads. The numerical results of the formulation are compared with thin plate, elasticity and Mindlin/Reissner solutions.

Librescu L and Khadeir A.A. (1988), presented a simple theory of bending in composite anisotropic plates that are laminated symmetrically about their mid-plane. This theory incorporates transverse shear deformation and transverse normal stress as well as higher-order effects and fulfilled the static conditions on the external boundary planes. Further, by using Levy type solutions, considered in conjunction with the state space concept, the state of stress and displacement of rectangular plates for a variety of edge conditions is determined and the results are compared to their first-order shear deformation and classical counter parts, obtained by using the same state-space technique.

Bhaskar K and Varadan T.K. (1989), proposed transverse shear stress continuity at the interface of the laminate with out increasing the number of unknown variables by using a piecewise displacement distribution. The analysis considers symmetric laminates subjected to anti-symmetric loading about the mid plane. The values of displacements and
stress/strain are closer to the exact solution for most values of z/h, without any improvement in transverse strain/stress.

Kant T and Manjunath B.S. (1989), presented a refined higher-order displacement models for the behaviour of symmetric and un-symmetric laminated beams based on $C^0$ finite element discretization. These theories incorporate a more realistic non-linear variation of longitudinal displacements through the beam thickness, by eliminating the use of shear correction coefficients. The computer program developed incorporates the realistic prediction of inter-laminar stresses from equilibrium conditions.

Kant T and Menon M.P (1989), presented a higher-order displacement model for the behaviour of symmetric and un-symmetric laminated composite and sandwich cylindrical shells, based on $C^0$ finite element discretization. The theory is developed for both thin and thick shells. These theories incorporate a non-linear variation of longitudinal displacement through the shell thickness and eliminate the use of shear correction coefficients.

Khadeir A.A. (1989), developed Levy-type solutions for anti-symmetric angle-ply laminated plates using Yang, Norris and Stavsky theory, taking state space concept into consideration. The solution is applicable to rectangular plates which are simply supported on two opposite edges and have arbitrary boundary conditions on the remaining ones. Results are presented over the effects of length to thickness ratio, degree of anisotropy, number of layers, ply-angles and transverse shear correction coefficients on the elastic state of stress.

Ahmed K. Noor and Scott Burton W (1990), presented an analytical solution for the stress and free vibration problems of rectangular multi-layered anisotropic plates. A mixed formulation is used with the fundamental unknowns consisting of six stress components and three displacement components of the plate. Each of the plate variables are decomposed into symmetric and anti-symmetric components in the thickness direction, which is expressed in terms of double Fourier series in the Cartesian surface coordinates. Extensive numerical results are presented showing the effects of variation in the lamination and geometric parameters of composite plates on the importance of transverse stress and strain components. The numerical studies shows that the transverse shear stresses and
strains increase, with the increase in the fiber orientation, the thickness ratio of the plate, the number of layers, and the degree of orthotropy of the individual layers.

Lin, Ye (1990), presented analysis of some characteristics of the distribution of interlaminar stresses near free edges in composite laminates. First, results on the effect of the nonlinear response of composite material are presented and identified whether the linear elastic laminate model has sufficient accuracy in the evaluations of interlaminar stresses. Second, the singularity problem of inter-laminar stresses is assessed by examining the dominant fields of singular stress potential and the effective modules concept. Third, characteristics of interlaminar stress distributions over the dimensions of the fibers diameter are discussed, based on a heterogeneous model. These results aided in better understanding of the nature of free-edge interlaminar stresses in composite laminates.

Reddy J.N. (1990), a review of all third order, two dimensional technical thesis of plates that satisfy vanishing of transverse shear stresses on bounding planes of the plate is presented and their equivalence is established. A general, strain-consistent third-order displacement field is proposed and associated Von-Karman theory for the static case is developed. The theory gives, a special case, all third order theories with parabolic distribution of transverse shear stresses.

Kant T. and Manjunath B.S (1990), a simple $C^0$ isoparametric finite element formulation based on a set of higher-order displacement models for the analysis of symmetric and asymmetric multilayered composite and sandwich beams subjected to sinusoidal loading is presented. These theories do not require the usual shear correction coefficients which are generally associated with the Timoshenko theory. The four-noded Lagrangian cubic element with kinematic models having 4, 5 and 6 degrees of freedom per node is used.

Lee K.H., Senthilnathan N.R., et al (1990), a simple layer wise higher-order zig-zag model is proposed for the bending of laminated composite plates. This model accounts for a cubic variation of the in-plane displacements in each layer and a parabolic variation of the transverse shear stresses across the laminate with zero values at the free surfaces. By imposing the continuity of the in-plane displacements and the transverse shear stresses at the interfaces, the number of variables is shown to be same as those given by Mindlin's
theory. The numerical results obtained for the cylindrical bending of symmetric laminates were compared with those from exact elasticity solutions.

Reaz A. Chaudhuri and Kamal R. (1991), the analytical or differential form of solutions to the problems of: (i) axi-symmetric angle-ply circular cylindrical planes of rectangular form and (ii) circumferentially complete circular cylindrical shells, subjected to transverse load and with SS2 type simply-supported boundary conditions at the edges, is presented. These problems under investigation have been solved, using a boundary discontinuous double Fourier series approach.

Barbero E.J. and Reddy J.N. (1991), extended the layer-wise laminate theory of Reddy to account for multiple delaminations between layers and the computational model is developed. Delaminations between layers of composite plates are modeled by jump discontinuity conditions at the interfaces. The strain energy release rate distribution along the boundary of delaminations is computed by a novel algorithm.

Mao K.M. and Sun C.T. (1991), proposed a global-local method to improve the efficiency of finite element analysis. This method, apart from the regular finite element method consists of three basic steps, i.e. the global-analysis, the local analysis and the refined global analysis. The first two steps involve a coarse finite element mesh to analyze the entire structure to obtain the nodal displacements, which will be subsequently used as displacement boundary conditions for local region of interest. These local regions with the prescribed boundary conditions were then analyzed with refined meshes to obtain more accurate stresses. In the third step, a new global displacement distribution based on the results of previous two steps was assumed, from which improved solutions for both stresses and displacements were produced. The above proposed method yields accurate solutions with significant savings in computing time when compared with regular finite element method.

Maenghyo Cho and Reid parameter R (1992), demonstrated an efficient higher – order plate theory for laminated composites. By superposing a cubic varying displacement field on a zig-zag linearly varying displacement theory for symmetric laminated composites is obtained. In this the displacement satisfies transverse shear stress continuity conditions at the interface between layers as well as shear free surface conditions. Finally
to demonstrate and compare with other theories, the analytical solution for cylindrical bending is obtained.

Cheung Y.K and Shenglin Di (1993), developed a simple quadrilateral hybrid stress multilayered laminated plate element using a rational approach which incorporate the energy constraints and higher-order shear deformation theory. They have solved a variety of laminated plate problems and the results are compared with other available solutions. The accuracy and efficiency of the hybrid element is verified by several examples.

Kant T and Manjunath B.S (1994), proposed new numerical algorithms for the accurate evaluation of transverse stresses in general composite and sandwich laminates. A set of higher-order theories with $C^0$ iso-parametric finite elements and exact three dimensional equilibrium equations are used. The integration of these equilibrium equations was carried out through exact surface fitting method, direct integration method and forward and central direct finite difference methods. The results obtained by these methods are closed to the available elasticity and other closed form solutions.

Kant T and Kommineni J.R. (1994), presented a $C^0$ continuous displacement based finite element formulation of a higher-order theory. This theory accounts for linear and geometrically non-linear analysis of symmetrically laminated composite and sandwich shells under transverse loads. The displacement model accounts for nonlinear and constant variation of tangential and transverse displacement components through the shell thickness. The assumed displacement model eliminates the use of shear correction coefficient. The refined higher-order shear deformation theory presented is essential for predicting accurate response especially for sandwich shells.

Yung M.W. and Jiann Q.T. (1994), developed an asymptotic theory for bending and stretching of anisotropic inhomogeneous laminated plates. The in homogeneities are considered to vary through the plate thickness and laminated plate belongs to an important class of inhomogeneous plate. Taking appropriate non-dimensionalization of the basic equations and expansion of the displacements and stresses in powers of a small parameter, obtained a set of differential equations of various orders, which can be integrated successively to determine the three-dimensional solutions for the anisotropic inhomogeneous plate under lateral tractions and edge loads.
Abhijit Mukherjee and Lalchand C. Menghani (1994), developed a higher-order theory for the shear deformations in composite plates. The element has been developed using the quadratic iso-parametric shape functions, which can accommodate curved boundaries. The stiffener element has been developed in such a fashion to make the mesh division free from the location of the stiffener. This is advantageous for optimization of the path of the stiffener in a stiffened plate structure.

Thomas D.M. and Weber J.P.H. (1994), produced the delamination of tapered laminated plates based on fracture mechanics. To show the effect of dropped ply thickness, number of delaminating surfaces, and dropped ply axial stiffness on the delamination load, some simple laminate examples are considered.

Byji Varghese & Abhijit Mukherjee (1997), a novel ply drop-off element has been developed for the analysis of tapered laminated composites. The element has the capability to accommodate the drop off within the element. So drop-off need not lie on the nodal lines. As a result, drop off location is independent of mesh division. This facilitated the use of a coarse mesh at the ply drop location even when the stagger distance is very small. As the dropped layer size is different from that of the elements, a special integration technique has been adopted for developing the stiffness matrices of dropped layers.

Aditi Chattopadhyay and Charles E. Seelay (1997), developed a refined higher-order laminate theory to analyze smart materials, surface bonded or embedded, in composite laminates. This analysis uses a refined displacement field which accounts for transverse shear stresses through the thickness non-linearities are introduced through the strain dependent piezoelectric coupling coefficients and the assumed strain distribution through the thickness. Analysis is implemented using the finite element method.

Wisheart M and Richardson M.O.W. (1997), developed an interface element specifically to model delamination in composite materials. The element is verified under three different modes of loading for both two and three dimensional models. Specific impact induced delamination modes were also investigated for a three ply pultruded composite system with the delamination initiated by inserting intra-laminar cracks manually into the model.
Jung-seok Kin, Chun-Gon Kim et al (2001), optimized the tapered composite laminates by a patch-wise lay-up design. In this approach, the weight of tapered composite laminates under strength constraints was minimized. For this purpose, stacking sequences and the number of plies were optimized. The design variables were the discrete ply angles such as $0^\circ$, $\pm 45^\circ$, and $90^\circ$ and the number of plies in each patch. The design results are compared with the uniform thickness laminates. It is shown that optimized lay-up considerably reduced total weight of composite laminates.

Cho M. and Kim J.S. (2001), developed a higher order zig-zag theory for laminated composite plates with multiple delaminations. By imposing top and bottom surface transverse shear stress-free conditions and interface continuity conditions of transverse shear stresses including delaminated interfaces, the displacement field with minimal degree of freedom is obtained. This displacement field can systematically handle the number, shape, size and location of delaminations. The dynamic equilibrium equations and variationally consistent boundary conditions are obtained through dynamic version of variational approach. The present higher-order zig-zag theory works as an efficient tool to analyze the static and dynamic behaviour of the composite plates with multiple delaminations.

2.3 VIBRATIONAL BEHAVIOUR OF LAMINATED COMPOSITE PLATES – A LITERATURE SURVEY:

Anderson R.G., Irons B.M and Zienkiewicz O.C. (1968), calculated frequencies and buckling loads of plates using simple displacement functions for the flexure of triangular plate elements. They attained good accuracy in frequency calculations, and made good predictions for stability, using coarse element subdivisions. They have used a powerful eigen value economizer technique which permits very fine subdivisions of elements with eigen value calculations of limited size.

Charles W. Bert and Byron L. Mayberry (1968), presented a linear analysis for determining the natural frequency of vibration of laminated anisotropic rectangular plates. They have considered the plate consisting of an arbitrary number of thin orthotropic layers, the major material symmetry axis of each layer oriented arbitrarily with respect to the longer plate edge. They have presented the numerical results for fully clamped boundary
conditions for symmetric and unsymmetrically laminated plates, using Rayleigh Ritz's free vibrational analysis.

Srinivas and Rao A.K. (1970), presented a unified exact analysis for the static and dynamics of class of thick laminates. They developed a three-dimensional linear small deformation theory of elasticity solution for the bending, vibration and buckling of simply supported thick orthotropic rectangular laminated plates by taking all the nine elastic constants of orthotropy. The solution obtained is exact and lead to simple infinite series for stresses and displacements in flexure, forced vibration and “beam column” type problems and to closed form characteristic equations for free vibration and buckling problems. For free vibration of plates, the present analysis yields a triply infinite spectrum of frequencies instead of only one doubly infinite spectrum by thin plate theory, or three doubly infinite spectra by Reissner/ Mindlin type analysis.

Srinivas S, Joga Rao C.V. and Rao A.K. (1970), derived a closed form characteristic equation for obtaining natural frequencies of thick, simply supported, homogeneous or laminated rectangular plates. The solution involves, determining a triply infinite sequence of eigen values from a doubly infinite set of closed form transcendental equations. As no restrictions are placed on the thickness variation of stresses or displacements, this formulation yields a triply infinite spectrum of frequencies. Also this analysis yields symmetric thickness modes. They have also extended analysis for laminated plates of isotropic materials.

Alfred T. Jones (1970), analyzed, vibration of a simply supported cross-ply laminated plates infinite in length. In this paper existence of plane strain conditions allow an exact solutions of the field equations. Curves of variation of exact natural frequencies versus wave length for several cases of a two ply laminated plate are calculated. The first two mode shapes are plotted for various wave lengths and compared to classical plate theory. In this the extensional vibration is approximated marginally only for very long wavelengths.

James A. Bennett (1971), investigated the large amplitude oscillations of a simply supported plate, constructed of plies alternately oriented at ±θ and un-symmetrically laminated about the mid surface. He indicated that for an angle-ply laminated plate, it is
possible to reduce the non-linear partial differential equation to ordinary differential
equations using Galerkin’s method. It is concluded that for the single mode solution, the
bending-stretching coupling term, do not directly enter the non-linear term in the
amplitude-frequency equation. The angle of the ply laminates determines the degree of
non-linearity of the plate. Although the bending stretching terms do enter the non-linear
terms for the multispacial mode solution these terms are small. The type of instability
response is dependent on the angle of the laminate but there are insignificant differences
between the orthotropic case and the coupling case.

Alfred T. Jones (1971), has exactly solved the three-dimensional elastic field
equations that describe the vibration of a class of thick orthotropic laminate. The class is
characterized by an arbitrary, in-plane, angle of inclination of the principal orthotropic axes
to the plate axes. The author has presented the frequency spectra and model functions for
two types of materials and five inclination angles. His work also showed that the classical
flexural plate theory is adequate for vibration with wavelengths greater than twenty times
the thickness.

Ahmed K Noor (1972), presented some of the results of his study over the reliability
and range of validity of two-dimensional plate theories. These theories are applied to the
low-frequency free vibration analysis of simply supported, bi-directional, multi layered
plates consisting of a large number of layers. He stated that for composite plates the error
in predictions of the CLPT is strongly dependent on the number and stacking of the layers
in addition to the degree of orthotropy of the individual layers and the thickness ratio of the
plate. For skew-symmetric laminated plates he has shown that the error in the fundamental
frequency sharply increases as the number of layers increases from two to four, and then
becomes insensitive to further increase in the number of layers.

Sun C.T. (1973), investigated the shock fronts in laminated plates subjected to
impact loadings by the ray theory. He derived the Kinematic conditions of compatibility
and dynamical conditions on the wave fronts. The propagation velocity, expression for the
position of the wave front and the ray geometry are presented. He found the direction and
propagations of wave front decay using the ray geometry. He also discussed the numerical
results for cross-ply laminates.
Lin C.C and King W.W. (1974), estimated the natural frequencies of un-symmetric cross-ply and anti-symmetric angle-ply rectangular plates. They have studied the free transverse vibrations of un-symmetrically laminated plates according to the classical Kirchoff's assumptions. The special character of the governing equations for these classes of laminates make applicable Bolotin's asymptotic analysis, which allows any homogenous boundary condition uniformly applied along an edge. This method is expected to yield greater accuracy for the higher modes.

Hinton E. (1976a, 1976b), extended the finite strip method, introduced by Cheung, to cater for transverse shear deformation effects. This paper mainly presents a note on a thick finite strip method for the free vibration of laminated plates. He also discussed the use of a parabolic plate bending element for the free flexural vibration analysis of symmetrically laminated plates in which transverse shear deformation and rotary inertia effects are considered. The basic steps used are identical to those used in a similar study involving the finite strip method.

Chandra R. (1976), has dealt with the large deflection flexural vibration of un-symmetric cross-ply laminated plates which are simply supported at two opposite edges and clamped at other two edges. He considered stress free and movable in plane boundary conditions. Non-linear frequency is obtained as a function of lamination parameters, material constants, aspect ratio, linear frequency and amplitude of vibration.

Cha C.Y. and Prabhakara M.K. (1978), presented an analysis for the non-linear flexural free vibrations of rectangular un-symmetric cross-ply and angle-ply plates. They have formulated a solution of dynamic Van-karman type equations of plates for general mode vibration. The non-linear frequency of a composite plate with all clamped and all simply supported edges increases with the amplitude and hence only a hardening type of non-linearity is noted. They have concluded that for a given amplitude the non-linear frequency decreases with the aspect ratio or the orientation angle of filaments but increases with total number of layers or the tensile moduli of the plate.

Bert C.W. and Chen T.C.C. (1978), solved the effect of shear deformation on vibration of anti-symmetric angle ply laminated rectangular plate for the case of all edges simply supported. They used the displacement formulation of the heterogeneous shear
deformation plate theory of Yong, Norris and Stavsky. The numerical results are studied by them for a highly directional composite materials (high modulus). The results are presented showing the parametric effects of aspect ratio, length to thickness ratio, number of layers, and lamination angle. They also investigated the effects of deleting rotatory inertia and in-plane inertia, singly and in combination. It is discussed that the relative effect of transverse shear deformation on the fundamental frequency is greater for anti-symmetric angle-ply plates than for homogeneous isotropic plates of the same dimensions.

Reddy J.N. (1979), has generalized the Mindlin's plate theory for isotropic plates to laminated anisotropic plates and included shear deformation and rotary inertia effects. He has presented the finite element solutions for rectangular plates of anti-symmetric angle-ply laminates whose material properties are typical of a highly anisotropic composite material. He used two sets of material properties that are typical of higher modulus fiber reinforced composites to show the parametric effects of plate aspect ratio, length to thickness ratio, number of layers and lamination angle.

Reddy J.N. and Kuppusamy T. (1983), described the three dimensional elasticity equations and the associated finite element model for natural vibrations of laminated rectangular plates. A number of cross-ply and angle-ply rectangular plates are analyzed for natural frequencies. They have concluded that for relatively thick plates, the plate element predicts frequencies higher than those predicted by the three dimensional element.

Bhimaraddi and Stevens L.K. (1984), developed a higher order theory for undamped dynamic response of orthotropic homogenous and composite laminated plates. The theory accounts for in-plane inertia, rotatory inertia, and shear deformation effects. This proposed method assumes parabolic variations across the thickness of plate for transverse shear strains. From the results it is concluded that thickness has a more pronounced effects on the behaviour of composite plates than on the behaviour of isotropic plates. The transverse shear deformation effects are more pronounced even in thin composite laminated plates.

Di Sciuva M (1985), has developed non-linear equations of motion for thick multilayered orthotropic plates. This is based on a piece wise linear displacement field which allows the contact conditions for the displacements and the transverse shearing
stresses at the interfaces. In order to assess the proposed theory the simple problem of the bending, free undamped vibration and buckling of three layered, symmetric cross-ply square plate, simply supported on all edges is investigated.

Reddy J.N. and Phan N.D. (1985), used a higher-order shear deformation theory to determine the natural frequencies and buckling loads of elastic plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotatory inertia. A comparison between solutions of simply supported plates, solution of three-dimensional elasticity theory, the first order shear deformation theory and the classical plate theory are made by them.

Levinson M. (1985), developed a three-dimensional solution for the free vibrations of simply supported, rectangular plates of arbitrary thickness within the linear theory of elastodynamics. The solution obtained in a semi-inverse fashion was the solution of the elastostatic problems for such plates, satisfies all of the boundary conditions of the problem in a point wise manner. The two types of modes of oscillations possible are analogues to the flexural modes of classical plate theory and the thickness twist modes which are called breathing modes of Mindlin plate theory. They have shown that the transcendental equation for flexural frequencies reduces, for thin plates, to the frequency equation of classical plate theory for a simply supported rectangular plate. The numerical results obtained are the predictions of Mindlin plate theory for the frequencies of classical type of flexural motion of simply supported rectangular plates.

Reddy J.N. and Putcha N.S. (1985), developed a mixed shear flexible finite element, with relaxed continuity, for the geometrically linear and non-linear analysis of layered anisotropic plates. The formulation based on a refined higher-order theory which satisfies the zero transverse shear stress boundary conditions on the top and bottom faces of the plate and require no shear correction coefficients. The element considered consists of eleven degrees of freedom per node which include three displacements, two rotations and six moment resultants.

Putcha N.S. and Reddy J.N. (1986), developed a mixed shear flexible finite element with relaxed continuity, for geometrically linear and nonlinear analysis of layered anisotropic plates. The element formulation is based on a refined higher-order theory
which satisfies the zero transverse shear stress boundary conditions on the top and bottom faces of the plate and require no shear correction coefficients. The element is evaluated for its accuracy in the analysis of stability and vibration of anisotropic rectangular plates with different lamination schemes and boundary conditions.

Bowlus J.A. , Palazotto et al (1987), conducted a study using Galerkin technique to determine the natural frequencies and mode shapes for symmetrically laminated plates considering the effects of shear deformation and rotary inertia. They have considered two different graphite epoxy symmetric plates for the analysis. They have taken three different boundary conditions for each plate, simply supported, clamped and two opposite sides simply supported. The conducted length to thickness ratio investigations indicate that length to thickness ratio reduce shear deformation effects significantly to lower the natural frequencies.

Owen D.R.J and Li Z.H. (1987), formulated an approximate solution for the bending, vibration and buckling of anisotropic laminates. They have presented a finite element local model and improved sub-structuring technique which are applicable to both the analysis of ‘local response’ and ‘global response’. The present model takes into account any lamina material properties and displacement boundary conditions at the lamina level. For static analysis, it reduces the total number of variables to those of one layer and for vibration and buckling analysis. It also reduces the equations to those associated with the transverse displacement variables. The predictions made by them for anisotropic laminated plate behaviour are compared with 3D elastic solutions.

Khdeir A.A. (1988), developed analytical procedure to investigate the free vibration of anti-symmetric angle-ply laminated plates. He has considered a generalized Levy type solution in conjunction with the state space concept, which enables to solve exactly the equations governing the laminated anisotropic theory considered by Yang, Norris and Stavsky. The theory is a generalization of the Mindlins theory for isotropic plates to laminated anisotropic plates and includes shear deformation and rotatory inertia effects. He concluded that, for all the boundary conditions involved, the effects of in-plane and rotary inertias on the dimensionless fundamental frequency are insignificant even for a relatively thick plates.
Khdeir A.A. and Librescu L (1988), analyzed the free vibration and buckling problems of rectangular cross-ply laminated plates, using the higher-order plate theory as well as the technique based on the state space approach. They have provided a mathematical tool to treat the response in buckling and free vibration of composite plates for a variety of boundary conditions.

Khdeir A.A. (1989), analyzed the free vibration and buckling of un-symmetrical cross-ply plates with a variety of boundary conditions, using a refined shear deformation theory. This theory incorporates transverse shear deformation and transverse normal stress effects. The results obtained are compared with classical and first order transverse shear deformation theories. He found that with simply supported edge conditions, the results obtained are in accordance with their counter parts.

Kant T and Mallikarjuna (1989), presented a higher-order theory for free vibration analysis of un-symmetrically laminated multilayered plates. This theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotary inertia effects. A simple $C^0$ finite element formulation for the vibration of anisotropic laminates is presented. They concluded that the effects of plate aspect ratio on the fundamental frequency and transverse shear moduli of stiff layers on the natural frequencies are more in thick plates than in thin plates.

Mallikarjuna and Kant T (1989), determined the natural frequencies of isotropic, orthotropic and layered anisotropic composite and sandwich plates, using a $C^0$ finite element formulation of the higher order theory. To show the parametric effects of plate aspect ratio, length-to-thickness ratio, degree of orthotropy, number of layers and lamination scheme, material properties of a typical high modulus fiber reinforced composites are used. The theory represents a quadratic variation of the transverse shearing strains and linear variation of the transverse normal strains through the plate thickness. A special mass matrix diagonalization scheme is adopted which conserves the total mass of the element and includes the effects due to rotary inertia terms. Apart from these it is advantageous that the increase in number of layers without changing the total thickness increases the fundamental frequency.
Mallikarjuna and Kant T (1989a, 1989b), used a ‘C⁰’ finite element formulation of
the higher-order theory to determine the natural frequencies of isotropic, orthotropic and
layered anisotropic composite and sandwich plates. They considered the material
properties of high modulus fiber reinforced composites to show the parametric effects of
plate aspect ratio, length to thickness ratio, degree of orthotropy, number of layers and
lamination schemes. The theory represents quadratic variation of the transverse shearing
strains and linear variation of the transverse normal strains through the plate thickness.
They concurred that, the increased in number of layers with out changing the total
thickness increases the fundamental frequency. They also presented a refined higher order
theory with simple C⁰ finite element formulation for the vibration of anisotropic laminates.
This theory accounts for parabolic distribution of transverse shear strains through the
thickness of the plate and rotary inertia effects. The predicted frequencies are compared
with first-order and classical plate theories by them.

Ahmed K Noor and Scott Burton W (1989), presented a two-phase computational
procedure for the accurate prediction of the frequencies, stresses and deformations in
multilayered composite plates. In the first phase they have predicted global response
characteristics using first-order shear deformation theory. In the second phase, equilibrium
equations and constitutive relations of the three-dimensional theory of elasticity are used.

Sivakumaran K.S.(1989), has carried out free vibration analysis to study the vibration
characteristics of undamaged annular and circular asymmetric composite plates. This
analysis includes bending-stretching coupling and rotatory inertia effects. He employed
Rayleigh-Ritz energy approach to obtain the approximate natural frequencies and
associated mode shapes. He documented the effects of the size of the plate, number of ply
groups, fiber orientation and hybridization on the natural frequencies of asymmetrically
laminated annular and circular plates.

Kant T and Mallikarjuna (1989), analyzed vibrations of laminated composite and
sandwich plates in conjunction with a C⁰ iso-parametric finite element formulation and
they used a special mass lumping procedure in the dynamic equilibrium equations. The
results obtained are compared with 3-D elasticity and Mindlin’s plate solutions.
Gajbir Singh, Venkateswara Rao G et al (1990), proposed a direct numerical integration method for accurate predictions of non-linear frequencies for rectangular anti-symmetric cross-ply composite plates. The perturbation method used widely, fails to yield any good results when bending extension coupling is large.

Ganapathi M, Varadan T.K. and Sarma B.S. (1991), studied the large amplitude free flexural vibrations of laminated plates using $C^0$ shear flexible QUAD-8 plate element. They solved the non-linear governing equations using direct iteration technique. Results are obtained for isotropic and cross-ply laminated plates with simply supported boundary conditions on immovable edges. They observed that the hardening behaviour increases for thick plates and orthotropic plates.

Palazotto A.N. and Peter E.L. (1991), developed a theory which is applicable to symmetrically laminated anisotropic circular cylindrical shell panels of arbitrary geometries. To determine the frequencies and critical buckling loads of the panel considered, an analytical study is conducted and higher-order constitutive relations are derived for the laminate. This method converges quickly in the determination of frequencies.

Cho K.N., Bert C.W. et al (1991), determined the natural frequencies, the relative stress and deflection distributions through the thickness of simply supported rectangular plate, using a higher-order plate theory. This theory approximates the in-plane and normal displacements by third and second order functions of the thickness coordinates. The theory has an advantage of predicting more free vibration modes (as in three dimensional elasticity theory) than the other approximate theories. By accounting for the second order distribution of transverse normal displacement through each layer thickness, accurate distribution of the relative transverse normal stresses and the deflection can be predicted.

Chandrashekara S. and Santhosh U. (1991), presented a mixed method of elastodynamics, the state space approach for analyzing the dynamic response of composite plates. They employed the principle of reducing the governing three-dimensional equations to a set of two-dimensional equations while keeping the field equations in their exact form. In the results natural frequencies for thick orthotropic laminae with varying angles of
inclination of the orthotropic axes are presented for two distinct materials and for large values of thickness.

Ahmed K. Noor and Scott Burton W (1992), presented analytical 3-D elasticity solutions for the free vibration and buckling of thermally stressed multilayered angle-ply composite plates. Sensitivity derivatives are also evaluated and used to study the sensitivity of the vibrational and buckling response to variations in different laminated plates. A Lagrangian description of the deformation, which accounts for the large displacements and rotations, is used. A mixed formulation is used with the fundamental unknowns consisting of the six perturbation stress components and the three perturbation displacements components of plate. Results are presented showing the effects of variations in material characteristics and fiber orientation of different layers. The effects of initial thermal deformations on the vibrational and buckling responses of the plate are also shown by them.

Bhimaraddi A and Chandrashekhara K. (1993), considered large amplitude vibration of imperfect angle-ply plates at elevated temperatures using the parabolic shear deformation theory. They have retained the strains due to initial imperfections, using Von-Karman type large deflection model. Results are obtained by using the single-mode approach to simply-supported plates, thus reducing five governing equations to a single non-linear time differential equation involving quadratic and cubic non-linearities. Results also demonstrated that the non-linear effects are higher in two ply anti-symmetric plates as compared to 4 and 6 ply plates.

Georges Akhras, Cheung M.S. et al (1993), developed a shear-deformable finite strip to analyze the static and vibrational behaviour of composite laminates. By selecting the proper displacement functions, the new strip includes all the effects of material anisotropy and suits arbitrary in-plane boundary conditions at the ends. They have applied this method not only to cross-ply laminates and symmetrical angle-ply laminates, but also to arbitrary angle-ply laminates.

Kant. T and Kommineni J.R. (1994), developed a higher-order shear deformation theory for large amplitude and free vibration analysis of fiber reinforced cross-ply composite and sandwich laminates, assuming cubic variations of in-plane displacement
components and a constant transverse displacement component through the thickness of the laminate. They adopted a displacement based finite element method of analysis using C^0 isoparametric nine noded quadrilateral elements of Lagrangian family. A mass matrix diagonalization scheme is employed which conserves the total mass of the element and includes the effects due to rotary inertia terms. The results of the present theory showed by them are considerable warping of the cross section for composite sandwich laminates.

Marur S.R. and Kant T. (1996), proposed a higher-order displacement model for the free vibration analysis of sandwich and composite beam fabrications. These theories are modeled by taking cubic variation of axial strain and eliminate the need for shear correction coefficient. They have assumed a quadratic shear strain variation across the depth of the cross section. Numerical results for various lamination schemes, boundary-conditions, and aspect ratios are carried out to compare with the first-order shear deformation theory.

Franco Correia V.M., Mota Soares C.M. et al (1997), dealt with the structural optimization of multilaminated composite plate structures of arbitrary geometry and lay-up, using single layer higher-order shear deformation theory discrete models. They have developed a structural and sensitivity analysis formulation for a family of C^0 Lagrangian elements. The objectives of design sensitivities considered are the maximization of natural frequencies of specified modes or minimization of the structural weight or volume.

Greenberg J.B. and Stavsky Y. (1999), analyzed the axi-symmetric vibrational behaviour of two concentric dissimilar isotropic composite plates. The major factors influencing the natural frequency of vibration of these structures are found to be i) the different material compositions of the two plates ii) the plate radius. They found that for certain combinations of material lay-ups higher frequencies can be achieved than for equivalent single plate structures.

Ji Wang, Yook – Kong Yong et al (1999), derived a finite element formulation of the piezoelectric vibrations of quartz resonators based on Mindlin plate theory. From the results, they found that this generalized formulation is straightforward and effective for the piezoelectric plate vibration problems and through the standard finite element procedure,
the vibration frequency, the vibration mode shapes and the electric potential distribution are obtained by them.

Kant T and Swaminathan K (2001), presented solutions to the natural frequency analysis of simply supported composite and sandwich plates. This model incorporates laminate deformations which account for the effects of transverse shear deformation, transverse normal strain/stress. The equations of motion are obtained using Hamilton's principle. They compared solutions of the laminated composite plates with the three dimensional elasticity solutions and found the percentage error with respect to three-dimensional elastic solutions is less compared to other shear deformation theories.

Ganapathi M and Patel B.P. (2002), investigated the non-linear free flexural vibrations of thick composite and sandwich plates, using a $C^0$ eight-noded plate element. The geometric non-linearly is introduced in the formulation based on the relevant Green's strain vector for the laminate. The effectiveness of this model over first and other higher-order theories, for non-linear dynamic analysis, is discussed.

Green Berg J.B. and Stavsky Y (2002), studied the axi-symmetric vibrational behaviour of two concentric dissimilar orthotropic composite annuli. Sixth order systems of equations of motion for each annulus are presented together with the matching conditions at their interface. Finite difference method is used to obtain the common frequency of vibration of the structure. The governing field equations are solved numerically using a fourth order finite difference method. They have shown that, in the absence of appropriate analytical equations, optimization and control of frequency of composite plate structures must make use of parametric studies.

Zhao Y.B. and Wei G.W. (2002), introduced the discrete singular convolution (DSC) for the vibration analysis of rectangular plate with non-uniform and combined boundary conditions. They proposed a systematic scheme for the treatment of boundary conditions. They obtained very accurate DSC results for the analytically solvable case of all side simply supported problem.
2.4 BRIEF REVIEW OF THE AVAILABLE LITERATURE ON THERMAL ANALYSIS OF COMPOSITE LAMINATED PLATES

Srinivas S and Rao A.K. (1972), discussed the analysis of thermally loaded laminated plates using the theory of elasticity. They analyzed the flexure of simply supported rectangular laminates made up of isotropic layers.

Padovon J (1975), investigated the effects of mechanical and thermal loads on the local stationary fields of generally laminated plates, based on three dimensional thermo elasticity theory. In particular, to model the stated problem, each individual lamina of the slab is considered to be composed of mechanically and thermally possible fully anisotropic materials whose fields satisfy 3-D elasticity and conduction theory. He used complex series expansions, together with the properties of complex adjoint differential forms and obtained a 3-D solution of the given model. In the results he emphasized the effects of material anisotropy on the governing fields of symmetric, alternating and single type laminates.

Tungikar V.B and Koganti M.Rao (1994), presented a three dimensional exact solutions for temperature distribution and thermal stresses in simply supported finite rectangular orthotropic laminates and is subjected to prescribed boundary conditions under combined thermal and mechanical loading, which will be used to check the accuracy of more generalized numerical tools. In the results they have shown: (i) the influence of the thermal field is very predominant and (ii) the thermal field induces a high transverse normal stress \( \sigma_z \) at the interfaces which may cause failure of the weak adhesive bond.

Savoia M and Reddy J.N. (1995), discussed the stress analysis of multilayered plates subjected to thermal and mechanical loads in the context of the three-dimensional quasi-static theory of thermo elasticity. They have derived, governing equations for a plate composed of monoclinic layers and are subjected to any given temperature and mechanical load distributions in terms of displacements. Solutions are obtained for cross-ply and angle-ply laminated rectangular plates subject to thermo mechanical loads, making use of a Navier like approach. In the results they have shown that the in-plane shear stresses at the corners are unbounded when plate faces are subjected to uniform heating.
Ali J.S.M., Bhaskar K and Varadan T.K. (1999), formulated a new displacement-based higher-order theory. The theory employs realistic displacement variations and is shown to be accurate for even thick laminates and for any combination of mechanical and thermal loading. The theory accounts for interface slope discontinuity of the in-plane displacements as well as thickness stretch/contraction effects. It has been proved that this theory is economical compared to a layer wise theory in which the number of unknowns remains constant irrespective of the number of layers.

Surot Thangjitham and Hyung Jip Choi (1991), considered a multilayered anisotropic medium under the state of generalized plane deformation for thermal stress analysis. They derived general solutions of thermo elasticity for layers with transversely isotropic, orthotropic, and monoclinic properties, utilizing a Fourier transform technique. They have presented a flexibility/stiffness matrix method for the heat conduction and thermo elasticity problems, subjected to arbitrary thermal and mechanical loadings applied on the boundary surfaces.

Theodore R Tauchert (1991), has made a survey of investigations concerned with the response of flat plates to thermal loadings. He focused on three major topics: (i) thermally induced bending (ii) buckling, post buckling and large deformation behaviours and (iii) vibrational characteristics associated with elevated temperatures and rapid heating. He considered both thin and moderately thick plates, having various forms and support conditions.

Dafedar J.B. and Desai Y.M. (2002), investigated buckling response of laminated composite plates subjected to mechanical and hygrothermal loads. For this they used analytical mixed theory based on the potential energy principle. They proposed two sets of higher order mixed models on the basis of an individual layer as well as equivalent single layer theory displacements including transverse stress continuities that are enforced in the formulation of models by incorporating displacement and transverse stresses. They have obtained modal transverse stresses and displacements as eigen vectors, so that the stresses **need not be evaluated separately.**
2.5 TRANSIENT ANALYSIS OF LAMINATED COMPOSITE PLATES – RELATED STUDIES:

John W Leech (1965), investigated the transient response of thin shells. In his study he considered certain "finite difference forms" of the equation of motion which governs the transient response of a flat plate. In particular, the problem arose as to what the limits on the ratio of the time mesh to the space mesh might be for an explicit finite-difference formula that would be useful for a numerical calculation of the transient response of plates.

Yu-Chung Lee and Herbert Reismann (1969), took the study of dynamic response of a simply supported rectangular plate, treated with in the framework of classical three dimensional elasticity theory. The plate is subjected to a suddenly applied, uniformly distributed transverse load over a rectangular area, the sides of which are parallel and perpendicular to the sides of the plate. The analysis affords a comparison of predictions of classical plate theory, improved plate theory and 3-D classical elasticity theory.

Tien Yu Tsui and Pin Tong (1971), discussed the stability of transient solution of moderately thick plate by finite difference method. Their work is an extension of transient response of "thin" flat plate given by Leech.

Kulkarni S.V and Pagano N.J (1972), presented an exact solution for the vibration of elastic composite laminates in cylindrical bending. They made a comparison of dispersion curves for multilayer symmetrical and unsymmetrical laminates with materials possessing high and low degrees of anisotropy at various fiber orientations, with those obtained from an approximate shear deformation theory. They have also developed equations for the wave propagation in an infinite medium consisting of a repeated pair of anisotropic layers.

Sun C.T. and Whitney J.M. (1972), investigated the effect of the heterogeneous shear deformation over the thickness of the plate on the dynamic behaviour of laminated plates and developed three theories for general laminated plates based upon different assumptions on the local transverse shear deformation and the interface conditions. They have presented a special formulation for laminates with midplane symmetry. A comparison is made between dispersion curves for plane harmonic wave propagations that are obtained from plate theories, with the curves obtained from the exact elasticity analysis.

Sun C.T. and Whitney J.M et al (1975), analyzed the dynamic response of an infinitely long simply supported composite plate under an uniform dynamic pressure, on the upper surface of the plate. considering two cases of dynamic pressures, viz., a) keeping the
magnitude of the uniform pressure unchanged. b) increasing uniform pressure linearly as a function of time and reported the dynamic response characteristics in terms of normal displacement, bending stress, inter laminar shear stress, and inter laminar normal stress, comparing with the corresponding quantities obtained statically.

Pica A and Hinton E (1980, 1981), presented a unified approach for the static and transient dynamic linear and geometrically nonlinear analysis of Mindlin plates with initial imperfections, taking into account the effects of transverse shear deformation and rotatory inertia. A finite element idealization is adopted and the quadratic Lagragian-Mindlin plate element is used together with selective integration. Their further studies include: a) the use of an alternative modified mass matrix b) the use of adaptive damping. c) comparisons of static solutions obtained using pseudo-transient (PT), Newton–Raphson (NR) and modified Newton–Raphson (MNR) schemes.

Reddy J.N. (1983a), investigated forced motions of laminated composite plates using a finite element that accounts for the transverse shear strains, rotary inertia, and large rotations. Numerical results of the nonlinear analysis of composite plates are presented showing the effects of plate thickness, lamination scheme, boundary conditions, and loading on the deflections and stresses. The present analysis does not account for material damping effects.

Reddy J.N. (1983b), employed a shear–flexible finite element to investigate the transient response of isotropic, orthotropic and layered anisotropic composite plates. He established numerical convergence and stability of the element using Newmark’s direct integration technique. He also investigated the parametric effects of the time step, finite element mesh, lamination scheme and orthotropy on the transient response.

Chen J.K and Sun C.T. (1984, 1985), used the finite element method in investigation of nonlinear transient response of initially stressed composite plates and developed a nine-noded isoparametric quadrilateral element to model laminated plates under initial deformation and stress. The boundary conditions are assumed to be simply-supported and immovable in the plane of plate along all the edges. They further worked on impact responses of composite laminates with and without initial stresses, using the finite element method, using a nine noded isoperametric quadrilateral element based on the mindlin plate theory. An experimentally established contact law which accounts for the permanent indentation is incorporated into the finite element program to evaluate the impact force. In
the time integration, the Newmark constant acceleration algorithm is used in conjunction with successive interactions with in each time step.

Khdeir A.A. and Reddy J.N. (1988), studied the transient response of simply supported anti-symmetric angle-ply rectangular plates subjected to arbitrary loading using the state variable technique to solve the equations of motion of first order transverse shear deformation theory as well as classical laminate theory. Solution obtained by the above two theories are considered to bring out the influence of the transverse shear deformation, the degree of anisotropy and the number of layers.

Mallikarjuna and Kant T (1988), presented a simple isoparametric finite element formulation based on higher-order displacement model for dynamic analysis of multi-layer symmetric composite plates with an explicit time marching scheme. The present higher-order theory proposed is more accurate for the evaluation of plate response of different types of dynamic loads. And also a special mass lumping scheme is adopted which conserves the total mass of the element and includes the effects due to rotary inertia terms and also reported the parametric effects of the time step, finite element mesh, lamination scheme and orthotropy on the transient response.

Kant T, Ravichandran R.V., Pandya B.N. and Mallikarjuna (1988), developed a higher-order shear deformable \( C^0 \) continuous finite element, and is employed to discuss the transient response of isotropic, orthotropic and layered anisotropic composite plates. The governing ordinary linear differential equations are integrated using the central difference explicit time integration scheme. They have presented numerical results for deflections and stresses for rectangular plates under various boundary conditions and loadings. Also the parametric effects of the time step, finite element mesh, lamination scheme and orthotropy on the transient response are investigated.

Kant T and Patel S (1990), presented a unified approach for the static and transient linear and geometrically non-linear analysis of two-dimensional problems. They have used a finite element idealization with 4, 8 and 9 noded isoparametric quadrilateral elements for space discretization. In this an explicit central difference time marching scheme is employed for time integration of the resulting discrete ordinary differential equations.
Mallikarjuna and Kant T. (1990), presented a finite element formulation for transient dynamics of a symmetrically laminated plate based on higher-order displacement model and 3-dimensional state of stress and strain. This higher-order theory incorporates linear variation of transverse normal strain and parabolic variation of transverse shear strains through the plate thickness. They have shown numerical results for deflections and stresses giving the effects of plate side to thickness ratio, material orthotropy, finite element mesh and lamination scheme.

Kant T, Varaiya J.H. and Arora C.P. (1990), derived an isoparametric finite element formulation based on higher-order displacement model for dynamic analysis of multilayer unsymmetric composite plate, using Newmark and Wilson schemes for time integration of discrete coupled second-order ordinary differential equation system of dynamic equilibrium, and found that for larger time step lengths the Newmark scheme is more accurate than Wilson scheme.

Kant T and Mallikarjuna (1991), discussed the impact responses of composite laminates with C⁰ finite elements and developed a nine-noded isoparametric quadrilateral element based on a higher-order theory and Von-Karman large deflection assumptions. They have illustrated the comparison of the contact force histories, deflections and strains in the plate using both the small and large deflection theory and reported in their results that the large deflection theory predicts smaller deflections.

Kant T and Kommineni J.R. (1994), illustrated a C⁰ continuous finite element formulation of higher-order shear deformation theory for predicting the linear and geometrically non-linear transient responses of composite and sandwich laminated shells. The displacement model used accounts for the non-linear cubic variation of the tangential displacement components through the thickness of the shell. The theory also doesn't require any shear correction coefficient. They have compared the finite element results in the linear and geometrically non-linear analysis with other closed form two dimensional solutions.

Marur S.R. and Kant T (1997), adopted a higher-order shear deformable theories, based on isoparametric elements for transient dynamic analysis of symmetric and unsymmetric sandwich and composite beams, employing special lumping scheme for the evaluation of
diagonal mass matrix and a central difference scheme is used for carrying out the integration of the equation of motion. For the unsymmetric composites, higher-order beam theory proposed yields higher in-plane and transverse displacements than those given by the rest.

Niklasson A.J. and Datta S.K. (2002), discussed the transient response of layered anisotropic plate due to an ultrasonic line force of finite bandwidth applied to the surface of the plate. In this, attention is focused on the transient response of homogenous and three-layered tapes assuming that the width is infinite and thickness of the super conducting layer is much smaller than the metal layer. The first-order approximation is used to simplify the problem to that of a homogenous isotropic plate subject to effective boundary conditions representing the thin anisotropic layers.