String theory has started divulging its mysteries in lore at an escalated rate in recent years, so much so that the beginning of this new era of string theory in 1994 is marked as the second superstring revolution [145]. In the aftermath of this revolution, duality symmetry between the hitherto known string theories, which in fact instigated the revolution, has been the prevalent topic of discussion in literature. In this thesis we shall study some aspects of duality symmetries of superstring theories and estimate some of its consequences as the first topic. The pursuit of understanding the different facets of duality symmetry and its connection to string theory as a whole, has led to the conception of a conjectural eleven-dimensional theory, M-theory, where M stands for mystery, magic, or in more mundane terms, membrane [73, 40]. We shall study some aspects of this yet unknown theory as the second topic. The third and final topic concerns a thermodynamic study of a few two-dimensional black holes that make their appearance in some theories of gravity in two dimensions, motivated by string theory.

Before the present era of string theory, it was believed that there were at least five different kinds of ten-dimensional (super)string theories, (since we shall only deal with superstring theories in this thesis, henceforth we shall part with the suffix super in most occurrences of the term) whose dynamics in low energies are governed by a supergravity theory. These are [63]

- Heterotic String Theory with gauge group \( E_8 \times E_8 \), EH for short, with its behavior in low energies described by the ten-dimensional \( N = 1 \) supergravity theory coupled to super-Yang-Mills fields with gauge group \( E_8 \times E_8 \).
- Heterotic String Theory with gauge group \( SO(32) \), OH for short, with its behavior in low energies described by the ten-dimensional \( N = 1 \) supergravity theory coupled to super-Yang-Mills fields with gauge group \( SO(32) \).
- Type-I String theory, with its behavior in low energies described again by the ten-dimensional
\( N = 1 \) supergravity theory coupled to super-Yang-Mills fields with gauge group \( SO(32) \).

- Type--IIA String Theory, with its behavior in low energies described by the ten-dimensional non-chiral \( N = 2 \) supergravity theory.

- Type--IIB String Theory, with its behavior in low energies described by the ten-dimensional chiral \( N = 2 \) supergravity theory.

The capital success of duality is in discerning that these string theories are not completely independent. On the contrary, the present-day notion is that all of these are connected by a web of transformations, namely duality transformations. String theory, thereby, is being contemplated as the generic name of a single theory, possessing non-perturbative features within. Not known in its totality though, it possesses a plethora of vacua connected by dualities in different regimes of the coupling and the moduli, and the five theories mentioned above are but five different vacua that give in to a perturbative formulation. This picture compels string theory to accommodate objects of various dimensions in addition to one-dimensional strings.

Let us start by recalling the notion of duality symmetry in string theory. Conventionally, a string theory is described \([63]\) by the motion of a string in a given space-time, flat or curved, in the presence of other background fields such as dilaton, axion and, in some instances, gauge fields. The two-dimensional surface swept out by the one-dimensional string is referred to as the world-sheet, while the background space-time is called the target space of string theory. All the five string theories mentioned above can be formulated in terms of the world-sheet fields, super-moduli i.e. the vacuum expectation values of the background fields, and topologies of a sigma-model corresponding to the world-sheet as embedded in a given target space. Moreover, in order for this description to qualify as a consistent quantum theory of superstrings, all of these theories are to be defined on a target space of dimension ten, let us denote it by \( \mathcal{M}^{10} \). The effective action for string theory is then derived as the action whose equations of motion are the same as the conditions of vanishing \( \beta \)-functions of the world-sheet theory against fluctuations in the background fields, which render renormalizability to string theory and assumed to be valid for consistent backgrounds of string theory. The effective action of a string theory, say \( \mathcal{S} \), is thus a ten-dimensional action comprising of the metric of \( \mathcal{M}^{10} \), the dilaton, the axion(s) and the moduli fields, if any. As mentioned earlier, the low energy limit of this effective action matches with some supergravity theory in ten dimensions.

Now, considering two string theories \( \mathcal{S}_1 \) and \( \mathcal{S}_2 \) defined on two target spaces \( \mathcal{M}_1^{10} \) and \( \mathcal{M}_2^{10} \), respectively (where \( \mathcal{S}_1 \) and \( \mathcal{S}_2 \) may be same or different), if there exists a transformation that relates \( \mathcal{M}_1^{10} \) with \( \mathcal{M}_2^{10} \) alongwith a redefinition of the corresponding background fields of the two theories, then the two theories \( \{ \mathcal{S}_1; \mathcal{M}_1^{10} \} \) and \( \{ \mathcal{S}_2; \mathcal{M}_2^{10} \} \) are said to be dual to each other. The group of transformations relating the two is called the duality group between the two theories. If the duality group relates a string theory to itself, i.e. \( \mathcal{S}_1 \) is the same as \( \mathcal{S}_2 \), it is a symmetry of
that string theory and the latter is called a self-dual theory. In ten dimensions the duality group includes the diffeomorphism group of $\mathcal{M}^{10}$ with suitable transformations on the other background fields. Other examples of such cases include the self-duality of the $E_8 \times E_8$ Heterotic string theory in four dimensions with one or four supersymmetries [49, 156].

In more realistic considerations, string theories are described on spaces with several compact directions: $\mathcal{M}^{10} = \mathcal{M}^{10-d} \otimes \mathcal{A}^d$, where $\mathcal{A}^d$ is a compact space of dimension $d$. In other words, the string theory is compactified on $\mathcal{A}^d$, or we are considering a string theory on $\mathcal{M}^{10-d}$. In this case there exists a relation that connects target spaces possessing a compact abelian isometry group, arising from their compact parts. This relation goes by the name of target space duality, or T-duality for short. The T-duality group depends on the compact space $\mathcal{A}^d$. Roughly speaking, this relates two theories defined on compact spaces whose volumes are inverse of each other. By way of an example, let the compact space be a one-dimensional circle, $\mathcal{A}^d = S^1$. T-duality then relates the corresponding quantum theories on circles of radii $R$ and $\frac{R'}{R}$, where $\frac{1}{2\pi a'}$ denotes the string tension. We should mention at this point that T-duality is non-perturbative on the world-sheet in the sigma-model coupling constant $\alpha'$, but turns out to be perturbative in the string coupling constant $g_s$.

Moreover, two string theories $\mathcal{S}1$ and $\mathcal{S}2$ may also be related by transformations which are perturbative in $\alpha'$, while non-perturbative in $g_s$, say an inversion of the string coupling, $g_s \rightarrow \frac{1}{g_s}$, for example. This will relate, say, $\mathcal{S}1$ at weak coupling to $\mathcal{S}2$ at strong coupling; and hence called weak-strong duality, or S-duality for short. Import of such a symmetry is immediately obvious: perturbative results in $\mathcal{S}1$ are related to non-perturbative results in $\mathcal{S}2$. However, a complete non-perturbative formulation of string theory to check the S-duality symmetry a-priori is not yet available. The best one can do in want of such a formulation is to assume S-duality to be a symmetry of the theory, and thence test the consistency of the predictions of string theory. A number of such consistency tests have been devised and no discrepancy is noted till date. In fact, one of the major triumphs of these considerations has been the works of Seiberg and Witten [153, 154], in propounding a four-dimensional supersymmetric model exhibiting confinement of quarks.

Thus, we have two possible kinds of duality symmetries, namely, T-duality, that relates a theory compactified on a space whose volume is small to one on a space whose volume is large; and S-duality, that relates a theory with small coupling to one with large coupling. To add to the story of dualities, it also happens that some string theories, especially when considered on spaces of lower dimensions, enjoy a symmetry that contains both S-duality and T-duality; this third kind of duality symmetry is known as U-duality. This relates a theory $\mathcal{S}1$ compactified on a space of small or large volume to another theory $\mathcal{S}2$ with strong or weak coupling, for example. Thus, in this sense, U-duality combines S-duality and T-duality. For example, the $E_8 \times E_8$ Heterotic string theory, EH, on $\mathcal{M}^{10} = \mathbb{M}^3 \otimes T^7$...
has the U-duality symmetry group $O(8,24;\mathbb{Z}) \supset SL(2;\mathbb{Z}) \otimes O(7,23;\mathbb{Z})$ in three dimensions, which contains both of its S-duality group $SL(2;\mathbb{Z})$ and the T-duality group $O(7,23;\mathbb{Z})$ as proper subgroups. We shall study some of the duality symmetries in detail in the next two chapters. Here let us mention some generalities about them.

**T: Target space duality**

Albeit string theory is deemed to contain non-perturbative features, by far the best practical means to describe field theories in general is through perturbative series in some perturbation parameter. String theories are formulated in the same spirit. It is assumed that the two-dimensional world-sheet swept out by the string is a Riemann surface of genus $g$, for all values of $g$. Intuitively this is a two-dimensional generalization of the Feynman diagrams of conventional field theories. The genus $g$ of the world-sheet is treated as the perturbation parameter. The five string theories mentioned above are the only ones known which can be described in a perturbative framework. In these theories T-duality hold order by order in string perturbation theory. That is why it is the best understood among the dualities [162]. A popular way of looking into the T-duality symmetry is as a transformation of the background or moduli fields on the world-sheet of the string. Let us consider a string theory compactified on a circle $S^1$ of radius $R$. One needs to consider two kinds of excitations in this theory — the Kaluza-Klein momentum excitations arising from the mode-expansion along the circle, and the winding number, meaning the number of times the string can wrap around the circle. The first one contributes by $\left(\frac{n}{R}\right)^2$ to the square of the mass of the string, while the second one contributes by $(mR\alpha'')^2$, where $m$ and $n$ are integers and $\frac{1}{2\pi\alpha''}$ denotes the string tension. Thus, the total contribution to the square of mass of the string due to compactification on the circle is

$$\left(\frac{n}{R}\right)^2 + (mR\alpha'')^2,$$

and the radius $R$ does not affect the other terms in the complete expression. This expression, and consequently the mass of the string, has a symmetry under the interchange of $m$ and $n$ with a simultaneous inversion of $R$,

$$m \leftrightarrow n \quad \text{and} \quad R \leftrightarrow \frac{R'}{R}.\]$$

Thus, the string theories are equivalent on circles of very small, $R \rightarrow 0$, and very large, $R \rightarrow \infty$, radii, provided the momentum modes $n$ and the winding modes $m$ are interchanged between the two cases. This is a manifestation of T-duality. It might be pointed out that the momentum modes $n$ are quantum concepts derived from the excitations within the circle, while the winding modes $m$ are obviously classical. Thus a consequence of T-duality transformation is to mix up classical and quantum features in string theories. Moreover, this picture exhibits why T-duality is special to string theories, and does not occur in particle theories. This is due to the fact that although particles can
have Kaluza-Klein momentum modes if the theory is considered on a compact space, they cannot have winding modes. However, this consideration does not preclude higher dimensional structures, known as p-branes, to make their appearance in considerations of T-duality. In the next two chapters we shall study dualities in the context of the effective theories, but we will have occasions to appeal to this picture.

**S: Strong-weak duality**

As mentioned earlier, S-duality is a transformation of the coupling of a theory. Starting from one theory, say $S_1$, existence of an S-dual theory implies that there exists a theory $S_2$, whose fields as well as coupling are determined in terms of those of $S_1$. One of the most interesting special cases of S-duality is when it inverts the coupling of a theory. It is a generalization of the electric-magnetic duality symmetry of the Maxwell-equations in classical electrodynamics [130, 64]. The Maxwell-equations enjoy a symmetry under the exchange of the electric field $E$ and the magnetic field $B$, in absence of any charged sources:

$$E \rightarrow B \quad \text{and} \quad B \rightarrow -E.$$ 

In order for this to become a symmetry even in presence of sources, one has to introduce electric as well as magnetic sources in Maxwell-equations. Dirac’s quantization condition then dictates the product of the electric and magnetic charges to be proportional to an integer. Recalling the connection of charges to coupling constants in the unified field theories, therefore, this implies a symmetry of the theory under the inversion of coupling constant. At weak coupling the electrically charged particles are weakly coupled with point-like interactions, while the magnetically charged ones are coupled strongly. The *magnetic fine structure constant* ($\alpha_m$) is determined by the Dirac’s condition in terms of the usual one ($\alpha$): $\alpha_m = \frac{n^2}{4\alpha}$, for some integer $n$. It was envisioned [117, 187, 131, 123] that in a theory in which the magnetically charged particles are the carriers of the basic quanta, the magnetic charges are weakly coupled and have point-like interactions, while, now the electrically charged ones are strongly coupled with non-singular cores [137]. S-duality is a generalization of this idea. Since S-duality involves transformation of the coupling constant of a theory, to test any prediction of S-duality one needs to find quantities in the theory which are determined by the charges. Moreover, these quantities must be relevant for the whole domain of values of the coupling constant that are reached by S-duality. Therefore one resorts to supersymmetric theories, for example, supersymmetric Yang-Mills gauge theories. For such theories, the mass of the states that saturate the Bogomol’nyi-Prasad-Sommerfield bound, or BPS-states for short, are completely determined by their charges. This is a consequence of supersymmetry itself [187] and does not depend on the dynamics of the system, and hence remains true even for large values of the coupling constant. What is more, S-
duality necessitates that the theory does not break down at some value of the coupling. That is, it does not have a phase transition. The theories with at least four supercharges $N$ have such a property due to non-renormalization theorems associated with them. Such considerations render the supersymmetric theories with four or more supercharges extremely important as a testing ground for duality in more scores than one. The following quantities are invariant under, say, an inversion of the coupling constant in $N = 4$ theories:

- Mass of the BPS-states
- Degeneracy of BPS-states
- Effective low-energy physics of the moduli that characterize the BPS-states.

The above discussion pertains to conventional supersymmetric field theories. Now that string theory contains gauge theories, one expects similar features in string theory too. Again using the power of the non-renormalization theorems one can compare physical quantities computable in both the theories and confirm their dual relationship. In fact, the self-duality of the $N = 1$ and $N = 4$ Heterotic string theories in four dimensions were conjectured in [49] and [156] respectively. Other sporadic examples were also known until in [79, 170, 185] a multitude of examples of duals of string theories with at least $N = 4$ supersymmetries were found. All these examples incited confidence in duality which finally led to promote duality to a principle in string theory. Usually there exists a single unique candidate for the dual of a theory, determined by the spectrum and symmetry. Thus duality can be used for computing physical quantities in a theory by mapping it to a simpler problem in the dual theory. This approach has furnished non-perturbative results, beyond the scope of the conventional perturbative formulation of string theory. However, in order to cash in on the power of duality, one needs to know the dual pair of theories in the first place. In want of an a priori derivation of dual pairs, one has to take recourse to indirect means. One of the simplest ways of discovering dual pairs is to consider two already known dual theories on orbifolds [157]. For example, let a theory $S_1$ compactified on a manifold $\mathcal{M}_1$ be known to be dual to another theory $S_2$ compactified on another manifold $\mathcal{M}_2$, in the sense that their duality has already been tested. Let us then consider compactifications of both the theories further on another manifold $\mathcal{K}$. Let $\mathcal{G}_1$ and $\mathcal{G}_2$ denote some discrete symmetry groups of the theories $S_1$ and $S_2$, respectively, one group being the image of the other. Then, by and large, $S_1$ compactified on $\mathcal{M}_1 \otimes \mathcal{K}$ quotiented out by $\mathcal{G}_1$ is dual to $S_2$ on $\mathcal{M}_2 \otimes \mathcal{K}$ quotiented out by $\mathcal{G}_2$. Although exceptions are known to this naive prescription, in cases where the quotienting by the discrete groups does not commute with the duality map, this is known to yield correct results in many instances [157, and references therein].
If two theories are known to be S-dual, then one can study the behaviors of these theories even when the coupling is strong. For example, the Type-IIA and Heterotic string theories both lead to a strong coupling theory related to the eleven-dimensional supergravity theory [79, 170, 185]. Both can be derived from this new theory by compactifying the eleventh dimension in such a way that the length of the eleventh compactified direction, say $R_{11}$, scales as $R_{11} \sim g_s^{2/3}$. The new eleven-dimensional theory was christened as “M-theory”. However, contrary to the usual field theories, the Lagrangian or action for M-theory is not known so far save for its low-energy behavior [191, is an attempt toward deriving it]. M-theory has been used to understand the behavior of other three string theories and their dualities. We shall expatiate on this aspect in Chapter 2.

It is to be noted that duality is a relation between theories. Therefore, it reduces the number of independent string theories, or string vacua. For example, considering the fact that Type-IIA and Type-IIB theories are T-dual to each other when both are compactified on a circle, reduces the number of independent string theories from five to four. Then the T-duality between EH and OH in nine dimensions leaves three independent string theories. Combined with the fact that Type-IIB theory goes over to Type-I theory upon quotienting by the reversal of orientation of its world-sheet, these reduce the number of independent string theories to two. Finally, the observation that Type-I and OH are S-dual of each other, which, as we shall discuss in §§ 2.1.4, can be looked upon as a prediction of M-theory, leaves us with a single independent string theory [162]. Thus M-theory is a major step toward the construction of a single string theory. In a scheme where the known string theories are merely different vacua of a single unified theory, duality is a symmetry of that unique string theory. In fact, in this sense duality can be thought of as a gauge symmetry of the theory. Although we will not have occasions to discuss it in this thesis, it must be mentioned at this point that the twelve-dimensional F-theory propounded in [174] is another major step in understanding the different dualities in string theory and constructing different string vacua connected by dualities.

The study of non-perturbative properties of string theories, and especially M-theory, received enormous impetus from the studies in p-branes. A p-brane is a p-dimensional subspace embedded in a space-time of higher dimension. Strings are 1-branes, particles are 0-branes in this terminology. It is known that the field theories of the excitations of p-branes for $p > 1$ are not renormalizable. Strings are special in this respect since the field theory on its world-sheet is renormalizable. Considering open strings, the usual formulation imposes Neumann boundary conditions at their dangling ends. However, T-duality requires that the T-dual open strings be described by Dirichlet boundary conditions at their ends [33]. Then, in order to retain Lorentz invariance, the ends of the string, which are not dangling anymore, must rest on a p-brane, the Dirichlet p-brane or a D-(p)-brane, for
short. The D-branes are allowed to move about in the background space-time. Then the excitations of the D-brane can be understood in terms of the renormalizable field theory of the strings attached to it [134, 135]. This furnishes a geometrical understanding of the Chan-Paton factors of open string theory: a Chan-Paton label is the label of the D-brane on which an open string stands. These D-branes can also be identified with the solitonic solutions of string theory. In Type-II theories the D-branes are looked upon as non-perturbative states carrying Rammond-Rammond charges. Thus, contrasted with the strings, they can be viewed as the analogues of the smooth classical configurations compared to the fundamental particles in gauge theories [137]. Their relevance for S-duality was understood in [134, 137]. Let us recount an example from [137] in this context. Various string theories have one-dimensional solitonic objects or 1-branes in their spectra. These can be interpreted as D-(1)-branes. In the weak coupling limit these states are heavier than the fundamental strings, also present in the spectrum. However, in the strong coupling limit, they become lighter, as a consequence of the mass formula for BPS-states, and it is much convenient to re-interpret the theory as being a theory of the D-branes as fundamental strings [137, and references therein]. The string theories are now known to contain D-(p)-branes in their spectrum for different values of p. The efforts to understand non-perturbative string theory as theories of D-branes led to the recent conjecture that non-perturbative formulation of string theories at strong coupling could be achieved by viewing it as a matrix-theory [10, and developments thereafter!], which is related to the membrane of M-theory in the infinite momentum frame.

Black holes

An interesting application of the D-brane technology has been in counting states of black holes in string theory [165, see also [75] for a review]. For last two decades and a half, it has been believed that black holes, which appear as solutions of Einstein’s equations in classical gravity, are thermodynamic systems, by virtue of possessing an entropy and having a temperature. However, an understanding the statistical origin of this entropy is an unanswered question. It is believed that an understanding of this requires an understanding of gravitational effects in the quantum regime, that is to say, quantum gravity. Since string theory contains gravity in its low-energy limit, and expected to purvey a theory of quantum gravity, it is also expected to provide an understanding of the microscopic origin of the entropy of black holes. Black holes were identified in string theory as belonging to the non-perturbative solitonic spectrum of the theory, which in turn, were connected to D-branes via duality. It was shown that using the D-branes one can count the quantum microstates associated with classical black hole configurations and entropies of extremal and near-extremal black holes were found from this counting to be the same as one-quarter of the area of the horizon — a result derived earlier in classical gravity. Apart from these recent developments, the thermodynamic
study of black holes is an interesting subject in its own right. The nexus of a theory of quantum
gravity or string theory and black hole thermodynamics can be envisaged to be the same as that
between statistical physics to thermodynamics.

The plan of the thesis is as follows. In Chapter 1 we shall discuss some aspects of T-duality and
S-duality symmetries in the string theory. In particular, we shall discuss in detail the methods
of finding out the symmetries with the example of $E_8 \times E_8$ Heterotic string theory in different
dimensions. We shall stress on the methods for finding the U-duality symmetry of Heterotic string
theory compactified to two dimensions following [100, 18]. We shall also discuss the analysis of Geroch
symmetries of Einstein equations, which was useful in the analysis of Heterotic strings. Finally, we
shall comment on the duality symmetries in other string theories. In Chapter 2 we shall continue the
study of duality through M-theory, the strong-coupling limit of Type-IIA theory. We shall discuss
how the five perturbative string theories and some of their dualities can be understood from M-theory.
Then we shall consider compactification of M-theory to find various models in six and two
dimensions following [101] and [102], respectively. In all the cases, as we shall see, the cancellation
of gravitational anomaly plays a crucial role. We shall close the chapter with some comments about
some other compactifications of M-theory. In Chapter 3 we shall embark on a little different topic,
namely the thermodynamics of black holes in two dimensions. We shall discuss the technique of
Noether charge for computing the entropy of black holes. Then we shall discuss a local or finite-space
formulation of black hole thermodynamics, which corresponds to a black hole being studied from a
finite spatial distance instead of the asymptotic infinity. Another way to put it is that the black hole
is trapped inside a box. This formulation is capable of dealing with asymptotically non-flat black
holes. We shall discuss the thermodynamics of some black-holes which arise in some string-motivated
gravity in two dimensions from theories having scalar fields with non-minimal coupling and with a
cosmological constant. We shall study the non-extremal and extremal cases, following [103] and
[104], respectively. We shall conclude the thesis with a summary of all the chapters.