CHAPTER - I

INTRODUCTION
I.1. Importance of Coulomb-nuclear interference Experiments:

There have been rapid developments in the Coulomb nuclear interference experiments during the last one and half decade. The experimentalists of various laboratories as CERN, LAMPE, SIN and TRIUMF have put forth sincere efforts to attain utmost preciseness in performing these experiments. For these experiments, pions of both charges are being used as a potential probe which allows detailed investigations of Coulomb effects. The simultaneous study of $\pi^+$ and $\pi^-$ experiments, once the Coulomb distortion effects are properly accounted for, should in principle yield valuable information on neutron densities. Probes like electron or nucleon beam are not that appropriate for such a purpose. Hence with high flux pion beams as well as improved pion spectrometers available in meson factories, the Coulomb-nuclear interference experiments are complementary of the conventional electron and nucleon scattering measurements for the determination of nuclear properties.

The nuclear effects obtained from these experiments will provide a check on the various assumed nuclear models. Specifically, the Coulomb-nuclear interference (CNI) experiments in the intermediate energy range, where the $\pi-N \,(3,3)$ resonance exists, bear much importance as they provide us with the knowledge on how pions interact with nuclei when pion-nucleon interaction is strongly dominated by a resonance.
They also help deducing the selection rules and probabilities of excitation of nuclear levels in the presence of such a resonance.

There has been no firmly established theoretical approach so far in this resonance region. Kisslinger's optical model\(^1\) is successful below 110 MeV. Glauber model\(^2\) works only for energies above 500 MeV. The recently developed isobar–hole model is also not free from flaws. The calculations of Hirata et al\(^3\) and Maillet et al\(^4\), assuming such model, overestimate the total cross sections. More recently the quark structure of the nucleon has been taken into consideration for theoretical calculations in pion nucleus interactions. As pointed out by Thomas\(^5\), the quantitative analysis of the data in the region of (3,3) resonance is extremely uncertain. He has argued that the understanding of low and medium energy nuclear physics can no longer proceed without recognition of the underlying quark structure of the nucleons themselves. Using a cloudy bag model\(^6\), Thomas and his collaborators have studied the \(\pi - N\) system. But the question of these exotic states is highly controversial. With the advent of developing experimental techniques, the CNI experiments, it is hoped, will lead us to a better conclusion as regards the behaviour of pion interaction with nuclei.


In many respects, scattering of charged pions from isoscalar nuclei is most interesting. The absence of spin–spin terms makes these systems simpler for study. Experiments for
measuring scattering cross sections for such processes in CNI region are being undertaken in several laboratories. The interference effects being most pronounced near the forward region, the cross section data near this region can be analysed to extract useful information in the forward direction. Binon et al have used a semiphenomenological form of parametrization for the complex nuclear amplitude for analysing the $\pi^- - ^4\text{He}$ and $\pi^- - ^{12}\text{C}$ scattering cross section data. The same form of analysis has been carried out by Scot et al and Mutchler et al for $^{12}\text{C}$ and by Albanese et al for $^{16}\text{O}$ targets. Though the method provides an ingenious way of obtaining the forward amplitude, still it suffers from the drawback of depending too heavily on the total cross section data. These data in the higher energy region being scanty, are often determined through extrapolation of the experimental results obtained at low energies. Moreover the semiphenomenological fit to the form of the amplitude may not be exactly valid over the entire physical region.

To extract the nuclear data from the CNI experiment results in a model-independent manner one can also look for the phase shift analysis method. Falomkin et al have made conventional phase shift analyses of pion-$^4\text{He}$ scattering data to obtain the real parts of forward amplitude in the energy range 24 to 180 MeV. But as it is obvious, the method is very much ambiguity-ridden. Specifically it is more so near and above the resonance region. As would be shown latter, it involves too
many parameters in this region. In Chapter IV, we have made an attempt to solve this problem by using a conformal mapping procedure.

Another model independent approach to the problem of extracting forward amplitude was first given by Ericson and Locher\(^\text{13}\). They exploited the analyticity property of the scattering amplitude. By then analyticity was fairly established in particle physics. Ericson and Locher tried to extend analyticity to nuclear scattering problems and hence used forward dispersion relations. They argued that the analyticity lore of elementary particle physics could be brought into nuclear physics, if one could suppose the nucleus to be a structureless particle with a definite set of quantum numbers. Then it behaves like any strongly interacting particle. It would be more appropriate to deal with small nuclei.

However, inspite of their best efforts Ericson and Locher and many contemporary workers\(^\text{14,15}\) in the same field have not been able to derive consistent results for the forward amplitude since the method depends too heavily on the total cross section data which are often scanty. At present, therefore we propose to utilize the analyticity properties of the scattering amplitude in a more suitable way to extract the forward amplitude and other nuclear data. This we have done in Chapters II and III.
I.3. Analytic Properties of hadron-nuclei scattering amplitudes:

For a scattering process involving two initial and two final spinless particles, there is only one amplitude which completely describes the process. This amplitude is a function of two variables, the laboratory energy $\omega$ and the scattering angle $\theta$. In much used relativistic notations they correspond to the two invariant Mandelstam variables $s$ and $t$.

For a scattering process (Fig. 1.1) written as

$$a + b \rightarrow c + d,$$

if $p_a$, $p_b$ are the four-momenta of the initial particles and $p_c$, $p_d$ are the four-momenta of the final particles, the three invariant Mandelstam variables are

$$s = (p_a + p_b)^2 = (p_c + p_d)^2,$$

$$t = (p_c - p_a)^2 = (p_d - p_b)^2,$$ (1.1)

and

$$u = (p_d - p_a)^2 = (p_c - p_b)^2.$$

$u$ is the dependent variable connected to the other two through the relation

$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$$ (1.2)

$m_a$, $m_b$, $m_c$ and $m_d$ being the corresponding masses. The quantity $s$ is the squared total energy in the c.m. system of the process $a + b \rightarrow c + d$. Analogically the quantities $t$ and $u$
correspond to the squared total energies in the c.m. system of the processes

\[ a + c \rightarrow \bar{b} + d \]
\[ a + \bar{d} \rightarrow c + \bar{b} \]

where \( \bar{b}, \bar{c} \) and \( \bar{d} \) are the antiparticles of \( b, c \) and \( d \) respectively.

For using analytic properties in any analysis, it is at first essential to find an algorithm for locating the nearest singularities of the scattering amplitude. One can begin with the assumption that the hadron–nucleus scattering amplitude is an analytic function of two independent complex variables and the location of the nearest singularities are determined by first few lower-order (generally up to the fourth order) Feynmann diagrams which are allowed by conservation laws. It is possible to express every Feynmann diagram in the form of an integral

\[
F_{\xi}(s,t) = \int d^4k_1 \ldots d^4k_l N/\Pi_j (q_j^2 - m_j^2 + i\epsilon)
\]

Here \( q_j \) denotes the four momentum of the \( j \)th propagator and \( m_j \) is the mass of the corresponding particle in virtual state. The numerator \( N \) represents the spin structure of the Feynmann diagram and \( k_j (j = 1, 2, \ldots l) \) are four-momenta connected with \( l \) independent loops in the diagram. These must be chosen in accordance with energy–momentum conservation law.
The scattering amplitude in the physical region of the external momenta is given by

\[ F(s,t) = \lim_{\xi \to 0} F(\xi s, \xi t). \]

The most convenient form of the integral (1.3) for the investigation of analytic properties of the corresponding Feynmann diagram is the so-called \( \xi \) -representation. It can be found by means of Feynmann's identity

\[ \frac{1}{u_1 \ldots u_n} = (n-1)! \int_0^1 d\xi_1 \ldots d\xi_n (1 - \sum_j \xi_j)/(\sum_j \xi_j u_j)^n \]

\[ \ldots \ldots (1.4) \]

where \( u_j = q_j^2 - m_j^2 \). This identity together with Eq. (1.3) gives \( \xi \) -representation in the following form

\[ F(\xi s, \xi t) = (n-1)! \int_0^1 d\xi_1 \ldots d\xi_n \int d^4k_1 \ldots d^4k_n \]

\[ x \frac{1}{(\sum_j \xi_j)/(\sum_j \xi_j u_j + i\epsilon)^n} \]

\[ \ldots \ldots (1.5) \]

Now to find singularities corresponding to any Feynmann diagram one must investigate analytic properties of integrals of type (1.5). This can be put into a set of algebraic conditions called Landau-Gutkosky equations in the manner
(i) either $\mathcal{L}_j = 0$ or $q_j^2 = m_j^2$ for each $j$

(ii) \[ \sum_{j=1}^{n} \pm \mathcal{L}_j q_j = 0 \] for each independent loop of the diagram where the sign is positive or negative depending on whether the direction of $q_j$ is along or towards a prechosen direction around the loop.

We will be interested only in singularities (poles and branch points) on the first Riemann sheet (physical sheet). In this case one has an extra condition.

(iii) $\mathcal{L}_j$ must be real positive numbers.

We would like to stress that the explicit knowledge of the effective Lagrangian for location of nearest singularities of the amplitude of hadron-nucleus interaction is not mandatory. It suffices to make the above practical procedure. First one should write down for all three channels ($s-$, $t-$ and $u-$) the allowed Feynmann diagrams up to fourth order. If among them there are some with stable intermediate particle state with respect to strong interactions in a pole diagram, then the scattering amplitude has poles on the physical sheet. These poles occur at squared total energies equal to the squared value of the corresponding masses. Nearest normal and anomalous thresholds of the scattering amplitude in all variables can be found from all possible reduced diagrams.
of fourth order diagrams, which can be obtained by contracting \( L_j = 0 \) lines to points.

An excellent work on study of analytic structure of nuclear scattering amplitudes has been given in a paper by Ericson and Locher\(^{13}\). They have exhaustively discussed how the analyticity properties have been useful in obtaining the real parts of the forward amplitude via forward dispersion relations.

Analyticity had found its wide use in extrapolation problems in elementary particle physics. The possibility of getting information about coupling constants was first noted by Chew\(^{17}\) and very soon Amado\(^{18}\) suggested that one could extrapolate the differential cross section data for pick up and stripping reactions to the pole in the \( \cos \theta \) -plane to determine the coupling constants and spectroscopic factors. A firm theoretical background involving analyticity of scattering amplitude was established by Lehmann\(^{19}\). Since then there has been rapid growth of the literature on its application in the field of elementary particle physics.

Application to extrapolation to poles in nuclear reactions came into existence at a much latter time. Schnitzer\(^{20}\) in 1965 made an attempt for extrapolating to the pole for the reaction \( ^{28}\text{Si} (d,p) ^{29}\text{Si} \). But he found it thoroughly disappointing since in nuclear reactions
the mutual location of the poles, cuts and the physical region were such that the extrapolation was much more difficult than anticipated. In pion-nucleus scattering, for example, there are in general several poles corresponding to the ground state and excited states of the neighbouring nuclei with the same nucleon number but differing in charge from the target nucleus. There were large distortions due to the nearby cuts. This fact almost stood as a barrier against extracting any useful information from the experimental data through the particle exchange poles.

However, Locher, neglecting the contributions from the unphysical cut determined the residues $r_{Hd}^3$ and $r_{dpn}$. In some cases the cuts were approximated by an effective pole or even by two or more number of poles to obtain the necessary information from a certain particle exchange pole. The fits were better with more number of poles. But such approximation of cuts to arbitrary number of pseudopoles had no physical meaning.

The problem of tackling the unwanted contribution from the cuts was solved only with the advent of the conformal mapping methods. Almost four years after the powerful technique of conformal mapping by Cutkosky-Deo and Ciulli evolved, Kisslinger thought of using it for the first time to obtain the coupling constant corresponding to the
proton pole from nd elastic scattering data. Soon the method was widely used in a great variety of problems involving scattering of hadrons from nuclei.

Excellent reviews of the applications of analyticity in nuclear problems have been written by Dubnička and Dumrais and Locher and Mizutani.

1.4. Optimized Polynomial Expansion of Analytic Function

As indicated earlier in the last section, the analyticity properties of scattering amplitudes help in extracting valuable information like various coupling constants or the residues at the particle exchange poles from the experimental cross section data. These properties could be used in a more meaningful way as pointed out by Frazer. In this respect, the technique evolved by Cutkosky and Dec and also by Giulli are more elegant. They have exploited analyticity maximally as a result of which the predicted value of an analytic function at some point has maximum stability with respect to the errors in the input information. In understanding the underlying principle of their method one must keep in mind the following facts.

Any analytic function can be expressed as a polynomial expansion of infinite order. This expansion can be truncated
at a certain step to approximate the analytic function only if the coefficients of expansion diminish most rapidly. The faster is the rate of convergence of the polynomial expansion, the greater is its stability against the error in the input data. That is for maximum rate of convergence, the error due to truncation would be minimum. The larger the natural domain of convergence of a series, the faster is its rate of convergence. But the domain of convergence may not include the entire cut plane over which the corresponding function is analytic. So to maximise the convergence rate one would like to include as much of the analytic cut plane as is possible into the domain of convergence. The best convergence is obtained if the entire analytic region merges into the region of convergence of the series and then expansions are 'optimal' in the sense that no region of analyticity is left out of the region of convergence.

So the basic idea behind the optimization problem consists in choosing a suitable function which maps the cut plane into the interior of the domain of convergence and then expand the amplitude in a polynomial series in terms of this new function. The choice of the correct form of this function is dictated by the geometry of the physical region as pointed out by Walsh. Here we state a theorem due to Walsh in the line of an Appendix of the paper by Ciulli.
Let \( \Gamma_1 \) be a closed limited point set whose complement is connected and regular (Fig. 1.2). If the function \( f(z) \) is single valued and analytic on \( \Gamma_1 \), there exists a greater number \( R' \) (finite or infinite) such that \( f(z) \) is analytic and single valued in every point interior to \( \Gamma_R \). If \( R' < R \) is arbitrary, there exist polynomials \( P_n(z) \) of respective degrees 0, 1, 2, ... such that \( 1 \leq R < R' \) \[
|f(z) - P_n(z)| \leq M(\frac{R}{R'})^n
\] is valid for \( z \) on \( \Gamma_\rho \) (or on \( \Gamma_1 \), for \( \rho = 1 \)); but there exist no polynomials \( P_n(z) \) such that (1.6) is valid for \( z \in \Gamma_\rho \) with \( R' > R \).

From the above theorem the complete domain of convergence corresponding to a series expansion could be immediately derived. The convergence properties of a sequence \( P_n(z) \), which approximates to \( f(z) \), can be obtained by finding the equipotentials in the electrostatic problem in which \( \Gamma_1 \) forms an earthed conductor \( (V = 0) \) in free space with a unit negative charge. We shall obtain a family of equipotential curves for each of which \( V(z) = \) a constant. If \( V \) be any finite potential smaller than each of the potential \( V(z_\alpha) \) at the singular points \( z_\alpha \) of \( f(z) \), then \( P_n(z) \) will converge uniformly to \( f(z) \) inside the closed region that is bounded by the equipotential \( V(z) = V \). The region bounded by the largest singularity free equipotential curve would be called the region of convergence. Moreover, the
error in the n th approximation (on \( \Gamma_1 \)) is bounded by 
\( e^{-nm} \). The equipotentials of (i) a point charge is a
circle, (ii) a finite line an ellipse, (iii) a semi—infinitie
line a parabola and (iv) an infinite line a strip. These are
also the shape of the region of convergence of equivalent
physical situations.

In problems of physical interest, for any analytic func-
tion \( f(x) \), we do not have the series expansion about a
point, rather we have it around a line segment from —1 to +1,
which is the physical region over which the function is known.
In such cases the region of convergence of the series is an
unifocal ellipse with the two focii at —1 and +1. The size
of the ellipse is bounded by the nearest singularity structure
in a complex plane. Considering in particular the case of
scattering problems, the amplitude is known over the physical
region \(-1 \leq x \leq 1\) (\( x = \) cosine of the scattering angle) and
can be continued analytically into the entire cut plane
except for the branch cuts along \(( -\infty, -x_-)\) and \((x_+, \infty)\).
Hence a series expansion for the amplitude would converge
to \( f(x) \) inside an ellipse whose boundary touches \( x_+ \) (or — \( x_-\)
if \(|x_+| \leq |x_-|\) (or \(|x_+| \geq |x_-|\)). Then the problem of
optimization reduces to conformally map these branches cuts
onto an unifocal ellipse and the physical region onto itself
by a transformation \( x \rightarrow z(x) \) such that the entire analytic
domain goes inside this new ellipse. The details of the
transformation would be given in section II.4. The rate of convergence of the series is determined by the parameter 
\[ R = e^c = a + (a^2 - 1)^{1/2}, \]
where 'a' is the semi-major axis of the ellipse and \( V_c \) is the potential on the curve \( C \) (the ellipse in this case). Since \( V_c \) includes the complete analytic domain, it is supposed to be the largest, leading to maximum rate of convergence.

An equivalent optimal mapping was proposed by Ciulli who mapped the cut x-plane onto an annular ring with inner and outer radii equal to 1 and \( R \) respectively in the new plane. The two mappings suggested by Cutkosky and Deo and Ciulli are equivalent in the sense that they have the same basis of Walsh's concept of maximal convergence of polynomial expansions. We, in our problem, would use the elliptic mapping for convenience.

A second and more important criterion which necessitates the urge for conformal mapping is the stability problem in case of analytic extrapolations as indicated by Cutkosky and Deo. In practice, neither exact nor complete values of the analytic function over the physical region are known. Since all observations are subjected to small errors, the experimental data are always error-associated. This creates problem and any physical theory has to be stable against these small errors. Ciulli, in his pioneering work has given a clear picture about how the stability of the series
expansion approximating a function breaks down on the boundary \( \Gamma \) (cuts). In the light of Nevalinna principle, the author has given the prescription for using conformal mapping to remove the reason for this stability breakdown.

Cutkosky\textsuperscript{31} has further extended the original idea of Cutkosky and Deo by choosing a more general class of functions for approximation for faster convergence and has suggested the use of a more precise convergence test function \( \Phi \) which extracts information from each expansion coefficient to the greatest extent. He has argued that the conformal mapping procedure can be used to extract correct information from a noise-ridden data. He has based on the fact that the noise differs from the true amplitude in not being analytic. Hence analyticity properties as reflected in the optimized convergence rate, provide a powerful way to separate the noise. Using some idealized models of the noise, Ciulli\textsuperscript{34,35} has given explicit calculations where he has illustrated clearly how the accuracy of estimation is improved by increasing the convergence rate. Since Cutkosky's method\textsuperscript{31} is a prescription for best representation of any data by functions with given analyticity properties, it is useful for estimating the uncertainties in phase shifts and other scattering parameters and for comparing the a priori plausibility of several sets of parameters which might appear to fit the data equally well.
1.5. Applications of Optimized Polynomial Expansion

The main applications of optimized polynomial expansion (OPE) in the field of particle physics have been in determination of coupling constants and residues at particle exchange poles, in phase shift and amplitude analysis, in determination of zeros and poles in a function and also in dispersion relations. In their first paper motivating the use of conformal mapping, Outkosky and Deo aimed at determining the residues of pion and $(\Lambda + \Sigma)$ poles from np and $k^+p$ scattering cross section data. Subsequently to determine coupling constants mapping procedure was used by Chao and Pietriinen, Schwela, Pazman et al. and Bisoi and Deo.

Phase shift analysis of low energy $k^+p$ scattering using OPE was at first carried out by Outkosky and Deo. Chao has used analytic approximation to pp scattering where he has used separate elliptic mappings for real and imaginary parts of the amplitude. He has also used the convergence test function of Outkosky. Miller et al. performing a phase shift analysis of $k^+p$ scattering by OPE method have obtained a 20% average improvement in the $\chi^2$ value over the conventional PSA. However, criticism of the OPE method has come from Griss and Fox who have suggested some modification to this. But then Outkosky and Sandusky, examining the work of Griss and Fox in
detail have not agreed with their results. The OPE method has been further subjected to trial by Arndt et al. who in a partial wave analysis of pion-nucleon scattering have obtained a higher $\chi^2$ value by OPE method compared to the corresponding conventional analysis. This surprising result has been attributed to the strong s-channel resonances. Similar situation has been encountered by Bisoi and Deo while determining $\text{J}^{\text{NN}}$ coupling constant from pion photoproduction data. They have remedied the situation by accounting for the direct channel resonances through reggeization of the crossed channel nucleon pole. Cutkosky and Shih have formulated a modified N/D method based on normed analytic approximation theory for interpolating the energy dependence of the phase shifts.

An extrapolation of the pp-forward scattering spin-nonflip amplitude has been carried out by Kanazawa. Dumbrais has used conformal mapping for a model independent determination of the real parts and zeroes of the pp and $\overline{\text{p}}p$ forward scattering spin dependent amplitudes. Kelly has carried out amplitude analysis for $\text{N}N$ scattering by the use of elliptic mapping.

Caprini et al. have used analytic continuation for the determination of bound, antibound and resonant states. Pietarinen has determined the phenomenological J-plane singularities by optimized conformal mapping methods.
OPE finds its extensive use in form factor analysis. An analysis of proton form factor data by analytic continuation to the $2\pi$ cut has been carried out by Cheung\(^{54}\). Later on extrapolations of the proton and pion form factor data to time-like region have been made effectively by Deo and Parida\(^{55}\). They have used parabolic mapping. Bisoi and Deo\(^{56}\) have used virtual photoproduction cross section data to obtain pion form factor where the conformal mapping has given results no better than those obtained normally. The reason has been attributed to the small number of terms required for a good fit.

The applications of OPE method in nuclear scattering problems have come into existence at a much later time. Only after Kisslinger\(^{27}\) successfully applied the method in \(n^2d\) scattering to handle the large distortions caused by the cuts, there were many who took advantage of the technique for obtaining the pole residues as well as the coupling constants. Dubnička et al\(^{57}\) have obtained the residue at the pion pole from \(3^3H\) and \(3^3He\) elastic scattering data. The residue turned out to be very weak as a result it was not possible to determine \(3^3H\Pi\) and \(3^3He\Pi\) coupling constants on the basis of these data. Subsequently Dubnička and Dumbrais\(^{58}\), basing on optimal polynomial expansion, have determined the residues \(12C^{11}Cn\) and \(14N^{13}Nn\) from differential cross section data for elastic \(n^{12}C\) and \(n^{14}N\) scattering respectively. They have also determined \(r_{dpn}\) from \(nd\) elastic scattering data.
Using $^3$He elastic scattering data Kisslinger has extrapolated to the deuteron pole to estimate $r_{^{3}He}^p$ and the corresponding spectroscopic factor. Kisslinger and Nichols were the first to use optimal polynomial expansion to study the Coulomb effects on spectroscopic factors for nuclear scattering problems. Borbély has also investigated the very important question of Coulomb effects in such kind of analyses. He has used the singularity subtraction method in order to obtain spectroscopic information about nuclei like $^{9}$Be, $^{16}$O and $^{31}$P. Dubnička and Dumbras have determined the $^4$He coupling strength from elastic $^d$He scattering, though they have not been able to get rid of the electromagnetic effect by using the trick of mapping the electromagnetic pole on an ellipse.

I.6. Pole Extrapolation

The problem of extrapolation of an analytic function from a segment in the physical region to a singular point outside it was first tackled by Chew in 1958. He proposed the extrapolation of the $n$-$p$ scattering data to the one-pion exchange pole to determine the pion-nucleon coupling constant. For this process, single pion exchange gives rise to a pole in the $\cos \theta$ plane at the unphysical value $\cos \theta_p$ (i.e at $x = x_p$). The method proposed was to plot the function $(\cos \theta - \cos \theta_p)^2 \frac{d\sigma}{d\Omega}$ against $\cos \theta$ and
extrapolate the curve to $\cos\omega = \cos\Omega_p$ to obtain the residue of the second order pole which is related to the coupling constant. Later on Taylor suggested the application of Chew type of extrapolation to various reactions involving strange particles. Similar extrapolations were used by Moravcsik and Cziffra for determination of the pion-nucleon coupling constant from photoproduction angular distribution. This method was advocated by Chew and Low for analysing scattering amplitudes involving unstable targets. In Chew's method one expands about a pole $x_p$ as

$$\left( x - x_p \right)^2 \left. \frac{d\sigma}{d\Omega} \right|_{\text{expt}} = \sum_n a_n (x - x_p)^n$$

from which one obtains $a_0$ as the residue of the pole. This in turn can yield the coupling constant.

This extrapolation procedure was subsequently improved by Cutkosky and Deo who suggested an expression of the form

$$T(x) = \prod_i \left( x - x_i \right) \left[ \left. \left( \frac{d\sigma}{d\Omega} \right) \right|_{\text{expt}} - \left. \left( \frac{d\sigma}{d\Omega} \right) \right|_{\text{Born}} \right]$$

$$= \sum_n C_n p_n(x)$$

where $p_n(x)$ are some orthogonal polynomials. The above expression suffices the absence of the poles $x = x_i$ only if the Born term contains the correct residue value. This singularity subtraction method was similar to that of
Ashmore et al\(^6\) only with the difference that the expansion can be obtained by the conventional least square techniques, in which \(p_n\) are orthonormalized according to the statistical weights of the data. Multiplication by zeroes of the first order assures that a lesser polynomial is needed for a good fit. Subsequently introduction of conformal mapping technique reduced the required order of the polynomial to a still lesser value.

Cutkosky and Deo method of extrapolation has been widely used by theoreticians to obtain coupling constants, the residues at poles and to determine form factors. Coming to the field of nuclear scattering problems Dubnička, Dumbrais and many others have found this method much suitable. In the present work we shall show in Chapter II and III how this method can be applied in a novel way to obtain the forward nuclear amplitude for charged pion scattering from light nuclei like \(^4\)He and \(^{12}\)C. For this, Coulomb nuclear interference experiment data have been suitably extrapolated to the electromagnetic pole which lies at the forward edge of the physical region.