CHAPTER - V

CONCLUSION
V.1. Summary of Results:

To summarize the results of this work, we mention that we have confirmed the existence of a Coulombic pole in the forward direction for charged pion scattering from atomic nuclei. This results in a sharp dip in the $\chi^2$ value corresponding to certain values of the real and imaginary parts of the forward amplitude as observed in chapters II and III. The extrapolation to this forward pole helps in extracting reliable values of various nuclear data in the forward direction. The residual nuclear amplitude which is distorted by a Coulomb phase factor deviates from the pure strong interaction amplitude. This deviation is more at lower energies and for higher $Z$ value. For targets like $^4$He ($Z = 2$) this variation is almost insignificant near and above the $(3,3)$ resonance region. But as discussed in Sec.II.6 the variation becomes appreciable below 75 MeV. For $^{12}$C nuclei as targets with $Z$ as high as 6, the residual effect is quite appreciable even in the $(3,3)$ resonance region as indicated by a large difference in the nuclear data obtained for positive and negative pion scattering.

The integrated elastic cross section which is ought to be free of any residual effect theoretically (rather mathematically), is seen to be different for positive and
negative pion scattering from $^4$He nuclei at energies below 75 MeV. This may be due to appreciable distortion effect at these low energies. But the exact amount of this distortion could not be estimated accurately due to lack of data sufficiently close to the forward region. Moreover, the distortion effect could not be studied for heavier targets like $^{12}$C since good data for $\pi^+$ and $\pi^-$ scattering at the same energy were not available. Again the integrated cross sections could not be estimated at energies where the data spreading out over the entire physical region were not available.

The total cross section for pion-helium scattering is found to be maximum within 150 MeV to 185 MeV which is close to the energy region of the $\pi$-$N (3,3)$ resonance existent at a total energy of 1232 MeV. The peak of this curve for pion-$^{12}$C scattering (Ref. Fig. 3.4) shifts to the lower energy side and occurs within the range 140 MeV to 160 MeV as determined by the present analysis of the negative pion scattering data. This is in confirmation with the experimental results of Binon et al. The results of positive pion scattering data analysis are not quite satisfactory since the data available are erratic and inadequate.
The results of our phase shift analysis of pion–\(^4\)He scattering using an optimized polynomial expansion are both satisfactory and confirmatory. Optimization helps in constructing a series of Tchebyshev polynomials which, when truncated at a certain suitable term, becomes the best approximate of the analytic amplitude. From this best approximate series various partial waves are projected out to give the phase shifts and inelastic parameters of all possible orders. The elegant use of the conformal mapping reduces the number of free parameters needed for a good fit to the data by six to eight at each energy near the \((3,3)\) resonance region. The ambiguities have not been found.

The results of this phase shift analysis are in agreement with the optical model calculation according to which pion absorption in the elastic channel is dominated by P–wave. But the phase shifts for the P–wave as obtained from the present analysis show a marked difference from the results of the conventional phase shift analysis performed in this energy region\(^8\). In stead of remaining positive upto 250 MeV as revealed by the conventional analysis, we find that the P–wave phase shift changes sign around 200 MeV. Moreover our P–wave phase shift at 200 MeV considerably differs from that of Binon et al\(^7\).
V.2. Scope for Further Work:

The present method of analysis being independent of any specified nuclear model can, in principle, be extended to analyse pion scattering by other heavier targets like those of $^{16}O$ and $^{40}Ca$. The experiments of Mutchler et al. and of Albanese et al. in the Coulomb nuclear interference region provide a good set of data to carry out the analysis for pion scattering from $^{16}O$ targets. With availability of differential cross section data for heavier targets, the pole extrapolation method would help in studying the variation of the complex forward amplitude as a function of both element and energy.

Although we have addressed ourselves to scattering processes for which neither the target nor the projectile has spin, the analysis can be extended to other elastic scattering processes like the scattering of protons from nuclei or of pions from nuclei with spin.

The present method of phase shift analysis involves minimum number of variable parameters. Hence the phase shifts for scattering at high energy of the order of a few GeV where the higher order partial waves are more meaningful can be obtained with least ambiguity. It will be instructive to carry out such analysis for pion-$^{12}C$ and pion-$^{16}O$. 
scattering near the (3,3) resonance region, where the conventional analysis would not produce any good result because of the need of a large number of parameters.

V.3. Concluding Remarks:

In concluding this piece of work, we again emphasize that the pole extrapolation method has been extremely helpful in extracting reliable information in the forward direction from the results of Coulomb nuclear interference experiments. The optimization procedure has improved the goodness of the fits to a considerable extent. The construction of an 'ideal' series as illustrated in Sec. II.5 has helped in eliminating the background noise of the data, thus making the singular term more visible for extrapolation.

At the end, it will be in order to mention the few limitations to which our analysis has been subjected. Nuclear parameters for pure strong interaction could have been obtained from the residual amplitudes in the forward direction for both positive and negative pions. But for this, differential scattering data for both types of pions at same energies are essential which are not there at higher energies as well as for heavier target nuclei. Secondly, as observed in Sec. II.5, due to lack of data very close to the forward region, the extrapolation has not been possibly smooth. Since for the available data, the number of terms needed for a
good fit is quite large (four to five), this leads to an appreciable uncertainly in the extrapolated results. Hence it has not been possible to have an accurate estimate of the Coulomb distortion effect which are so important at these low energies, where experimental data for both types of pions are available at same energies and same angles. Lastly, we have also faced the problem for studying the distortion effect for targets with higher Z value as the differential cross section data spreading over the entire physical region are not there which could have enabled us to determine the integrated elastic cross sections which are supposed to be free of any residual effect and hence could be directly compared with results of experiments.

So we urge upon the experimentalists to carry out the experiments still closer to the forward as well as to the backward region. Moreover improved experimental data should be made available near the (3,3) resonance region at same energies for both π⁺ and π⁻ for a study of the variation of purely strong interaction parameters with energy.