CHAPTER-16

R.C. COLUMNS UNDER BIAxIALLy ECCENTRIC LOADs

16.1. General:

In the study of R.C. rectangular columns subjected to uniaxial eccentric loads, the concrete in the compression side of neutral axis is also rectangular in shape. But in the case of rectangular reinforced concrete columns subjected to biaxially eccentric loads, the shape of the compression zone varies from a triangular to pentagon depending on the position of the load due to the variation of neutral axis in position and direction. Hence the validity of the confined concrete theory which is developed based on the studies on prismatic confined concrete specimens is sought in the present chapter.

In the square R.C. columns under biaxially eccentric loads the direction of the neutral axis is known i.e., almost perpendicular to the radial line joining the position of the applied load and the centroid of the square section. The main variables are the eccentricity of the load (i.e., axial load level) and degree of confinement (diameter and pitch of binders). The load is eccentrically applied only along 45° bisector line to adjacent sides i.e., along one of the diagonal directions of the cross-section.

The experimental moment-curvature and load-lateral deflection curves are presented for all the columns.
FIG 16.1  MOMENT-CURVATURE CURVES.
FIG 16

M/\pi \beta D^2

\phi_0 \times 10

0 4 8 12 16 20 24 28

BE 5

BE 6

FIG 16.2  MOMENT-CURVATURE RESULTS
FIG: 16-3  MOMENT-CURVATURE CURVES
FIG. 16.4 MOMENT-CURVATURE CURVES.
FIG. 16.5  MOMENT-CURVATURE CURVES
FIG: 16.7 LOAD - DEFLECTION CURVES.
FIG 16.8 LOAD-DEFLECTION CURVES.
FIG 16.9 LOAD-DEFLECTION CURVES.
FIG. 16-10 LOAD-DEFLECTION CURVES
FIG 16–11 LOAD–DEFLECTION CURVES
eccentric loads are analysed with the proposed idealised stress-strain curve for confined concrete in the Chapter-5 of the present work. The analytical ultimate loads, curvatures thus computed are compared with the experimental values. The theoretical failure strains are compared with the actual experimental results.

The columns are also analysed with the confined concrete theory developed by Ramaswamy Reddy and compared with the experimental results.

To have a comparison, the ultimate strength of the columns are analysed with the formulae developed by Bresler and Ramamurthy based on unconfined concrete theory.

In case of columns subjected to biaxially eccentric loads, the equilibrium conditions to be satisfied are:

(i) the algebraic sum of the internal and external forces must be zero.

(ii) the algebraic sum of the internal and external moments with reference to any two orthogonal axes must be zero.

For a square R.C. sections, the moments are taken on the two principal axes. Usually there are there unknowns
for a given section and eccentricities of the load position:

(a) position of the neutral axis.
(b) inclination or direction of neutral axis.
(c) magnitude of the neutral axis depth.

Hence the above three equilibrium conditions are required to solve the three unknowns.

In the present experimental work, the 24 square columns are subjected to eccentric loads along the bisector line of the two adjacent sides i.e., along one of the diagonals (i.e., along the 45° line between the principal axes of the sections). According to the extensive investigation carried out by Ramamuthy[56], the following assumption is made in addition to the assumptions already made under section 13.2.

"The neutral is assumed to be perpendicular to the line joining the load position to the centroid of the square cross-section".

Then the unknowns are reduced to two only and only two equilibrium conditions are applied to get the position and magnitudes of the neutral axis.

16.3.1: Analysis of columns under biaxial bending using proposed Trilinear stress-strain curve for confined concrete:

The theoretical ultimate load, curvature, moment are
computed using the proposed trilinear stress-strain curve for confined-concrete. The cross-section of a column together with the strain distribution and stress distribution along the section are shown in Fig. 16.1.

The equilibrium of forces and moments are given by:

\[
\begin{align*}
P &= C_c + A_{s1} f_{s1} + A_{s2} f_{s2} + A_{s3} f_{s3} + A_{s4} f_{s4} + A_{s5} x f_{s5} \quad \ldots (16.1) \\
M &= C_c \bar{y} + A_{s1} f_{s1} (d_5 - d_1) + A_{s2} f_{s2} (d_5 - d_2) + A_{s3} f_{s3} x (d_5 - d_3) + A_{s4} f_{s4} (d_5 - d_4) \ldots (16.2)
\end{align*}
\]

where,

- \(C_c\) = compressive force resisted by concrete.
- \(A_{s1}, A_{s2}\) - cross sectional area of the longitudinal reinforcement.
- \(f_{s1}, f_{s2}, f_{s3}, f_{s4}, f_{s5}\) - stresses in the longitudinal reinforcement.
- \(\bar{y}\) = is the C.G of the \(C_c\) from the centre of the extreme tension reinforcement.

\(d_1, d_2, d_3, d_4, d_5\) are depth of the various reinforcing bars as shown in Fig. 16.1.
FIG. 16 I+ STRESS BLOCK PARAMETERS.
Every position of the neutral axis corresponding to a certain ultimate capacity of the column (P) and the corresponding ultimate moment of resistance M (i.e., interaction relation). Based on this principle a number of P and M and curvature values are computed assuming the different positions of neutral axis using IBM 370/155 computer. The theoretical ultimate load, curvature for the corresponding experimental eccentricities are interpolated from the above values and are compared with the experimental values as shown in Table 16.3.

16.3.2: Analysis of Columns under biaxial bending using Ramaswamy Reddy's confined concrete theory.

The theoretical ultimate load and curvature are computed using the stress strain curve for confined concrete proposed by Ramaswamy Reddy (21) and the corresponding cross-section with the stress and strain distribution are shown in Fig. 16.2.

The equilibrium conditions for the forces and moments are given by:

\[ P = C_c + A_{s1} f_{s1} + A_{s2} f_{s2} + A_{s2} f_{s3} + A_{s2} f_{s4} + A_{s1} f_{s5} \]  
\[ (16.3) \]

\[ M = C_c \bar{y} + A_{s1} f_{s1} (d_5-d_1) + A_{s2} f_{s2} (d_5-d_2) + A_{s2} f_{s3} (d_5-d_3) + A_{s2} f_{s4} (d_5-d_4) \]  
\[ (16.4) \]

The notation of the various terms used in the Fig. 16.2 and equations (16.3) and (16.4) are same as given under section 16.3.1.
FIG 6.15 STRESS BLOCK PARAMETERS USING RAMSDAY-REDDY'S STRESS STRAIN CURVE
The theoretical ultimate loads and the corresponding curvatures are computed similar to the procedure explained in the section 16.3.1.

The experimental values are compared with the theoretical values as shown in Table 16.4.

16.3.3: Bresler's Theory:

Using the Hognestad's theory already discussed in section 13.5, the square column sections are analysed to get the axial load and moment on the principal axes (Pux and Puy) for the eccentricity of the load given by
\[ e_x = e_y = e_r / \sqrt{2} \] i.e., components of the radial eccentricity er.

Knowing the values of Pux & Puy, the ultimate load for an R.C. column under biaxial eccentricity Pur according to Bresler's formula is given by:
\[ \frac{1}{P_{ur}} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_o} \quad \ldots (16.5) \]

Where \( P_o \) is the axial load capacity of the column given by the equation.
\[ P_o = 0.85 f_c' A_c + A_s f_{sy} \quad \ldots (16.6) \]
For square columns, the formula (16.5) reduces to

\[ \frac{P_{ur}}{P_{ux}} = \frac{P_0}{2P_0 - P_{ux}} \quad \ldots \quad (16.7) \]

Thus computed using the equation (16.7).

The analytical results are compared with the experimental results as shown in Table 16.6.

16.3.4: Ramamurthy's Theory:

As in the section 16.3.3, the axial load and moments (\(P_{ux}, P_{uy}, M_{ux}\)) are computed using Hognestad's theory for uniaxial bending for the eccentricity of the load given by

\[ \epsilon_1 = \epsilon_2 = \frac{\sqrt{2}}{\sqrt{3}} \quad \text{i.e., components of the radial eccentricity.} \]

According to Ramamurthy's theory that the ultimate radial moment for a square H.C. section is given by

\[ M_{ur} = M_{ux} \quad \ldots \quad (1 - \alpha/45) \quad \ldots \quad (15.6) \]

where \(M_{ux}\) = uniaxial ultimate moment capacity of the column section on the principal axis.

\[ \alpha = \text{in degrees is the inclination of the line joining the centroid of the sections to the point of application of the load.} \]

The analytical results thus computed are compared with the experimental results as shown in Table 16.6.
### Table 16.1

Experimental results of R.C. square columns (Biaxially Eccentric loads)

<table>
<thead>
<tr>
<th>Designation</th>
<th>$f_c^1$ (kg/cm²)</th>
<th>$%$ of steel</th>
<th>$\Delta e_0$ (cm)</th>
<th>Ultimate long. load $P_u$ (tonnes)</th>
<th>$M_u$ (tonnes.cm)</th>
<th>$P_{u}$ (tonnes)</th>
<th>Type of failure</th>
<th>Long. bars buckled or not</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. BE1</td>
<td>200</td>
<td>18.70</td>
<td>0.392</td>
<td>5.67</td>
<td>107.30</td>
<td>0.126</td>
<td>T</td>
<td>N.B.</td>
</tr>
<tr>
<td>2. BE2</td>
<td>&quot;</td>
<td>18.70</td>
<td>0.191</td>
<td>5.64</td>
<td>106.00</td>
<td>0.13</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>3. BE3</td>
<td>&quot;</td>
<td>12.50</td>
<td>1.716</td>
<td>11.41</td>
<td>152.00</td>
<td>0.254</td>
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<td>&quot;</td>
</tr>
<tr>
<td>4. BE4</td>
<td>&quot;</td>
<td>12.50</td>
<td>0.605</td>
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<td>139.00</td>
<td>0.237</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>5. BE5</td>
<td>300</td>
<td>3.00</td>
<td>0.74</td>
<td>55.30</td>
<td>210.00</td>
<td>0.835</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>6. BE6</td>
<td>&quot;</td>
<td>5.00</td>
<td>1.174</td>
<td>29.31</td>
<td>184.00</td>
<td>0.442</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>7. BE7</td>
<td>&quot;</td>
<td>4.00</td>
<td>1.333</td>
<td>36.02</td>
<td>181.00</td>
<td>0.635</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>8. BE8</td>
<td>&quot;</td>
<td>4.00</td>
<td>1.405</td>
<td>36.30</td>
<td>195.00</td>
<td>0.537</td>
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<tr>
<td>9. BE9</td>
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<td>12.00</td>
<td>0.555</td>
<td>9.56</td>
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<td>0.163</td>
<td>T</td>
<td>&quot;</td>
</tr>
<tr>
<td>10. BE10</td>
<td>&quot;</td>
<td>8.00</td>
<td>0.660</td>
<td>19.00</td>
<td>164.00</td>
<td>0.325</td>
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### TABLE - 16.2

Comparison of ultimate loads in R.C. square columns (Biaxial Eccentricity)

<table>
<thead>
<tr>
<th>$g - P_u$ (tonnes)</th>
<th>Theoretical Ultimate loads $P_u$ (tonnes)</th>
<th>Value of Confinement index ($C_1$)</th>
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<tbody>
<tr>
<td></td>
<td>Proposed theory</td>
<td>Rameswamy's theory 2/7</td>
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<td>-------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>2</td>
<td>5.67</td>
<td>6.52 1.02 4.93 1.15</td>
</tr>
<tr>
<td>3</td>
<td>5.34</td>
<td>5.92 0.93 6.03 1.16</td>
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<td>4</td>
<td>11.41</td>
<td>8.17 1.40 7.09 1.60</td>
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<td>10.63</td>
<td>8.79 1.21 7.69 1.33</td>
</tr>
<tr>
<td>6</td>
<td>56.30</td>
<td>28.59 1.96 23.95 2.35</td>
</tr>
<tr>
<td>7</td>
<td>29.91</td>
<td>17.37 1.72 14.41 2.03</td>
</tr>
<tr>
<td>8</td>
<td>36.02</td>
<td>30.33 1.18 26.19 1.38</td>
</tr>
<tr>
<td>9</td>
<td>36.30</td>
<td>28.96 1.25 24.92 1.49</td>
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</table>

377
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<th></th>
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<th>5</th>
<th>6</th>
<th>7</th>
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<th>10</th>
<th>11</th>
<th>12</th>
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<td>1</td>
<td>25.80</td>
<td>30.19</td>
<td>0.86</td>
<td>24.98</td>
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<td>15.90</td>
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<td>1.29</td>
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<td>1.12</td>
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<td>4.94</td>
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<td>1.85</td>
<td>4.22</td>
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<tr>
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<td>11.08</td>
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<td>6.50</td>
<td>1.70</td>
<td>5.87</td>
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<td>6.90</td>
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<td>13.50</td>
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<td>14.50</td>
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<td>1.09</td>
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<td>1.25</td>
<td>7.15</td>
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<td>6.39</td>
<td>1.31</td>
<td>2.99</td>
<td>2.28</td>
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<td>14.99</td>
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<td>1.17</td>
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<td>21.90</td>
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<td>16.92</td>
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<td>12.75</td>
<td>1.92</td>
<td>13.10</td>
<td>1.87</td>
<td>1.77</td>
<td>1.76</td>
</tr>
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</table>

**Average Fc:**

|   | 1.13 | 1.39 | 1.55 | 1.62 |
### TABLE - 16.3

Comparison of ultimate strains, curvatures in R.C. square columns (Biaxial Eccentricity)

<table>
<thead>
<tr>
<th>n</th>
<th>Experimental strains in concrete</th>
<th>Theoretical strain in concrete</th>
<th>Experimental curvatures $\phi_D$</th>
<th>Theoretical curvatures $\phi_D$</th>
<th>Neutral Axis depth (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>at $P_u$ at 0.9 $P_u$</td>
<td>Proposed theory</td>
<td>at $P_u$ at 0.9 $P_u$</td>
<td>Proposed theory</td>
<td>Expt. Proposed theory</td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td>0.0564</td>
<td>0.032</td>
<td>*</td>
<td>0.1572 0.153</td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td>0.0549</td>
<td>0.032</td>
<td>*</td>
<td>0.1693 0.153</td>
</tr>
<tr>
<td>4</td>
<td>*</td>
<td>0.0553</td>
<td>0.051</td>
<td>*</td>
<td>0.1650 0.150</td>
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<tr>
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<td>*</td>
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<td>0.031</td>
<td>*</td>
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<td>7</td>
<td>0.00457</td>
<td>0.0123</td>
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<td>0.0112 0.0317</td>
<td>0.063 0.063</td>
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<td>0.00565</td>
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<td>0.0112 0.0259</td>
<td>0.045 0.041</td>
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<td>0.030</td>
<td>0.015</td>
<td>0.129 0.116</td>
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<td>11</td>
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<td>0.0123</td>
<td>0.032</td>
<td>0.0101 0.0276</td>
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CHAPTER 17

CONCLUSIONS AND SCOPE FOR FURTHER RESEARCH

17.1 Conclusions:

The following conclusions are drawn based on the experimental and analytical investigation of the present work.

Rectangular columns under uniaxial eccentricity:

Ultimate Loads:

1. The ultimate load values obtained by the proposed confined concrete theory (i.e., using trilinear stress strain curve) are in close agreement with experimental values. It is found that \( \frac{P_u \text{ expt}}{P_u \text{ cal}} = 1.214 \) (Vide Table 13.1).

2. The ultimate loads obtained by the unconfined concrete theory (Hognestad's theory) and Ramaswamy Reddy's confined concrete theory are slightly conservative compared to the proposed theory (Vide Tables 13.1, 13.2, 13.3). It is found that \( \frac{P_u \text{ expt}}{P_u \text{ cal}} = 1.32 \) and 1.283 in the two cases respectively.

Deformations (Curvatures and lateral deflections):

3. The curvatures at ultimate in the eccentrically loaded columns for a given axial load level are found to be linearly related to the factor called confinement index \( C_i \) when \( C_i \leq 1 \) which is the maximum practicable value of confinement that can be achieved in any column.
4. The increase in curvature of an eccentrically loaded column for a given axial load level can be achieved by suitably modifying the confinement index which is a function of the material properties, least lateral dimension of the column, spacing of the binders, volumetric ratio of binder to the concrete in compression zone.

5. The flat portion of moment curvature and the load-lateral deflection curves are found to be larger for higher values of confinement index for a given axial load level (Fig. 12.1 to 12.23).

6. The curvatures predicted by the proposed theory is realised when the ultimate load falls to 90 to 85 percent (Vide Table 13.1).

It may be stated here that the Hoggestad's theory (unconfined concrete) gives the curvature only at ultimate load (Vide Table 13.3).

7. In the case of columns under tension type of failure, the ductility factor which is the ratio of ultimate curvature to curvature at first yield is linearly related to confinement index for a given axial load level (Vide Table 13.5).
8. Higher the axial load level, steeper the descending portion of the load-lateral deflection curve for any column, and irrespective of the value of confinement index (Vide Fig. 12.16 to 12.29).

9. It is observed that magnitude of the lateral deflection of a column at ultimate load or post ultimate load does not follow any set pattern (Vide Tables 12.1 to 12.3 and Fig. 12.16 to 12.29).

**Concrete strain on the compression face of the column:**

10. The maximum value of compressive strain in concrete reached in any column is in the order of 0.01 when the load has fallen to 90 to 85% of the ultimate load (Vide Table 13.6).

**Type of binders:**

11. There is no appreciable difference between a rectangular binder (i.e., tie) and rectangular helical binder in improving the strength, curvature, lateral deflection and strain of confined concrete members.

**Percentage of longitudinal reinforcement:**

12. Within the range of 2 to 3.8% that is used in the present experiments, the longitudinal reinforcement has no significant effect on the behaviour of confined concrete.
Mode of Failure:

13. When the eccentricity of the load on a column is close to the balanced eccentricity, compressive type of failure can be converted into a tension type of failure, by increasing the value of confinement index (Vide Table 13.7).

Square Columns under biaxially eccentric loads:

14. The experimental ultimate loads are in close agreement with the proposed confined concrete theory (Vide Table 16.1). The value of $P_u \text{expt}/P_u \text{cal}$ is equal to 1.13 in this case whereas it ranges from 1.39 to 1.62 in the theories proposed by Ramamurthy, Ramaswamy, Reddy, and Bresler.

15. The strains and curvatures predicted by confined concrete theory are found to be 3 to 6 times higher than the experimental ones. It may be attributed due to the high values of confinement index resulting on account of the triangular shape of the compression zone though the amount of lateral reinforcement is kept the same (Vide Table 16.2). Further investigation is necessary before any conclusions are drawn in this respect.
17.2. Scope for further research:

(i) Effect of cover in the confined columns.

(ii) Deformations in confined columns under biaxial eccentricity of loads.

(iii) Effect of cross-ties in confined columns.

(iv) Behaviour of confined columns subjected to dynamic and repeated loading.

(v) Behaviour of confined columns under sustained loading.

(vi) The effect of rate of loading in confined columns.
REFERENCES


33. Hognestad, E. Hanson, N.W., "Concrete stress distribution in ultimate strength design", Journal of the American Concrete Institute, December, 1955, Proceedings Vol. 52, p. 455 to p. 479.


