APPENDIX III-1

MAINLINE PROGRAMME FOR TYPE TWO (FIGURE 4.2) AND TYPE THREE (FIGURE 4.3) SOLUTIONS FOR EXTRUSION WITH SLIPPING FRICTION. SLIPPING FRICTION ON DIE FACES AND SMOOTH CONTAINER WALLS.

REAL LADA, LAD1, LAD, NU1, NU
REAL U (10, 10), RC1(10, 1), RC2(10, 1), FRIC(10)
REAL GT(10, 10), SAP(10, 10), TE(10, 10), TT(10, 10), SNU(10, 10), TA(10, 10),
TB(10, 10), Q(10, 10), QT(10, 10), PB(10, 10), SST(10, 10),
ISSN(10, 10), SST(10, 10), GSS(10, 10), CSQ(10, 10), GSQ(10, 10), GSU(10, 10), GSG(10, 10),
GCPB(10, 10), RE1(10, 1)
REAL QB(10, 10), P(10, 10), PB1(10, 10), PQB(10, 10), GPQ(10, 10), GTS(10, 10),
PFB(10, 10), GU(10, 10), GPQ(10, 10), GP(10, 10), PRI(10, 1), GZ(10, 1),
RHZ(10, 1), TP(10, 10), TH(10, 10)
REAL SL1(10, 1), OJ(10, 1), AH(10, 1), BH(10, 1), SL3(10, 1),
SL4(10, 1), SSL4(10, 1), SL5(10, 1), OBR(10, 1), JH1(10, 1), JH(10, 1),
IQ(10, 1), EBR(10, 1), EH(10, 1), SL6(10, 1), SL7(10, 1)

NO = 6
N = 10
CON = 3.14159/180.0
FRIC (1) = 0.0
FRIC (2) = 3.0
FRIC (3) = 5.75
FRIC (4) = 8.75
FRIC (5) = 11.0 + (47.5/60.0)
FRIC (6) = 15.0
FRIC (7) = 18.0 + (26.0/60.0)
FRIC (8) = 22.0 + (13.0/60.0)
FRIC (9) = 26.5 + (8.0/60.0)
FRIC (10) = 32.0 + (5.0/60.0)
DO 1 I1 = 1, 10
DO 2 J1 = 1, 20
APHA1 = FRIC(I1)
TETA1 = FLOAT(J1)

NU1 = 90.0
LADA = 45.0 - APHA1
LAD1 = 90.0 - LADA
NU = NE1 x CON
APHA = APHA1 x CON
TETA = TETA1 x CON
LADA = LADA x CON
LAD1 = LAD1 x CON
BETA = NU + TETA + APHA
ETA = TETA + APHA

LAD = 2.0 x LADA
ANG = 45.0 x CON
AL1 = ANG - TETA
PM = SIN (LADA)/COS(LADA)
DO 10 I = 1, N
DO 11 J = 1, N
U (I, J) = 0.0
11 CONTINUE
10 CONTINUE
DO 12 I = 1,N
  U(I,1) = 1.0
12 CONTINUE
RC1 (1,1) = 1.0
DO 14 I = 2,N
  RC1 (I,1) = 0.0
14 CONTINUE
RC2 (1,1) = PM
DO 15 I = 2,N
  RC2 (I,1) = 0.0
15 CONTINUE
CALL GMAT (N,TETA,LAD1,GT)
CALL SMAT(N,Apha,SAP)
CALL TMAT (N,Eta,TE,TT)
CALL SMAT (N,NU,SNU)
CALL TMAT (N,BETA,TA,TB)
CALL QMAT (N,TETA,BETA,R,Q,QTB)
CALL PSTAR(N,-BETA,PB)
CALL MMULT(N,N,SAP,TT,STT)
CALL MMULT(N,N,STT,SNU,SSM)
CALL MMULT(N,N,GT,SST,GSS)
CALL MMULT(N,N,GSS,QTB,GSQ)
CALL MADDA(N,GSQ,SR,U,GSU,-1.0,1.0,1.0)
CALL MADDA(N,GSS,GT,GSG,1.0,1.0,1.0)
CALL MMULT(N,N,GSU,PB,GPB)
CALL MMULT(N,1,GPB,RH1)
CALL AMIN(N,GPB)
CALL MMULT(N,1,GPB,SL1)
CALL MMULT(N,1,PB,SL1,SL5)
CALL MMULT(N,1,QTB,AL,AD)
DO 16 I = 1,N
  BH (I,1) = AD(I,1) - RC1 (I,1)
16 CONTINUE
CALL MMULT (N,1,TB,BH,THB)
CALL MMULT (N,1,SNU,TBH,SL3)
CALL MMULT (N,1,TT,SL3,SL4)
CALL MMULT (N,1,SAP,SL4,SSL4)
DO 17 I = 1,N
  SL5 (I,1) = SSL4 (I,1) - RC1 (I,1)
17 CONTINUE
C CALCULATION OF HYDROSTATIC PRESSURE
PO = 0.0
P1 = PO - 2.0 x TETA
CALL FORC(N,SL1,XX1,YY1,FX1,FY1,TETA,PO,-1.0,-1.0,1.0)
F1 = FX1 x SIN (A1) - FY1 x COS (A1)
FT = COS (ANG) - P1 x SIN (ANG) - F1
CALL RDXY (N,SL1,XX1,YY1,X1,Y1,TETA,-1.0)
H = X1 x COS (A1) + Y1 x SIN (A1) + COS (ANG)
PO = FT/H
CALL FORC(N,SL1,XXA,YYA,FXA,FYA,TETA,PO,-1.0,-1.0,1.0)
FTA = COS (ANG) - (PO - 2.0 x TETA) x SIN (ANG) + FYA x COS (A1)
INDEX = 1
GOTO 20

50 D = W1 + W2 + W3 + COS(LADA)
F3 = SIN(LADA) + P3 x COS(LADA)
FEX = F1 - F2 + F3
RHO1 = 1.0/HE
GOTO 201

52 RAT1 = RAT
WRITE(NO,107) RHO1, RAT
107 FORMAT(1H ,5HRHO1 ,F10.6,5X,4HRAT ,F10.6)
IF (RHO1 -0.0) 1,1,20

C CALCULATION FOR SECOND SLIP LINE
20 CALL QSTAR(N,BETA,QB)
CALL PMAT (N ,BETA, TETA,R ,P,PBT)
CALL MMULT(N,N,PB, QB,PQB)
CALL MMULT(N,N,QTB,PQB,QPO)
CALL MMULT(N,N,GT,STT,GTS)
CALL MADDA(N,PBT,QPQ,PQP,-1.0,1.0,1.0)
CALL MADDA(N,GTS,U,GU,1.0,1.0,1.0)
CALL MMULT(N,N,GSS,PQP,GPQ)
CALL MMULT(N,1,PBP,RC1,PR1)
CALL MMULT(N,1,GU,RC2,GR2)
DO 25 I = 1,N
RH2 (I,1)= GR2(I,1) - PBP(I,1)
25 CONTINUE
CALL MMULT(N,1,GPB,RH2,SL1)
CALL MMULT(N,1,QBT,RC1,QR1)
DO 26 I = 1,N
JH1 (I,1) - SL1(1,1) - QBR(I,1)
26 CONTINUE
CALL MMULT(N,1,PBT,RC1,PBR)
DO 27 I = 1,N
BH (I,1) = PBR(I,1) + QTJ(I,1)
27 CONTINUE
CALL MMULT(N,1,TB,BH,TBH)
CALL MMULT(N,1,SNU,TBH,EH)
DO 28 I = 1,N
SL3 (I,1) = EH (I,1) - RC2(I,1)
28 CONTINUE
CALL MMULT(N,1,TT,SL2,SL4)
CALL MMULT(N,1,SAP,SL4,SL5)

C CALCULATION OF HYDROSTATIC PRESSURE
PO = 0.0
CALL FORC(N,SL1,XXI,YY1,FX1,FY1,TETA,PO,-1.0,-1.0,1.0)
F1 = FX1 x SIN(A1) - FY1 x COS(A1)
FT = SIN(A1) - F1
CALL RDXY(N,SL1,XX1,YY1,X1,Y1,TETA,-1.0)
\[ H = X_1 \cos(A_1) + Y_1 \sin(A_1) + \cos(A_1) \]
\[ PO = IT/HE \]
\[ \text{CALL FORC}(N, SL1, XXA, YYA, FXA, FYA, TETA, PO, -1.0, -1.0, 1.0) \]
\[ FTA = \sin(A_1) - PO \cos(A_1) + FYA \cos(A_1) - FXA \sin(A_1) \]
\[ \text{INDEX} = 2 \]
\[ \text{GOTO 200} \]
51 \[ D = W1 + W2 + W3 + 1.0/\cos(LADA) \]
\[ F3 = \sin(LADA) + P2 \cos(LADA) + PM \cos(LADA) + P2 \sin(LADA) \]
\[ FEX = F1 - P2 + F3 \]
\[ \text{RHO2} = 1.0/H \]
\[ \text{GOTO 201} \]
53 \[ \text{RAT2} = \text{RAT} \]
\[ \text{PROD} = \text{RAT1} \times \text{RAT2} \]
\[ \text{WRITE}(NO, 207) \text{RAT2}, \text{RHO2}, \text{PROD} \]
207 \[ \text{FORMA} \text{FORMAT}(1H, 5H \text{RAT2}, F10.6, 5X, 5H \text{RHO2}, F10.6, 5X, 5H \text{PROD}, F10.6) \]
\[ \text{GOTO 2} \]
200 \[ \text{WRITE}(NO, 102) \text{TETA1}, \text{APHA1} \]
102 \[ \text{FORMAT}(1H, 6HTETA1, F10.6, 5X, 6H \text{APHA1}, F10.6) \]
\[ \text{WRITE}(NO, 110) \text{ft}, \text{F1}, \text{H}, \text{PO}, \text{FTA} \]
110 \[ \text{FORMAT}(1H, 3HF1, F10.6, 5X, 2HH, F10.6, 5X, 3H \text{PO}, F10.6, 5X, 14HF1A, F10.6) \]
\[ \text{CALL TMAT}(N, \text{APHA}, TP, TH) \]
\[ \text{CALL MMULT}(N, 1, TP, SL4, SL6) \]
\[ \text{CALL MMULT}(N, 1, GT, SL5, SL7) \]
\[ \text{CALL RDXY}(N, SL5, XX5, YY5, X5, Y5, TETA, -1.0) \]
\[ \text{CALL RDXY}(N, SL7, XX7, YY7, X7, Y7, TETA, -1.0) \]
\[ \text{CALL RDXY}(N, SL6, XX6, YY6, X6, Y6, \text{APHA}, -1.0) \]
\[ W1 = X5 \times \cos(LADA) + Y5 \times \sin(LADA) \]
\[ W2 = X7 \times \cos(LADA) - Y7 \times \sin(LADA) \]
\[ W3 = X6 \times \sin(\text{ANG}) - Y6 \times \cos(\text{ANG}) \]
\[ P2 = PO + 2.0 \times \text{BETA} \]
\[ P3 = P2 - 4.0 \times \text{TETA} \]
\[ P4 = P3 + 2.0 \times \text{APHA} \]
\[ \text{CALL PRES1}(N, SL5, \text{TETA1}, J1, P3, LADA, LAD1, LAD, -1.0, -1.0, \text{F1}) \]
\[ \text{CALL FORC}(N, SL6, XX6, YY6, FX6, FY6, APHA, P4, -1.0, -1.0, 1.0) \]
\[ F2 = FX6 \times \cos(\text{ANG}) + FY6 \times \sin(\text{ANG}) \]
\[ \text{GOTO (50, 51)}, \text{INDEX} \]
201 \[ B = \text{B+D} \]
\[ \text{EXP} = \text{FEX}/B \]
\[ \text{BW} = B/H \]
\[ \text{DW} = D/H \]
\[ \text{RED} = D/B \]
\[ \text{RAT} = D/H \]
\[ \text{AM} = \cos(LAD) \]
\[ \text{PRED} = \text{FEX}/D \]
\[ \text{WRITE}(NO, 103) \text{W1, W2, W3, D, B, H} \]
103 \[ \text{FORMAT}(1H, 3HW1, F10.6, 5X, 3HW2, F10.6, 5X, 3HW3, F10.6, 5X, 2HD, \]
\[ F10.6, 5X, 2HH, F10.6, 5X, 2HH, F10.6) \]
\[ \text{WRITE}(NO, 104) \text{F1, F2, F3, FEX, EXP, RED} \]
104 FORMAT (1H, 3HF1, F10.6, 5X, 3HF2, F10.6, 5X, 3HF3, F10.6, 15X, 4HFEX, F10.6, 5X, 4HEXP, F10.6, 5X, 4HRAD, F10.6)
WRITE (NO, 106) BW, DW, WRED, AM)
106 FORMAT (1H, 3HBW, F10.6, 5X, 3HDW, F10.6, 5X, 5HPRED, F10.6, 15X, 3HAM, F10.6)
GOTO (52, 53), INDEX
2 CONTINUE
1 CONTINUE
STOP
END.
APPENDIX III-2

MAINLINE PROGRAMME FOR TYPE IV SOLUTION (FIGURE 4.4) FOR EXTRUSION WITH SLIPPING FRICTION

REAL LADA1, LADA, LAD1, LAD, NU1, NU
REAL FRIC(10), RC1(10,1), RZ2(10,1), U(10,10)
REAL TA(10,10), TB(10,10), SN(10,10), RZ(10,10), GT(10,10), GZ(10,10), 
1 GE(10,10), RE(10,10)
RE(10,1), RE(10,1), SF(10,1), FS(10,1), LF(10,1), OS(10,1), LT(10,1), 
1 TL(10,1)
NO = 6
N = 10
CON = 3.14159/180.0
FRIC (1) = 0.0
FRIC (2) = 3.0
FRIC (3) = 5.75
FRIC (4) = 8.75
FRIC (5) = 11.0 + (47.5/60.0)
FRIC (6) = 15.0
FRIC (7) = 18.0 + (26.0/60.0)
FRIC (8) = 22.0 + (13.0/60.0)
FRIC (9) = 26.5 + (8.0/60.0)
FRIC (10) = 32.0 + (5.0/60.0)

C ALPHA KEPT FIXED AND TETA VARIED
C FOR INITIAL CALCULATION ZETA EQUALS ZERO
DO 10 J1 = 1,10
DO 20 II = 2,20,2
ZETAI = 0.0
APHA1 = FRIC(J1)
TETA1 = FLOWT(11)
NU1 = 90.0 - ZETAI - TETA1
BETA1 = 90.0 + APHA1
ETA1 = APHA1 + ZETAI
ZI1 = APHA1 + ZETAI + TETA1
LADA1 = 45.0 - APHA1
LAD1 = 90.0 - LADA1
ZETA = ZETAI x CON
APHA = APHA1 x CON
NU = NU1 x CON
BETA = BETA1 x CON
ETA = ETA1 x CON
ZI = ZI1 x CON
LADA = LADA1 x CON
LAD1 = LAD1 x CON
LAD = 2.0 x LADA
ANG = 45.0 x CON
ANG1 = (45.0 - ZETA1) x CON
DO 10 I = 1,N
DO 11 J = 1,N
U(I,J) = 0.0
CONTINUE
CONTINUE
DO 12 I = 1,N
U(I,I) = 1.0
12 CONTINUE
RC1 (1,1) = -1.0
DO 14 I = 2,N
RC1 (I,1) = 0.0
14 CONTINUE
PM = SIN(LADA)/COS(LADA)
RC2 (1,1) = PM
DO 15 I = 2,N
RC2 (I,1) = 0.0
15 CONTINUE
CALL RMAT(N,BETA,TA,TB)
CALL SMAT(N,NU,SN)
CALL RMAT(N,ZI,RZ)
CALL GMAT(N,ETA,LADA,GT)
CALL GMAT(N,ZI,LADA,GZ)
CALL GMAT(N,ETA,LADA,GE)
CALL AMAT(N,GE)
CALL RMAT(N,ETA,RET)
CALL MMULT(N,1,TB,RC1,CE)
CALL MMULT(N,1,SN,OE,RE)
DO 16 I = 1,N
SF (I,1) = RE(I,1) - RC2(I,1)
16 CONTINUE
CALL HFLILT(N,1,RZ,SF,FS)
CALL MMULT(N,1,GT,FS,LK)
CALL MMULT(N,1,GS,FS,OS)
CALL MMULT(N,1,GE,OS,LT)
CALL MMULT(N,1,RET,LT,TL)
PI = 1.0 + 2.0 x BETA
P2 = PI + 4.0 x TETA
P3 = P2 + 2.0 x ETA
CALL RDXY (N,TL,XX1,YY1,X1,Y1,ETA,-1.0)
CALL RDXY (N,LK,XX2,YY2,X2,Y2,TETA,1.0)
CALL RDXY (N,FS,XX3,YY3,X3,Y3,TETA,1.0)
W1 = X1 x SIN(ANG1) - Y1 x COS(ANG1)
W2 = X2 x COS(LAD1) + Y2 x SIN(LAD1)
W3 = X3 x COS(LADA) - Y3 x SIN(LADA)
D = W1 + W2 + W3 + 1.0/COS(LADA)
B = D + COS(ANG)
CALL PRESS1 (N,FS,TETA1,II,P1,LAD1,LADA,LAD,0.0,0.0,F1)
CALL FORC(N,TL,XX1,YY1,FX1,FY1,ETA,F3,0.0,0.0)
F2 = FX1 x COS(ANG1) + FY1 x SIN(ANG1)
F3 = SIN (LADA) + W1 x COS(LADA) + PM x (COS(LADA) + PM x SIN(LADA))
FEX = F1 - F2 + F3
EXP = FEX/B
RED = D/B
PRED = FEX/D
BW = B/COS(ANG)
aw
DW = D/COS(ANG)
AM = COS(LAD)
WRITE(NO,101) ZETA1,APHA1,TETA1,NU1,BETA1,ETA1
101 FORMAT(1H 6HZETA1 I10.6,5X,6HAPHA1 F10.6,5X,6HTETA1 ,
1F10.6,5X,4HNU1 ,F10.6,5X,6HBETA1 ,F10.6,5X,5HETA1 ,F10.6)
WRITE(NO,102) W1,W2,W3,D,S,ZI1,LADA1
102 FORMAT(1H 3HW1 F10.6,5X,3HW2 F10.6,5X,3HW3 F10.6,5X,
12HD F10.6,5X,2HB F10.6,5X,4HZI1 F10.6,5X,6HLADA1 F10.6)
WRITE(NO,103) F1,F2,F3,FEX,EXP,RED
103 FORMAT(1H 3HF1 F10.6,5X,3HF2,F10.6,5X,3HF3,F10.6,5X,4HFEX ,
1F10.6,5X,4HEXP,F10.6,5X,4HRED,F10.6)
WRITE (NO,104) PRED,BW,DW,AM
104 FORMAT(1H 5HPRED F10.6,5X,3HBW,F10.6,5X,3HDW,F10.6,5X,
13HAM,F10.6)
20 CONTINUE
10 CONTINUE
STOP
END
CALCULATION OF UPPER BOUND LOADS FOR PROPOSED VELOCITY FIELDS

In this section, equations for the lengths and the magnitudes of the velocity discontinuities of the proposed velocity fields are derived. The upper bound to the extrusion load for any field was calculated by equating the work done by the punch to that against the velocity discontinuities. In all cases the nature of the constraints were similar and they were eliminated by a change of variables as explained in 4.6.2.1.

Extrusion through smooth square dies

The proposed velocity fields for the case of extrusion through smooth square dies are shown in figures 4.20 to 4.23. Calculation of upper bound load for the velocity field of figure 4.20 (type I) was discussed in 4.6.2.1. The lengths and the magnitudes of the velocity discontinuities for the velocity field II (figure 4.21) were calculated by substituting \( \theta_5 = 0 \) and \( \theta_6 = 0 \) in equations (4.31) and (4.32) and for velocity field III substituting \( \theta_5 = -\theta_5 \) in the above equations. For velocity field II all constraints were eliminated by defining the new variables \( y_i \), where,

\[
\theta_i = \pi/2 (\exp (-y_i^2))
\]

For velocity field III, however, it was necessary to satisfy the additional constraints,
$$\theta_3 > \theta_5 \quad \text{and,} \quad \theta_2 > \theta_5$$

(III.1)

The constraints, $\theta_3 > \theta_5$, was eliminated by expressing $\theta_3$ and $\theta_5$ such that

$$\theta_3 = \pi/2 \left( \exp \left( -\frac{\theta_3^2}{2} \right) \right)$$

and,

$$\theta_5 = \pi/2 \left( \exp \left( -\frac{\theta_5^2}{2} - \frac{\theta_3^2}{2} \right) \right)$$

(III.2)

and with this formulation the optimum value of $\theta_5$ was found to be always less than $\theta_2$.

Referring to figure 4.23 the velocity field consists of only one rigid triangle and the extrusion load is a function of the single variable, $\theta$. From the velocity field (figure 4.24(a)) and the hodograph (figure 4.23(b)), the upper bound load is given by

$$F/k = \Pi (\cosec \theta \sec \theta + \cosec \phi \sec \phi)$$

(III.3)

where,

$$\sin \phi = \frac{H}{(H^2 + \cot^2 \theta)^{1/2}}$$

and,

$$\cos \phi = \cot \theta / (H^2 + \cot^2 \theta)^{1/2}$$

(III.4)

Substitution of equation (III.4) in equation (III.3) yields,

$$F/k = (\cosec \theta \sec \theta + \frac{H \cdot \cot \theta}{H^2 + \cot^2 \theta})^H$$

(III.5)

For least upper bound, $dF/d\theta = 0$. Differentiating equation (III.5) and equating to zero we obtain,
\[ \tan \theta = 1/ \sqrt{H} \quad (III.6) \]

From equation (III.5) and equation (III.6), the least extrusion pressure is given by,

\[ (F/\kappa H)_{\text{min}} = \frac{2(1 + H)}{\sqrt{H}} \quad (III.7) \]

Extrusion through rough square dies

The proposed velocity fields for the case of extrusion with complete sticking on the die faces are shown in figure 4.24 to figure 4.29. Referring to figure 4.24 (a) (velocity field I), the length of the velocity discontinuities are as follows:

\[ \begin{align*}
AB &= 1/\sin \theta_1 \\
AC &= \sin(\theta_1 + \theta_2)/\sin \theta_1 \sin(\theta_2 + \theta_5) \\
BC &= \sin(\theta_1 - \theta_5)/\sin \theta_1 \sin(\theta_2 + \theta_5) \\
AD &= (H - 1)/\sin \theta_3 \\
\end{align*} \]

and,

\[ CD = (\cot \theta_1 + BC \cos \theta_2 - (H - 1) \cot \theta_3)/\cos \theta_4 \]

where,

\[ \theta_4 = \tan^{-1}((H - BC \sin \theta_2)/(AC \cos \theta_5 - AD \cos \theta_3)) \quad (III.8) \]

From the corresponding hodograph (figure 4.24(b)), the magnitudes of the velocity discontinuities are given by,

\[ \begin{align*}
U_{AD} &= \sin \theta_4 / \sin(\theta_3 + \theta_4) \\
U_{CD} &= \sin \theta_3 / \sin(\theta_3 + \theta_4) \\
\end{align*} \]


\[ U_{BC} = (H - 1) \sin \theta_1 / \sin (\theta_1 + \theta_2) \]

\[ U_{AB} = (H - 1) \sin \theta_2 / \sin (\theta_1 + \theta_2) \]

and,

\[ U_{AC} = (U_{AD} \sin \theta_3 - U_{AB} \sin \theta_1) / \sin \theta_5 \]  

(III.9)

Upper bound to the extrusion load, \( F \), is calculated from the relation,

\[ F/k = A_3 \times U_{AB} + B_3 \times U_{BC} + A_4 \times U_{AC} + A_5 \times U_{AD} \]

\[ + C_5 \times U_{CD} \]  

(III.10)

Once again the extrusion load, \( F \), was expressed as function of \( \theta_1, \theta_2, \theta_3 \), and \( \theta_5 \). The constraints in the present problem were similar to those for velocity field I for the case of extrusion through smooth square dies (figure 4.20) and, they were eliminated using new variables \( y_1 \) as defined in equation (4.35).

The lengths and magnitudes of the velocity discontinuities for velocity field II (figure 4.25) were calculated by substituting \( \theta_5 = 0 \) in equations (III-8) and (III-9) and for velocity field III (figure 4.26) substituting \( \theta_5 = - \theta_5 \). The constraints in case of the above fields were also similar to those for the corresponding fields for smooth dies (velocity field II (figure 4.21) and velocity field III (figure 4.22)) and they were eliminated using the procedure discussed in the last section.

Upper bound to the extrusion load for velocity field IV (figure 4.27) is calculated in the following manner:-
Referring to the physical plane (figure 4.27(a)), the length of the velocity discontinuities are given by,

\[
\begin{align*}
AB &= \frac{1}{\sin \theta_1} \\
BC &= \frac{H}{\sin \theta_2} \\
\text{and}, \\
AC &= \frac{(H - 1)}{\sin \theta_3}
\end{align*}
\]  

(III.11)

where,

\[
\theta_3 = \tan^{-1}\left(\frac{H-1}{\cot \theta_1 - H \cot \theta_2}\right)
\]

From the hodograph (figure 4.27(b)), the magnitudes of the velocity discontinuities are,

\[
\begin{align*}
U_{BC} &= \frac{\sin \theta_3}{\sin (\theta_2 + \theta_3)} \\
U_{AC} &= \frac{\sin \theta_2}{\sin (\theta_2 + \theta_3)} \\
\text{and}, \\
U_{AB} &= \frac{H \sin \theta_3}{\sin (\theta_1 + \theta_3)}
\end{align*}
\]

(III.12)

The upper bound load, \( F \), is calculated from the equation,

\[
F/k = AB \times U_{AB} + BC \times U_{BC} + AC \times U_{AC}
\]

(III.13)

It is easy to see that \( F \) is a function of the field angles \( \theta_1 \) and \( \theta_2 \). The constraints were eliminated by defining new variables \( y_1, y_2 \) such that,

\[
\begin{align*}
\theta_1 &= \frac{\pi}{2} \left( \exp (-y_1^2 - y_2^2) \right) \\
\text{and}, \\
\theta_2 &= \frac{\pi}{2} \left( \exp (-y_2^2) \right)
\end{align*}
\]

(III.14)
Referring to figure 4.28(a), the lengths of the velocity discontinuities for velocity field $V$, are as follows:

$$OA = \frac{1}{\sin\theta_1}$$

$$AB = \frac{\sin (\theta_5 - \theta_1)}{\sin \theta_1 \sin (\theta_2 + \theta_5)}$$

$$OB = \frac{\sin (\theta_1 + \theta_2)}{\sin \theta_1 \sin (\theta_2 + \theta_5)}$$

$$BC = \frac{\sin (\theta_1 + \theta_2) \sin \theta_5}{\sin \theta_1 \sin (\theta_2 + \theta_5)}$$

$$BD = \frac{(H - OB \sin \theta_5)}{\sin \theta_4}$$

and,

$$CD = \frac{H}{\sin \theta_6}$$

where,

$$\theta_6 = \tan^{-1} \left( \frac{H}{(BC \cos \theta_5 - BD \cos \theta_4)} \right)$$

(III-19)

From the hodograph (figure 4.28(b)), the magnitudes of the velocity discontinuities are given by,

$$U_{CD} = \frac{\sin \theta_4}{\sin (\theta_6 + \theta_4)}$$

$$U_{BD} = \frac{\sin \theta_6}{\sin (\theta_6 + \theta_4)}$$

$$U_{BC} = \frac{\sin \theta_4 \sin \theta_6}{\sin \theta_3 \sin (\theta_6 + \theta_4)}$$

$$U_{OB} = (1 - U_{CD} \cos \theta_6 + U_{BC} \cos \theta_3) \sin \theta_2 / \sin (\theta_2 + \theta_5)$$

$$U_{AB} = (1 - U_{CD} \cos \theta_6 + U_{BC} \cos \theta_3) \sin \theta_5 / \sin (\theta_2 + \theta_5)$$

and,

$$U_{OA} = \frac{H \sin \theta_2}{\sin (\theta_1 + \theta_2)}$$

(III-16)

The upper bound is calculated from the equation

$$\frac{F}{k} = CD \times U_{CD} + BD \times U_{BD} + BC \times U_{BC} + OB \times U_{OB} + AB \times U_{AB} + OA \times U_{OA}$$

(III.17)
and using equation (III.15), equation (III.16) and equation (III.17), \( F \) is expressed as function of the field angles.

The constraints were eliminated by defining the field angles, such that,
\[
\begin{align*}
\theta_1 &= \pi/2 \exp(-y^2_i), \ i = 2, \ldots, 5 \\
\text{and,} \quad 
\theta_1 &= \pi/2 \exp(-y^2_1 - y^2_5)
\end{align*}
\] (III.18)

(The optimum value of \( \theta_5 \) was always greater than \( \theta_3 \)).

The more complicated configuration consisting of four rigid triangles is the velocity field VI as shown in figure 4.29. From the physical plane shown in figure 4.29(a), the length of the velocity discontinuities are given by,

\[
\begin{align*}
AB &= l / \sin \theta_1 \\
AC &= \sin(\theta_1 + \theta_2) / \sin \theta_1 \sin(\theta_2 + \theta_5) \\
BC &= \sin(\theta_1 - \theta_5) / \sin \theta_1 \sin(\theta_2 + \theta_5) \\
CE &= AC \sin(\theta_4 + \theta_5) / \cos(\theta_4 + \theta_7) \\
AE &= AC \cos(\theta_5 - \theta_7) / \cos(\theta_4 + \theta_7) \\
AD &= (H - 1) / \sin \theta_3 \\
\text{and,} \\
DE &= (H - CE \cos \theta_7 - BC \sin \theta_2) / \sin \theta_6 \\
\text{where,} \\
\theta_6 &= \tan^{-1} ((H - CE \cos \theta_7 - BC \sin \theta_2) / (AE \cos \theta_4 - AD \cos \theta_3)) \\
\text{(III-19)}
\end{align*}
\]

From the hodograph (figure 4.29(b)), the magnitudes of the velocity discontinuities are as follows:-
\[ U_{DE} = \frac{\sin \theta_3}{\sin (\theta_3 + \theta_6)} \]
\[ U_{AD} = \frac{\sin \theta_6}{\sin (\theta_3 + \theta_6)} \]
\[ U_{BC} = \frac{(H-1) \sin \theta_1}{\sin (\theta_1 + \theta_2)} \]
\[ U_{AB} = \frac{(H-1) \sin \theta_2}{\sin (\theta_1 + \theta_2)} \]
\[ U_{AE} = \frac{U_{DE} \cos(\theta_6 - \theta_7)}{\cos(\theta_4 + \theta_7)} \]
\[ U_{CE} = \frac{U_{DE} \sin(\theta_4 + \theta_6)}{\cos(\theta_4 + \theta_7)} \]

and,
\[ U_{AC} = \frac{(U_{BC} \cos \theta_2 - U_{CE} \sin \theta_7)}{\cos \theta_5} \]

The upper bound load is calculated from the relation,
\[ F/lc = AB \times U_{AB} + BC \times U_{BC} + AC \times U_{AC} + AE \times U_{AE} + CE \times U_{CE} + AD \times U_{AD} + DE \times U_{DE} \quad (III.21) \]

The constraints were eliminated by defining the variables \( y_1 \) in the following manner:-
\[ \theta_1 = \frac{\pi}{2} \exp (-y_1^2) \]
\[ \theta_2 = \frac{\pi}{2} \exp (-y_2^2) \]
\[ \theta_3 = \frac{\pi}{2} \exp (-y_3^2) \]
\[ \theta_4 = \frac{\pi}{2} \exp (-y_3^2 - y_4^2) \]
\[ \theta_5 = \frac{\pi}{2} \exp (-y_5^2) \]

and,
\[ \theta_7 = \frac{\pi}{2} \left(1 - \exp (-y_3^2 - y_4^2)\right) \exp(-y_6^2) \]
Extrusion with slipping friction on the die faces:

When slipping friction is assumed ($\gamma = mk$) at the interface between the die and the extruding billet, there will be slipping and sticking regions in contact with the die as explained in 4.2. The velocity fields which satisfy these conditions are shown from figure 4.30 to figure 4.34.

The upper bound load for velocity field I (figure 4.30) is calculated in the following manner:

From the physical plane shown in figure 4.30(a), the lengths of the velocity discontinuities are as follows.

\[
AB = \frac{1}{\sin \theta_1}
\]

\[
BC = \frac{\sin (\theta_1 - \theta_5)}{\sin \theta_1} \frac{\sin (\theta_2 + \theta_5)}{\sin \theta_1}
\]

\[
AC = \frac{\sin (\theta_1 + \theta_2)}{\sin \theta_1} \frac{\sin (\theta_2 + \theta_5)}{\sin \theta_1}
\]

\[
CD = \frac{AC \cos \theta_5}{\cos \theta_4}
\]

\[
AD = CD \sin \theta_4 - AC \sin \theta_5
\]

\[
ED = \frac{(H - AD - 1)}{\sin \theta_4}
\]

and,

\[
EC = \frac{(H - BC \sin \theta_2)}{\sin \theta_6}
\]

where,

\[
\theta_6 = \tan^{-1}\left(\frac{H - BC \sin \theta_2}{AC \cos \theta_5 - ED \cos \theta_3}\right) \quad (III.23)
\]

From the hodograph shown figure 4.30(b) the magnitudes of the velocity discontinuities are given by,

\[
U_{EC} = \frac{\sin \theta_2}{\sin (\theta_3 + \theta_6)}
\]

\[
U_{ED} = \frac{\sin \theta_6}{\sin (\theta_3 + \theta_6)}
\]
\[ U_{CD} = \sin\theta_6 \cos\theta_5 / \sin (\theta_3 + \theta_6) \cos \theta_4 \]
\[ U_{BC} = (H-1) \sin\theta_1 / \sin (\theta_1 + \theta_2) \]
\[ U_{AB} = (H-1) \sin\theta_2 / \sin (\theta_1 + \theta_2) \]
\[ U_{AC} = (1 + U_{BC} \cos\theta_2) / \cos\theta_5 \]
and,
\[ U_{AD} = U_{ED} \sin\theta_3 + U_{CD} \sin\theta_4 \]

The upper bound load is calculated from the equation,

\[ F/k = AB \cdot U_{AB} + BC \cdot U_{BC} + AC \cdot U_{AC} + CD \cdot U_{CD} \]
\[ + ED \cdot U_{ED} + BC \cdot U_{EC} + m_1 \cdot AD \cdot U_{AD} \quad \text{(III.25)} \]

where, \( m_1 \) is the assumed friction factor for the billet-die interface. Using equation (III.23) (III.24) and (III.25), \( F \) was expressed as a function of the field angles. The constraints in the present problem were similar to those for velocity field I (figure 4.20) for the case of smooth dies and were eliminated by defining the field angles as in equation (4.35). (The constraint, \( \theta_5 \leq \theta_4 \) was found to be always satisfied).

The lengths and the magnitudes of the velocity discontinuities for velocity field II (figure 4.31) were calculated by substituting \( \theta_5 = 0 \) in equation (III.23) and (III.24) and for velocity field III (figure 4.32) by substituting \( \theta_5 = -\theta_5 \), in the above equations. The nature of constraints for the above two velocity fields were similar to those for the corresponding fields for the case of extrusion through smooth dies.
(velocity field II (figure 4.21) and velocity field III (figure 4.22)) and were eliminated using the procedure discussed in the previous sections.

Velocity field IV as shown in figure 4.33, consists of three rigid triangles and the upper bound load for the present field is calculated in the following manner:

From the physical plane shown in figure 4.33(a), the lengths of the velocity discontinuities are given by,

\[
\begin{align*}
AB &= l/sin\theta_1 \\
BC &= cot\theta_1/cos\theta_2 \\
CD &= \frac{(H - cot\theta_1 tan\theta_2)}{sin\theta_4} \\
AC &= cot\theta_1 tan\theta_2 - 1 \\
BD &= H/sin\theta_3
\end{align*}
\]

where,

\[
\theta_4 = tan^{-1}\{(H - cot\theta_1 tan\theta_2)/(cot\theta_1 - Hcot\theta_3)\}
\]

From the hodograph shown in figure 4.33(b) the magnitudes of the velocity discontinuities are,

\[
\begin{align*}
U_{BD} &= sin\theta_4/sin(\theta_3 + \theta_4) \\
U_{CD} &= sin\theta_3/sin(\theta_3 + \theta_4) \\
U_{BC} &= sin\theta_3 cos\theta_4/sin(\theta_3 + \theta_4) cos\theta_2 \\
U_{AC} &= U_{CD} sin\theta_4 + U_{BC} sin\theta_2 \\
U_{AB} &= H/cos\theta_1
\end{align*}
\]

(III.26)
Upper bound to the extrusion load is obtained from the equation,

\[ F/k = AB \times U_{AB} + BC \times U_{BC} + CD \times U_{CD} + BD \times U_{BD} \]
\[ + m_1 \times AC \times U_{AC} \]  \hspace{1cm} (III.28)

It may be seen that the problem is transformed to an unconstrained optimisation problem by defining the field angles in the following manner:

\[
\begin{align*}
\theta_1 &= \pi/2 \left( \exp \left( -y_1^2 - y_2^2 - y_3^2 \right) \right) \\
\theta_2 &= \pi/2 \left( \exp \left( -y_2^2 - y_3^2 \right) \right) \\
\theta_3 &= \pi/2 \left( \exp \left( -y_3^2 \right) \right)
\end{align*}
\]  \hspace{1cm} (III.29)

The upper bound bound load for velocity field V (figure 4.34) was calculated in the following manner:

Referring to figure 4.34(a), the lengths of the velocity discontinuities are given by,

\[
\begin{align*}
AB &= 1/\sin \theta_1 \\
AC &= \sin(\theta_1 + \theta_2)/\sin \theta_1 \sin (\theta_2 - \theta_5) \\
BC &= \sin (\theta_1 + \theta_5)/\sin \theta_1 \sin (\theta_2 - \theta_5) \\
AD &= AC \sin (\theta_4 + \theta_5)/\cos \theta_4 \\
CD &= AC \cos \theta_5/\cos \theta_4 \\
DF &= CD \cos (\theta_4 - \theta_7)/\cos (\theta_3 - \theta_7) \\
CF &= CD \sin (\theta_4 - \theta_3)/\cos (\theta_3 - \theta_7) \\
DE &= (H - AD)/\sin \theta_6 \\
\end{align*}
\]  \hspace{1cm} (III.30)

and,

\[
EF = (H - CF \cos \theta_7 - BC \sin \theta_2)/\sin \theta_8
\]

where,

\[
\theta_8 = \tan^{-1}((H-CF \cos \theta_7 - BC \sin \theta_2)/(DF \cos \theta_3 - DE \cos \theta_6))
\]
From figure 4.34(b), the magnitudes of the velocity discontinuities are,

\[ U_{\text{EF}} = \frac{\sin \theta_6}{\sin (\theta_6 + \theta_8)} \]
\[ U_{\text{DE}} = \frac{\sin \theta_8}{\sin (\theta_6 + \theta_8)} \]
\[ U_{\text{BC}} = (H - 1) \frac{\sin \theta_1}{\sin (\theta_1 + \theta_2)} \]
\[ U_{\text{AB}} = (H - 1) \frac{\sin \theta_2}{\sin (\theta_1 + \theta_2)} \]
\[ U_{\text{DF}} = \frac{U_{\text{EF}} \cos (\theta_8 - \theta_7) \cos (\theta_3 - \theta_7)}{\cos \theta_4} \]
\[ U_{\text{CF}} = \frac{U_{\text{EF}} \sin (\theta_8 - \theta_3) \cos (\theta_3 - \theta_7)}{\cos \theta_3} \]
\[ U_{\text{CD}} = \frac{1 + U_{\text{CF}} \sin \theta_7}{\cos \theta_4} \]
\[ U_{\text{AC}} = \frac{1 + U_{\text{BC}} \cos \theta_2}{\cos \theta_3} \]

and,
\[ U_{\text{AD}} = U_{\text{CF}} \cos \theta_7 + U_{\text{CD}} \sin \theta_4 \]

Upper bound to the extrusion load is calculated from the relation,

\[ \frac{F}{k} = AB \times U_{\text{AB}} + AC \times U_{\text{AC}} + BC \times U_{\text{BC}} + CD \times U_{\text{CD}} + DF \times U_{\text{DF}} + CF \times U_{\text{CF}} + DE \times U_{\text{DE}} + EF \times U_{\text{EF}} + m_1 \times AD \times U_{\text{AD}} \] (III.32)

The constraints were eliminated by defining the field angles in the following manner:

\[ \theta_1 = \frac{\pi}{2} \exp (-y_1^2) \]
\[ \theta_2 = \frac{\pi}{2} \exp (-y_2^2) \]
\[ \theta_3 = \frac{\pi}{2} \exp (-y_3^2 - y_4^2) \]
\[ \theta_4 = \frac{\pi}{2} \exp (-y_4^2) \]
\[ \theta_5 = \frac{\pi}{2} \exp (-y_5^2 - y_6^2) \]
\[ \theta_6 = \frac{\pi}{2} \exp (-y_6^2) \]

and,
\[ \theta_7 = \left( \frac{\pi}{2} - \theta_2 \right) \exp (-y_7^2) \]