CHAPTER V

A CLASS OF SLIPLINE FIELD SOLUTIONS FOR EXTRUSION THROUGH
WEDGE SHAPED DIES WITH SLIPPING FRICTION

5.1. INTRODUCTION

In the slipline field solutions for drawing/extrusion through smooth wedge-shaped dies it is observed that the field with straight lines and circular arcs which applies for lower reductions, fails for certain higher reductions when the velocity discontinuity terminates on the exit slipline (Hill and Tupper\(^{(70)}\)). Solutions for these reductions involving curved exit sliplines were proposed by Green\(^{(24)}\) from an analogy with the problem of plane strain compression between smooth parallel plates where, simple straightline fields occur for integral values of width to thickness ratios and, for intermediate values, fields with curved exit sliplines apply. These indirect solutions for the case of drawing through frictionless dies have been analysed by Dewhurst and Collins\(^{(29)}\) and more recently by Venter, Hewitt and Johnson\(^{(33)}\), who have computed the mean drawing stress for a number of die geometries and also have established the oscillatory behaviour of the redundant work factor, similar to that observed in the case of plane strain compression between smooth parallel plates\(^{(24)}\).
For short dies, the slipline fields can be constructed starting from straight lines and circular arcs and these fields for the case of smooth dies have been analysed by Hill and Tupper, Venter, Hewitt and Johnson, and by Chenot, Felgeres, Lavarenne and Salencon, who have obtained results for the more complicated problem of axisymmetric extrusion. The effect of friction on drawing/extrusion pressure for these geometries has been studied by Hill and Green, Bishop and Johnson. Upper bound to the mean pressures for these cases have also been computed by Green, Westwood and Wallace.

In this chapter the above solutions have been extended to the case of extrusion with slipping friction at the die metal interface ($\gamma = mk$). All solutions are analysed over the complete range of reductions to which they apply and, mean extrusion and die pressures are computed for a number of die geometries and for different frictional conditions at the interfaces. The results are also compared with those obtained from other approximate theories.

5.2. SLIPLINE FIELDS

The four slipline field configurations associated with extrusion of metals through wedge-shaped dies with slipping friction at the interfaces ($\gamma = mk$) and designated as type I, type II, type III and type IV solutions are illustrated in
figure 5.1, figure 5.3, figure 5.6 and figure 5.7 respectively. For a given die angle \( \alpha \) and for a given value of the friction factor \( m \) each of these fields extend over a definite range of reduction.

Type I solution (figure 5.1) applies when the fractional reduction \( r \) is less than \( 2\sin\alpha/(\sqrt{1+m} + 2\sin\alpha) \). The maximum reduction corresponds to the limiting field of figure 5.2 for which \( \Theta = 0 \). For reductions greater than the above value the slipline field of figure 5.2 can be continued in the manner demonstrated in figure 5.3 (type II solution). The limiting configuration of this solution is attained when the angular range of the sliplines equals \( (\alpha + \gamma) \) where, \( m = \cos2\lambda = \sin2\gamma \) (figure 5.4). For reductions greater than this limiting case the above solution cannot be continued as demonstrated in figure 5.5 since, the velocity discontinuity propagated along the die entry would terminate on the exit slipline \( OA \), instead of at the singular point \( O \), which, would be incompatible with the rigid motion of the extruded product. Nevertheless, for these reductions the slipline field solution of figure 5.5 would provide a lower bound to the mean extrusion pressure as explained in chapter III and chapter IV (figure 3.9 and figure 4.5(a)).

Beyond the limiting configuration of figure 5.4, a different class of solutions apply as indicated by Green\(^{24}\). These solutions involving curved exit sliplines are the type III
FIG. 5.1 (a). SLIPLINE FIELD TYPE I

FIG. 5.1 (b). HODOGRAPH
FIG. 5.2. LIMITING CONFIGURATION FOR TYPE I FIELD

FIG. 5.3 (a). SLIPLINE FIELD TYPE II.
FIG. 5.5. IMPROPER CONTINUATION OF TYPE III FIELD. VELOCITY DISCONTINUITY TERMINATES ON THE EXIT SLIPLINE.
and type IV solutions as shown in figure 5.6 and figure 5.7 respectively. The solution of Fig. 5.6 applies when the reduction is greater than that corresponding to the limiting field of figure 5.4 while, the solution of figure 5.7 applies when the reduction is less than that for the slipline field shown in figure 5.8 which, is again of direct type. The characteristic feature of these solutions is that they involve velocity discontinuities along certain slipline curves and also tramline regions, where one family of sliplines are straight. For a particular reduction depending on the die angle \( \alpha \) and the friction factor \( \mu \), however, the velocity discontinuities and the tramline regions vanish simultaneously so that both solutions merge into a single eigen field shown in figure 5.9. Solutions for still greater reductions can be obtained by continuing construction beyond the limiting field of figure 5.8. These solutions, however, are more of theoretical interest since, dies of such geometries are rarely used for extrusion or drawing of metals.

5.3. METHOD OF ANALYSIS

In the present study, the above solutions were analysed with the help of the matrix method, for different frictional conditions on the die faces. For the direct solutions of figure 5.1 and figure 5.3 the radius of curvature of the different slipline curves could be readily calculated starting from the circular arc AC. For the indirect solutions of figure 5.6 and figure 5.7, the matrix inversion procedure was followed to
FIG. 5.6 (a) SLIPLINE FIELD TYPE III

FIG. 5.6 (b) HODOGRAPH
FIG. 5.8. LIMITING CONFIGURATION FOR TYPE IV SOLUTION.

FIG. 5.9 (a) TRANSITION EIGEN FIELD BETWEEN TYPE III AND TYPE IV SOLUTIONS.
FIG. 5-10. SHEET BULGING AT LOW REDUCTIONS.
determine the base slipline. In the following sections the
governing equations for these solutions are derived and equa-
tions also presented for evaluation of coordinates and load
variables.

5.3.1. Slipline field type I

Referring to figure 5.1(a), CA and CD are circular arcs of
radius $\rho_1$ and $\rho_2$ respectively. Hence, with the sign convention
used for the slipline curvature these slipline curves can be
represented by,

$$
\begin{align*}
CA & = \rho_1 c \\
CD & = -\rho_2 c
\end{align*}
$$

(5.1)

where, $\rho_2 = \rho_1 \tan \lambda$ and $c$, as before, represents a circle of
unit radius.

From the slipline net CABD, slipline curves, BA and DB
can be calculated by constructing singular fields about GA and
CD. Thus,

$$
\begin{align*}
BA & = (\rho_1 Q_\theta - \rho_2 P_\theta) c \\
DB & = (\rho_1 P^*_\theta - \rho_2 Q^*_\theta) c
\end{align*}
$$

(5.2)

The equations for the evaluation of force and geometrical
parameters for this field are given as,

Thickness of the extruded sheet $h = X_{BA} \sin \pi/4 + Y_{BA} \cos \pi/4
+ \rho_1 \sin (\pi/4 + \theta)$

Thickness of the billet $H = X_{DB} \cos(\pi/4-\xi) - Y_{DB} \sin(\pi/4-\xi)$

+ $\rho_2 \cos(\pi/4-\xi)$
Extrusion force \( \frac{F}{k} = F_{DB} \sin(\pi/4 - \Psi) + F_{DB} \cos(\pi/4 - \Psi) + \varphi_2 (P_D \cos(\pi/4 - \Psi) - \sin(\pi/4 - \Psi)) \),

where, the field angle \( \Psi \) is given by relation, \( \Psi = \pi/4 + \alpha + \theta - \lambda \) and, \( P_D \) is the hydrostatic pressure at \( D \).

### 5.3.2. Slipline field type II

Referring to figure 5.3(a), AC is a circular arc of radius \( \varphi_1 \). Hence,

\( AC = - \varphi_1 \) c

The radius of curvature of other slipline curves are calculated with the help of the following equations

\[
\begin{aligned}
FC &= - \varphi_1 S_{\varphi} T_{\varphi}^{-1} c \\
FE &= - \varphi_1 R_{\varphi} T_{\varphi} T_{\varphi}^{-1} c \\
GD &= - (\varphi_1 S_{\varphi} T_{\varphi}^{-1} + \varphi_2) c \\
DG &= - R_{\varphi} (\varphi_1 S_{\varphi} T_{\varphi}^{-1} + \varphi_2) c \\
HG &= - \gamma \gamma (\varphi_1 S_{\varphi} T_{\varphi}^{-1} + \varphi_2) c \\
\end{aligned}
\]

where, \( \gamma = \gamma + \gamma \)

The equations yielding the force and geometrical parameters are,

Thickness of the extruded sheet \( h = \varphi_1 \cos \pi/4 \)

Thickness of the billet

\[
\begin{aligned}
H &= X_{HG} \cos(\pi/4 - \Psi) \\
&- Y_{HG} \sin(\pi/4 - \Psi) \\
&+ \varphi_2 \cos(\pi/4 - \Psi) \\
&+ X_{FE} \cos(\pi/4 - \Psi) \\
&- Y_{FE} \sin(\pi/4 - \Psi) \\
\end{aligned}
\]
Extrusion force \( F/k = \frac{FX_{HG}}{k} \sin(\pi/4 - \Psi) + \frac{FY_{HG}}{k} \cos(\pi/4 - \Psi) \) 
\( + \frac{FX_{P}}{k} \cos(\pi/4 - \Psi) + \frac{FY_{P}}{k} \sin(\pi/4 - \Psi) \) 
\( + \frac{FX_{P}}{k} \sin(F/\pi - \Psi) \)

where, \( P_{P} \) is the hydrostatic pressure at \( F \).

5.3.3. Slipline field type III

We designate the base slipline \( MK \) by \( \sigma_{2} \) and the slipline curve \( OB \) by \( \sigma_{1} \) (figure 5.6). Therefore,

\[
\begin{align*}
AB &= Q_{\Psi} \sigma_{1} \\
CD &= Q_{\Psi} \sigma_{1} - \sigma_{1} c \\
ED &= T_{\Psi}^{-1}(Q_{\Psi} \sigma_{1} - \sigma_{1} c)
\end{align*}
\]

(5.4)

Also,

\[
\begin{align*}
LK &= Q_{\Psi} \sigma_{2} \\
JI &= Q_{\Psi} \sigma_{2} + \sigma_{2} c \\
EI &= T_{\Psi}^{-1}(Q_{\Psi} \sigma_{2} + \sigma_{2} c)
\end{align*}
\]

(5.5)

where, \( \Psi = \pi/4 + \alpha - \lambda + \Theta \)

Moreover, \( FB, OB \) and \( HK, MK \) meet the prescribed rough boundary (die face). Therefore,

\[
\begin{align*}
FB &= G_{\Psi}^{-1} \sigma_{1} \\
HK &= G_{\Psi}^{-1} \sigma_{2}
\end{align*}
\]
where, \( \mu = \pi/2 - \lambda \) and, \( G \) is the rough boundary operator. Crossing over the tramline region, \( GD \) and \( GI \) are given by,

\[
\begin{align*}
GD &= G^{-1} \lambda \sigma_1 + \sigma_1 c \\
GI &= G^{-1} \lambda \sigma_2 - \sigma_2 c
\end{align*}
\]  

(5.6)

From \( \text{EIGD} \), slipline curves \( EI, ED \) are calculated from \( GD, GI \) with the help of the following equations

\[
\begin{align*}
EI &= P_{\psi \psi} GD - Q_{\psi \theta} GI \\
ED &= P_{\psi \theta} GI - Q_{\theta \psi} GD
\end{align*}
\]  

(5.7)  

(5.8)

Substituting for \( GD, GI \) from equation (5.6), eliminating \( ED \), \( EI \) with the help of equation (5.4) and equation (5.5) respectively, equation (5.7) and equation (5.8) reduce after some simplification to the following two equations:

\[
\begin{align*}
B \sigma_1 - A \sigma_2 + \sigma_1 P_{\psi \psi} c + \sigma_2 (Q_{\theta \psi} - T^{-1}) c &= 0 \\
C \sigma_1 - D \sigma_2 + \sigma_1 (Q_{\theta \psi} - T^{-1}) c + \sigma_2 P_{\psi \theta} c &= 0
\end{align*}
\]  

(5.9)  

(5.10)

where,

\[
\begin{align*}
A &= Q_{\psi \theta} G^{-1} + T^{-1} Q_{\psi \theta} \\
B &= P_{\psi \psi} G^{-1} \\
C &= Q_{\theta \psi} G^{-1} + T^{-1} Q_{\theta \psi} \\
D &= P_{\psi \theta} G^{-1}
\end{align*}
\]  

(5.11)
Equations (5.9) and (5.10) are the two equations for the two unknown column vectors $\sigma_1$ and $\sigma_2$. Solving these equations for $\sigma_2$ we obtain,

\[
(C G_{\theta} P^*_{\phi} R_{\theta} A - D) \sigma_2 = \mathcal{P}_1 (C G_{\theta} - (Q_{\phi\phi} - T_{\phi}^{-1})) c \\
+ \mathcal{P}_2 (C G_{\theta} P^*_{\phi} R_{\phi} (Q_{\phi\phi} - T_{\phi})) - P_{\phi\phi} c \quad (5.12)
\]

Thus the base slipline $MK$ is obtained by inverting the operator $(C G_{\theta} P^*_{\phi} R_{\theta} A - D)$. The equations for the force and the geometrical parameters for this slipline field are as follows:

Thickness of the extruded sheet $h = X_{0A} \cos(\pi/4 - \theta) + Y_{0A} \sin(\pi/4 - \theta) + \mathcal{P}_1 \sin \pi/4$

Thickness of the billet $H = X_{ML} \cos(\pi/4 - \theta) - Y_{ML} \sin(\pi/4 - \theta) + \mathcal{P}_2 (P_J \cos \pi/4 - \sin \pi/4)$

Extrusion load $F/k = F_{XML} \cos(\pi/4 - \theta) + F_{YML} \sin(\pi/4 - \theta) + \mathcal{P}_2 (P_J \cos \pi/4 - \sin \pi/4)$

where, $P_J$ is the hydrostatic pressure at the point $J$.

For a frictionless die, $\lambda = \mu = \pi/4$, equation (5.12), then, reduces to,

\[
(C T_{\phi}^{-1} P^*_{\phi} R_{\phi} A - D) \sigma_2 = \mathcal{P}_1 (C T_{\phi}^{-1} - (Q_{\phi\phi} - T_{\phi}^{-1})) c \\
+ C T_{\phi}^{-1} P^*_{\phi} R_{\phi} (Q_{\phi\phi} - T_{\phi}) - P_{\phi\phi} c \quad (5.13)
\]
This is the solution discussed in detail by Dewhurst and Collins. The matrix operators, A, B, C and D for this field are obtained from equation (5.11) using the relations, $G_{\lambda\psi}^{-1} = T_{\psi}$ and $G_{\Theta}^{-1} = T_{\Theta}.$

5.3.4. Slipline field type IV

Referring to figure 5.7(a), the base slipline LK is designated by $\sigma_2$ and the slipline curve BD by $\sigma_1$. BA and LN are circular arcs represented by,

$$BA = \sigma_1 c$$

and,

$$LN = -\sigma_3 c$$

where,

$$\sigma_3 = \sigma_1 \tan^2 \lambda$$

From the slipline net BACD, CD is calculated from BA and BD. Thus,

$$CD = \sigma_1 P_{\psi\Theta} c + Q_{\Theta\psi} \sigma_1$$

Hence,

$$ED = T_{\psi}^{-1}\left(\sigma_1 P_{\psi\Theta} c + Q_{\Theta\psi} \sigma_1\right)$$

and,

$$GR = T_{\psi}^{-1}\left(\sigma_1 P_{\psi\Theta} c + Q_{\Theta\psi} \sigma_1\right) - \sigma_3 c$$

Similarly, from the slipline net LKMN, MK is calculated from LK and LN. Thus,

$$MK = Q_{\psi\Theta} \sigma_2 - \sigma_3 P_{\psi\Theta} c$$

Therefore,
\[ JK = T^{-1}_\varTheta \left( Q_{\Psi\Theta} \sigma_2 - \sigma_2^* \ P_{\Theta\Psi} \ c \right) \]

and,
\[ GH = T^{-1}_\varTheta \left( Q_{\Psi\Theta} \ \sigma_2 - \sigma_2^* \ P_{\Theta\Psi} \ c \right) + \sigma_2^* \ c \]

Also,
\[ OR = \sigma_1 + \sigma_2^* \ c \]
\[ IH = \sigma_2 - \sigma_2^* \ c \]

Hence,
\[ FR = G^{-1}_\varTheta \left( \sigma_1 + \sigma_2^* \ c \right) \]
\[ PH = G^{-1}_\Lambda \Psi \left( \sigma_2 - \sigma_2^* \ c \right) \]

Also,
\[ GR = P_{\Psi\Theta} \ PH - Q_{\Theta\Psi} \ FR \]
\[ GH = P_{\Theta\Psi} \ FR - Q_{\Psi\Theta} \ PH \]

Using equations (5.17), (5.16), (5.15) and (5.14) two homogeneous algebraic equations are obtained for the two unknown slipline vectors \( \sigma_1 \) and \( \sigma_2 \). These are written as

\[
\begin{align*}
C \sigma_1 - D \sigma_2 + \sigma_1^* T^{-1}_\Psi \ P_{\Psi\Theta} \ c + \sigma_2^* \left( D + Q_{\Theta\Psi} G^{-1}_\Lambda \Psi \ -I \right) c &= 0 \\
B \sigma_1 - A \sigma_2 + \sigma_2^* \left( B + Q_{\Psi\Theta} G^{-1}_\Lambda \Psi \ -I \right) c + \sigma_2^* T^{-1}_\Theta \ P_{\Theta\Psi} \ c &= 0
\end{align*}
\]

(5.18)

where, \( I \) is the unit matrix and \( A, B, C, D \) are the matrix operators given by equation (5.11).
Solving the two algebraic equations in (5.18) for the base slipline \( \sigma_2 \), we obtain,

\[
(C_{\theta \phi} P_{-\psi}^* R_{\phi} A \Delta D) \sigma_2 = \mathcal{F}_2 (C_{\theta \phi} P_{-\psi}^* R_{\phi} (B + Q_{\theta \phi} G_{\lambda \psi}^{-1} - I)) c
\]

\[
+ \mathcal{F}_3 C_{\theta \phi} P_{-\psi}^* R_{\phi} T_{\theta}^{-1} P_{\theta \psi} c
\]

\[-\mathcal{F}_1 T_{\psi}^{-1} P_{\theta \phi} c \quad (5.19)
\]

As in case of type III solution, the base slipline in the present case is also obtained by inverting the matrix operator \((C_{\theta \phi} P_{-\psi}^* R_{\phi} A \Delta D)\). The equations for the calculation of the force and geometrical parameters for this field are as follows:

Thickness of the extruded sheet, \( h = (X_{AC} + \mathcal{F}_1) \cos(\pi/4 - \phi) + Y_{AC} \sin(\pi/4 - \phi) \)

Thickness of the billet, \( H = (X_{NM} + \mathcal{F}_3) \cos(\pi/4 - \psi) - Y_{NM} \sin(\pi/4 - \psi) \)

Extrusion force, \( F/k = F X_{NM} \sin(\pi/4 - \psi) + F Y_{NM} \cos(\pi/4 - \psi) \)

\[-\mathcal{F}_1 (P_N \cos(\pi/4 - \psi) - \sin(\pi/4 - \psi)) \]

where, \( P_N \) is the hydrostatic pressure at N.

For a frictionless die, equation (5.19) reduces to

\[
(C T_{\theta}^{-1} P_{-\psi}^* R_{\phi} A \Delta D) \sigma_2 = \mathcal{F}_1 (C T_{\theta}^{-1} P_{-\psi}^* R_{\phi} (B + Q_{\theta \psi} T_{\theta})
\]

\[+ T_{\theta}^{-1} P_{\theta \psi} - I) - (D + Q_{\theta \psi} T_{\theta})
\]

\[+ T_{\psi}^{-1} P_{\theta \phi} - I)) c \quad (5.20)
\]
This is the solution discussed in detail by Venter, Hewitt and Johnson\(^{(33)}\). Obviously, the matrix operators, \(A\), \(B\), \(C\) and \(D\) in equation (5.20) are the same as in equation (5.13).

5.3.5. Equations for eigen fields

It may be seen that the matrix on the left hand side of both equation (5.12) and equation (5.19) is identical and that when \(\theta_1\) tends to zero, these equations reduce to a single homogeneous equation, given as,

\[
(C_{\theta \psi} P^*_\gamma R^*_\theta A - D) \sigma_2 = 0 \tag{5.21}
\]

which is the governing equation for the transition eigen field shown in figure 5.9(a) of the two types of solution. The vector \(\sigma_2\) is, hence, the eigen vector of the matrix \(D^{-1} C_{\theta \psi} P^*_\gamma R^*_\theta A\). For a smooth die, \(\lambda = \mu = \pi/4\), equation (5.21) reduces to

\[
(C T^{-1}_\theta P^*_\gamma R^*_\theta A - D) \sigma_2 = 0 \tag{5.22}
\]

This is the governing equation for the transition field between the two solutions due to Green\(^{(24)}\).

5.4. COMPUTATION

The procedure followed in the present case was similar to that described in Chapter III and Chapter IV in connection with the slipline field solutions for compression and extrusion respectively. For a given set of field angles, the base slipline
for type III solution was determined from equation (5.12) and for type IV solution from equation (5.19) by inversion of the matrix operator \( (C G \rho \Phi^* P \mathcal{F}^* r \mathcal{G} A - D) \). The radius of curvature of other slipline curves and the geometrical details of the fields were then calculated with the help of standard subroutines. In both cases, the hydrostatic pressure at point C (figure 5.6(a) and figure 5.7(a)) was obtained from the condition that the extruded product is stress free. The hydrostatic pressures at the origin of the bounding sliplines ML (type III solution) and NM (type IV solution) were then evaluated using Hencky's equation. The total traction on the bounding slipline was then calculated from which, the mean extrusion and die pressures were determined for the given geometry.

In this manner, results from the above indirect solutions were computed for various values of \( m \) between 0.1 and 0.9 and, for semi-wedge angle 'a' of the dies between five and sixty degrees. In view of small angular range of slipline curves, 6 x 6 truncated matrices were used for calculation. All programmes were run on an I.B.M. 1130 computer and the time taken for each calculation was approximately five minutes.

The geometry of each field is specified by the field angles \( \theta, \lambda \), semi-wedge angle 'a' and the length \( \varphi \) of the exit slipline \( (\varphi = \pi/4 + a + \theta - \lambda) \). \( \varphi \) being a scale factor was set equal to unity while, the value of \( \lambda \) was decided by
the friction factor \( m \) between the die metal interface 
\( (m = \cos 2\lambda) \). The values of \( \lambda \) corresponding to different 
values of \( m \) were same as given in table 3.1. The range of 
possible values of \( \Psi \) corresponds to the range of reductions 
for which the field is valid. For initial set of calculations, 
\( \lambda \) was taken equal to \( \pi/4 \) and the results were checked against 
those of Venter, Hewitt and Johnson\(^{33} \) for the frictionless 
case.

For the direct solutions (type I and type II), the 
radius of curvature of the various slipline curves were cal-
culated from the circular arc CA using standard matrix opera-
tors and superposition principle. The force and the geometri-
cal details of the fields were then evaluated with the help 
of the equations presented in 5.3.1. and 5.3.2.

5.5. GEOMETRICAL AND BULGE LIMITS

As the value of \( \Theta \) (and \( \Psi \)) increases, the reduction ratio 
as calculated from type I solution decreases. Since a slipline 
cannot meet a frictionless or a free surface at an angle less 
than \( \pi/4 \), the maximum permissible value of \( \Theta \) for a smooth 
container equals

\[
\Theta = \pi/4 + \lambda - \phi
\]

(5.23)

and the reduction corresponding to the above value of \( \Theta \) is 
the geometrical limit as shown in figure 5.12 to figure 5.19
From geometrical considerations, the reduction from type I solution cannot be less than the above value.

As reduction decreases, the pressure on the die increases. When the die pressure exceeds a certain critical value, the die simply acts as an indentor and the deformation mode changes to that shown in figure 5.10. The limiting reduction where, the above transition takes place, is the bulge limit. For reduction less than the bulge limit, the deformation is only local and the plastic region spreads round the edge of the die to the surface of the extruded sheet. For a smooth die, the critical die pressure, \( q \), at bulge limit is given by (33),

\[
q = 2k \left( 1 + \frac{\pi}{2} + \alpha \right) \quad (5.24)
\]

and for a rough die, by the relation,

\[
q = k \left( 1 + \sin 2\lambda + \frac{3\pi}{2} + 2\alpha - 2\lambda \right) \quad (5.25)
\]

The bulge limit was found to exceed the geometrical limit only for values of \( \alpha \) equal to five and ten degrees and for values of \( m \) equal to 0 and 0.1. For all other values of \( \alpha \) and \( m \) geometrical limit exceeded the bulge limit.

It may also be seen from equation (5.23) that for dies with semi-wedge angle \( \alpha \) greater than \( \pi/4 \), \( \Theta \) becomes negative for values of \( \lambda \) less than \( (\alpha - \pi/4) \). In such cases the above solutions fail to apply.
5.6. RESULTS AND DISCUSSION

For any given value of $A$, as $\Theta$ (and $\Psi$) increases, the reduction

$$(1 - h/H)$$

increases from its limiting value corresponding to the slipline field of figure 5.4 for type III solution, but decreases from its limiting value corresponding to the slipline field of figure 5.8 for type IV solution (refer figure 4.14). For both solutions, $\varphi_1$ decreases for fixed thickness, $h$, of the extruded sheet as $\Theta$ increases and at a critical angle of $\Theta (= \Theta_E)$, $\varphi_1$ eventually vanishes so that both solutions merge into the single eigen solution shown in figure 5.9. For any given wedge angle, $\alpha$, and the friction factor, $m$, $\Theta_E$ was found by extrapolation as discussed in Chapter IV (refer figure 4.15). The variation of $\Theta_E$, with $m$, is shown in figure 5.11 for different values of wedge angle $\alpha$. Each $\Theta_E$ is found to decrease from a maximum corresponding to the smooth dies ($m = 0$) to zero for perfectly rough dies ($m = 1$) where, the range of validity of these indirect solutions vanishes and the direct solution of figure 5.3 (type II) is valid for all reductions $\varphi$. Similar variation of $\Theta_E$, with $m$, was also observed earlier in case of square dies (figure 4.16). For the same value of $m$, however, the transition eigen field for a die with greater wedge angle occurs at a lower value of $\Theta$ than that for a die with smaller wedge angle.
Fig. 5.11. Graph showing variation of the critical angle for the eigen field with friction factor for various die angles.
The variation of mean extrusion and die pressures with reduction is shown in figure 5.12—figure 5.19 for semi-angle, \( \alpha \) between five and sixty degrees and for friction factor, \( m \), between 0 and 0.9. In all cases, the reduction corresponding to the geometrical limit has been taken as the lower limit and the bulge limit, which slightly exceeds the geometrical limit for small wedge angles (33) has been omitted for clarity. It may be seen from the above figures that the extrusion pressure increases due to friction at the billet-die interface and the effect is more evident at higher reductions. For any value of \( m \), however, while the extrusion pressure increases monotonically with reduction, the die pressure first decreases from its limiting value corresponding to the geometrical limit, passes through a minimum corresponding to the limiting field of figure 5.2 and, then increases through type II solution and, in a quasi-oscillatory manner through type III and type IV solutions similar to that observed for the slipline field solutions for compression discussed earlier in Chapter III. Also, the lower bounds from the slipline field solution of figure 5.5 are found to be very close to the exact values except for small die angles and for low values of \( m \).

The effect of die angle \( \alpha \) on mean extrusion pressure is shown in figure 5.20 to figure 5.23 for friction factor \( m \) between 0 and 0.9 and for fractional reductions between ten and eighty percent. It may be seen from these figures that for
Fig. 5.12. Graph showing variation of mean extrusion and die pressures with percent reduction for different friction factors on die face, semi-wedge angle $\alpha = 5$ degrees.
Fig. 5.13. Graph showing variation of mean extrusion and die pressure with reduction. Semi-wedge angle of the die $\alpha = 10$ degrees.
Fig. 5.14. Graph showing variation of mean extrusion and die pressures with reduction.

**Semi-wedge angle** \( \alpha = 15 \text{ degrees} \)
Fig. 5.15. Graph showing variation of extrusion and die pressures with reduction for different friction factors on die face semi-wedge angle $\alpha = 20$ degrees.
Fig 5.16. Graph showing variation of extrusion and die pressures with reduction percent for different friction factors on die face. Semi-wedge angle $\alpha = 25$ degrees.
Fig. 5.17. Graph showing variation of mean extrusion and die pressures with percent reduction for different friction factors on die face, semi-wedge angle $\alpha = 30$ degrees.
Fig. 5.18. Graph showing variation of extrusion and die pressures with percent reduction for different frictional conditions on die face. Semi-wedge angle $\alpha = 45^\circ$. 
Fig. 5.19. Graph showing variation of extrusion and die pressures with percent reduction for different friction factors on die face. Semi-wedge angle $\alpha = 60$ degrees.
Fig 5.20. Graph showing the variation in the value of optimum die angle $\alpha$ as a function of the friction factor 'm' and reduction percent.
Fig. 5.21. Graph showing variation in the value of optimum die angle with variation in friction factor $m$ and reduction.
Fig 5.22. Graph showing variation in the value of optimum die angle with variation in friction factor m and reduction.
Graph showing variation in the value of optimum die angle with variation in friction factor $m$ and reduction.

Fig. 5.23.
any reduction depending on the value of \( m \), there is an optimum
die angle for which the extrusion pressure is minimum and, its
value for any reduction increases with increase in the value of \( m \).

In Table 5.1 given below the results from the present analysis
are compared with those of Chenot et al. (85) for the case of
axisymmetric extrusion with smooth dies. It is rather interesting
to note that for reductions even up to sixty percent the maximum
discrepancy in the two results does not exceed seven percent.

**Table 5.1**

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<th>Semi-wedge angle ( \alpha ), degrees</th>
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5.7. APPROXIMATE METHODS

Some approximate methods are presented below, which are frequently used for determination of mean extrusion pressure in case of wedge-shaped dies.

5.7.1. Technology theory

In the present method, a plane section during extrusion is assumed to remain plane and the mean pressure is calculated from equilibrium of forces across any section. The method has been discussed in detail by Rowe [7]. Referring to figure 5.24, the equation of equilibrium in the direction of punch travel is written as,

\[ \sigma_x \, dh_i + h_i \, d \sigma_x - q \, dh_i - mk \, \cot \alpha \, dh_i = 0 \quad (5.26) \]

Using Tresca's yield criterion, \( q - \sigma_x = 2k \) and integrating equation (5.26) with the boundary condition \( \sigma_x = 0 \) at die exit \( (h_i = h) \) the relation for mean extrusion pressure \( P \) is given as,

\[ \frac{P}{k} = (2 + m \, \cot \alpha) \ln \left( \frac{H}{h} \right) \quad (5.27) \]

Using Siebel's correction [93] for change in the direction of flow at die entry and exit, the final equation for mean extrusion pressure is written as,

\[ \frac{P}{k} = (2 + m \, \cot \alpha) \ln \left( \frac{H}{h} \right) + \alpha \quad (5.28) \]

At the optimum die angle \( \frac{dP}{d\alpha} = 0 \) and equation (5.28) yields to,
FIG. 5.24. FORCE SYSTEM FOR TECHNOLOGICAL ANALYSIS

FIG. 5.25. HILL AND GREEN ANALOGY (86) BETWEEN PLANE-STRAIN COMPRESSION AND DRAWING/EXTRUSION.
\[
\sin \alpha_{\text{opt}} = \left( m \ln \left( \frac{H}{h} \right) \right)^{1/2}
\] (5.29)

Or, in terms of the fractional reduction \( r \), equation to the optimum die angle is given by,
\[
\sin \alpha_{\text{opt}} = \left( m \ln \left( \frac{1}{1 - r} \right) \right)^{1/2}
\] (5.30)

5.7.2. Analogy with plane strain compression

Green and Hill (86) developed the analogy between plane-strain compression and drawing in terms of the parameters \( c/d \) (figure 5.25), where \( c \) is the length of the circular arc which has its centre at the virtual apex of the channel and joins the mid points of the two zones of contact and \( d \) is the length of contact with each die. Referring to figure 5.25, we have
\[ c = \left( \frac{-R_1 + R_2}{2} \right) 2\alpha \]
\[ d = R_1 - R_2 \]

and fractional reduction,
\[ r = \frac{R_1 - R_2}{R_1} \]

Hence,
\[ \frac{c}{d} = \left( \frac{2}{\frac{r}{1-r}} \right) \alpha \] (5.31)

Green and Hill (86) and more recently Venter et al. (33) have shown that the redundant work correction in drawing/extrusion is a function of the geometrical parameter \( c/d \). Thus, in the
absence of friction, the drawing/extrusion stress, \( t \), is given by \((86)\)

\[
\frac{t}{2h} = f \ln \left( \frac{1}{1 - \tau} \right)
\]

(5.32)

where, \( f \) is the redundant work factor and equals the mean die pressure for plane-strain compression for height/thickness ratio equal to \( c/d \). For any value of the ratio \( c/d \), the redundant work factor may be found by extrapolation from the list of values supplied by Green \((94)\) or by Hill and Kim \((126)\).

The mean extrusion pressures calculated from equation (5.32) are compared with the exact values (slipline field analysis) in table 5.2 for a number of reductions and for die semi-angle \( \alpha \) between fifteen and ninety degrees. Referring to the above table, it may be seen that for wedge-shaped dies the error increases with increase in the value of the fractional reduction \( \tau \), but even at a reduction level of eight percent the discrepancy in the two values is only of the order of three percent. For square dies, however, the error is more at lower reductions and it decreases with increase in the value of the fractional reduction \( \tau \).

Further, if \( t' \) is the drawing stress in the presence of friction, Green \((94)\) has shown that the ratio \( (t'/t) \) calculated from stress evaluation approach and from slipline field analysis are equal within one percent. Extending Green's procedure \((94)\)
to the case of plane strain extrusion, the mean extrusion pressure in the presence of friction can be calculated more simply by using equation (5.27) and (5.32) or equations (5.28) and (5.32). Hence (equation (5.27) and equation (5.32)),

\[ \frac{p}{k} = 2 \left( 1 + \frac{mcot\alpha}{2} \right) \text{ln} \left\{ \frac{1}{1 - r} \right\} \]  

(5.34)

Or (equation (5.28) and (5.32),

\[ \frac{p}{k} = \frac{\left( 2 + \frac{mcot\alpha}{2} \right) \text{ln} \left( \frac{1}{1 - r} \right) + c}{2 \text{ ln} \left( \frac{1}{1 - r} \right) + c} \]  

(5.35)

With Siebel's correction\(^{(95)}\) taken into account. For wire drawing, alternative equations considering friction and work hardening has also been suggested by Wistreich\(^{(95,73)}\) and R.W. Johnson\(^{(7,96)}\).

It may be mentioned here that the mean extrusion pressure in case of plane-strain and axisymmetric extrusion can be calculated from the equation\(^{(76-79)}\),

\[ \frac{p}{y} = a + b \text{ ln} \left( \frac{1}{1 - r} \right) \]  

(5.36)

For the given geometry and frictional condition, however, the empirical constants, a and b, must be known a priori.

5.7.3 Upper bound analysis

Upper bound to mean extrusion pressure for extrusion through smooth wedge-shaped dies have been obtained by Hill\(^{(97)}\) and Green\(^{(88)}\). The problem has also been discussed in detail by Johnson and Nellor\(^{(14)}\). We choose a system of tangential velocity discontinuities as shown in figure 5.26. From the physical
The lengths of the velocity discontinuities are given by,

\[
\begin{align*}
AB &= \frac{h}{\sin \theta_1} \\
AC &= \frac{H}{\sin \theta_2} \\
BC &= \frac{(H - h)}{\sin \alpha}
\end{align*}
\]

where,

\[
\cot \theta_2 = \frac{(H - h) \cot \alpha - h \cot \theta_1}{H}
\]

From the hodograph shown in (figure 5.26(b)), the magnitudes of the velocity discontinuities are,

\[
\begin{align*}
U_{AB} &= \frac{\sin \alpha \sin \theta_2}{\sin \theta_1 \sin(\theta_2 - \alpha)} \\
U_{AC} &= \frac{\sin \alpha}{\sin(\theta_2 - \alpha)} \\
U_{BC} &= \frac{\sin \theta_2}{\sin(\theta_2 - \alpha)}
\end{align*}
\]

From equations (5.37) and (5.38) upper bound to mean extrusion pressure is written as

\[
p/k = \frac{\sin \alpha \sin \theta_2}{\sin(\theta_2 - \alpha) H} \left( \frac{h}{\sin^2 \theta_1} + \frac{H}{\sin^2 \theta_2} + \frac{(H - h)}{\sin^2 \alpha} \right)
\]

A better upper bound for larger reductions may be obtained using velocity fields, with two rigid triangles such as shown in figure 5.27. From the physical plane (figure 5.27(a)), the
KINEMATICALLY ADMISSIBLE VELOCITY FIELDS FOR CALCULATION OF UPPER BOUND.
length of the tangential velocity discontinuities are as follows

\[ AB = \frac{h}{\sin \theta_1} \]
\[ AC = \frac{h_1}{\sin \theta_2} \]
\[ BC = \frac{(h_1 - h_1)}{\sin \alpha} \]
\[ DE = \frac{H}{\sin \theta_2} \]
\[ CD = \frac{h_1}{\sin \theta_1} \]

and,
\[ EC = \frac{(H - h_1)}{\sin \alpha} \]

Also, from the triangles CDE and ABC we have,

\[ CD = \frac{\sin \left( \theta_2 - \alpha \right)}{\sin \left( \theta_1 + \alpha \right)} \]
\[ AC = \frac{\sin \left( \theta_1 + \alpha \right)}{\sin \left( \theta_2 - \alpha \right)} \]

Hence
\[ \frac{\sin \theta_1 \sin \left( \theta_2 - \alpha \right)}{\sin \theta_2 \sin \left( \theta_1 + \alpha \right)} = \sqrt{\frac{h}{H}} \]

and,
\[ \cot \theta_2 = \cot \alpha - \left( \cot \theta_1 + \cot \alpha \right) \frac{\sqrt{h}}{H} \]

From the hodograph shown in figure 5.27 (b), the magnitudes of the tangential velocity discontinuities are given by,
\[ U_{AB} = \frac{\sin \alpha}{\sin(\theta_1 + \alpha)} \frac{H}{h} \]
\[ U_{AC} = \frac{\sin \alpha \sin \theta_1}{\sin(\theta_1 + \alpha) \sin \theta_2} \frac{H}{h} \]
\[ U_{BC} = \frac{\sin \theta_1}{\sin(\theta_1 + \alpha)} \frac{H}{h} \]
\[ U_{DE} = \frac{\sin \alpha}{\sin(\theta_2 - \alpha)} \]
\[ U_{CD} = \frac{\sin \alpha \sin \theta_2}{\sin(\theta_2 - \alpha) \sin \theta_1} \]
\[ \text{and}, \quad U_{EC} = \frac{\sin \theta_2}{\sin(\theta_2 - \alpha)} \]

Using equations (5.40) and (5.42) upper bound to mean extrusion pressure is written as,
\[ p/k = \frac{\sin \alpha}{H \sin(\theta_1 + \alpha)} \left( \frac{\sin^2 \theta_1}{\sin^2 \theta_1} + \frac{h_1}{\sin^2 \theta_1} \frac{H}{h} + \frac{m(h_1 - h)}{\sin^2 \alpha} \frac{H}{h} \right) \]
\[ + \frac{\sin \alpha}{H \sin(\theta_2 - \alpha)} \left( \frac{\sin^2 \theta_2}{\sin^2 \theta_2} + \frac{h_1}{\sin^2 \theta_1} + \frac{m(H - h_1)}{\sin^2 \alpha} \right) \]

(5.43)

Substituting equation (5.41) in (5.43) it is seen that the two terms in the right hand side of equation (5.43) are equal so that the mean extrusion pressure is given by,
\[ p/k = \frac{2 \sin \alpha \sin \theta_2}{H \sin(\theta_2 - \alpha)} \left( \frac{\sin^2 \theta_2}{\sin^2 \theta_2} + \frac{h_1}{\sin^2 \theta_1} + \frac{m(H - h_1)}{\sin^2 \alpha} \right) \]

(5.44)
For a smooth die \((m = 0)\), the best upper bound either from equation (5.39) or from equation (5.44) corresponds to,

\[
\frac{\sin \theta_2}{\sin \theta_1} = \sqrt{\frac{H}{h}}
\]

For the rough die situation, however, no such simple relation between \(\theta_1\) and \(\theta_2\) could be found and the best upper bound either from equation (5.39) or from equation (5.44) was obtained graphically.

5.7.4. Comparison with slipline field analysis

The results of calculation from the above approximate theories are presented in tables 5.3 to 5.6 and in figures (5.28) to (5.31) where the prediction of the mean extrusion pressure \(p/k\) from the approximate theories is compared with the prediction from the slipline field analysis for die semi-angle, \(\alpha\), between 15 to 60 degrees and for values of \(m\) between 0.1 and 0.9. It may be noticed that of the various approximate theories, Green's method\(^{(94)}\) provides the most accurate estimate of mean extrusion pressure: the discrepancy even at a reduction level of eighty percent being only of the order of five to ten percent. At lower reductions equation (5.35) incorporating Siebel's correction yields better results while at higher reductions the results from equation (5.34) are superior. Moreover, the results calculated from the upper bound analysis are found to be inferior to those from Green's method."
Fig 5.28. Graph showing comparison of results calculated from slipline field analysis with those obtained from approximate theories.
Fig 5.29. Graph showing comparison of results calculated from slipline field analysis with those obtained from approximate theories.
Fig. 5.30. Graph showing comparison of results calculated from slip line field analysis with those obtained from approximate theories.
Fig. 5.31. Graph showing comparison of results calculated from slipline field analysis with those obtained from approximate theories.
at all reduction levels. It may also be seen that equation (5.28) (Technological theory) consistently over estimates the extrusion pressure except at lower reductions while, equation (5.27) provides reliable results at higher reductions only. The prediction of optimum die angle from equation (5.30) is also found to be in error by more than ten percent.

In this analysis of wire drawing process, Green* considered only smaller die angles and lower reductions with Coulomb friction at the interfaces. It is interesting to note that Green's analysis also extends to the case of extrusion involving greater die angles and reductions and with constant frictional traction at the interfaces ($\tau = mk$). This is especially important since the stress evaluation approach is generally considered less reliable at large die angles. Recently Durban has extended the earlier solution of Shield and has proposed a more accurate analysis of the process considering interface friction and hardening of the billet.

5.8. CONCLUSIONS

1. The mean extrusion pressure increases with reduction. For type III and type IV solutions, the variation is quasi-oscillatory, similar to that observed for compression solutions.
2. Extrusion pressure increases in the presence of friction and the effect is more evident at larger reductions.
3. For a given die angle, $\alpha$, and friction factor, $m$, the die pressure is a minimum for the transition field between type I
and type II solutions (figure 5.2),

4. For smooth dies ($m = 0$), the maximum discrepancy between extrusion pressures for axisymmetric extrusion and that for sheet extrusion does not exceed about seven percent up to a reduction level of about sixty percent.

5. The value of the optimum die angle increases with increase in reduction and with increase in the value of the friction factor, $m$.

6. Of the various approximate theories, Green's method provides the most accurate estimate of mean extrusion pressure-the discrepancy being only of the order of five to ten percent for the range of reductions and die angles studied.

5.9. PLOTTING OF SLIPLINE FIELDS

In the following sections, slipline field configurations (type III and type IV solutions) for some die geometries are presented. The plotting procedure was similar to that described for the slipline field solutions in Chapter III and Chapter IV. In all cases, the values of the field angles and the reduction ratio to which the field applies is also mentioned.
$\alpha = 10^\circ$
$\theta = 5^\circ$
$Y = 18^\circ$
$\chi = 42^\circ$

$\beta = 5.0$
$\eta = 0.1$
RED $= 36.77\%$

Fig. 5.32. Slipline field type III
Fig. 5.33. Slipline field type III

\[ \alpha = 20^\circ \quad f = 4.0 \]
\[ \theta = 3^\circ \quad m = 0.3 \]
\[ \psi = 31.75^\circ \quad \text{RED} = 61.78\% \]
\[ \lambda = 36.25^\circ \]
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<th>c/d</th>
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*Results calculated with Siebel's correction (Equations (5.29) and (5.37)).
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* same as before.
TABLE 5.5

SEMI-WEDGE ANGLE $\alpha = 45$ degrees

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* same as before.
TABLE 5.6

SEMI WEDGE ANGLE $\alpha = 60$ DEGREES

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* same as before.