SYNOPSIS

Recently, there has been increasing interest in both the construction and the question of the existence of a solution to the following problem: Given a map \( F : \mathbb{R}^n \to \mathbb{R}^n \), find a vector \( z \in \mathbb{R}^n \) such that

\[
\begin{align*}
\text{(CP)} \\
z &> 0, \quad F(z) > 0, \\
z^T F(z) &= 0.
\end{align*}
\]

Because of the complementary nature of its solution, the above problem, in general, is known as the complementarity problem. When \( F \) is of the form \( F(z) = Mz + q \), where \( M \) is an \( nxn \) matrix and \( q \) is an \( n \)-vector, the (CP) is referred to as the linear complementarity problem; otherwise, it is called the nonlinear complementarity problem. An interesting generalization of the (CP), where the usual non-negative partial ordering of \( \mathbb{R}^n \) is replaced by partial orderings generated by a cone, in \( \mathbb{R}^n \), and its polar, has been considered by several authors. This generalized complementarity problem may be stated as follows: Find a \( z \in \mathbb{R}^n \) satisfying the system

\[
\begin{align*}
\text{(GCP)} \\
z &\in C, \quad F(z) \in C^r, \\
z^T F(z) &= 0.
\end{align*}
\]

(viii)
where $C$ is a closed, convex cone in $\mathbb{R}^n$, $C^*$ the polar cone and $F$ is a map from $\mathbb{R}^n$ into itself. The importance of the (CP) lies in the fact that its form is fairly general in the sense that several problems in different fields such as mathematical programming, game theory, economic equilibrium, mechanics, structural engineering etc., may, by appropriate choice of the function $F$, be so posed. In mathematical programming, a problem in the form of the (CP) arises when the primal and dual constraints are composed together with complementary slackness. Thus, the problem of the existence of solutions to a pair of dual programs can be reduced to a complementarity problem. It is also well known that the problem of finding a Kuhn-Tucker stationary point can be posed as a complementarity problem.

In the past decade, a number of important results have been established, dealing with both the computational and the theoretical (existence and uniqueness) aspects of the (CP). In some of the works, attempts have been made to apply existence theorems for the complementarity problem to the existence and uniqueness of optimal solutions to programming problems. In the present work, we have generalized some earlier results and developed some new results in the theory of the
complementarity problem. Further, complementarity problems associated with quadratic and nonlinear programming problems have been discussed and existence results for these problems have been applied to show the existence of optimal solutions to the respective programming problems.

The thesis is divided into five chapters.

Basic notations and geometric preliminaries are given in Chapter 0.

Chapter I presents some new theorems on the existence of a solution to the (CP), and discusses how existence theorems for the (CP) can be used to study the problem of the existence of an optimal solution to a convex program.

Chapter II deals with the (GCP). In Section 3 of this chapter, we study the (GCP) when \( F(z) = Mz + q \), \( M \in \mathbb{R}^{nxn} \) is copositive with respect to \( C \) and \( q \in \mathbb{R}^n \). A duality theorem for quadratic programming over cone domains is proved, using an existence theorem for this complementarity problem. Section 4 is devoted to the case when \( F \) is nonlinear. The main results in this section are two existence theorems which generalize several previously known theorems for the (GCP).
Chapter III is devoted to the study of the generalized complementarity problem associated with a programming problem over cone domains. This study leads to some interesting and useful results on the existence of an optimal solution to the programming problem under consideration.

The last chapter is aimed at developing an existence theorem for a nondifferentiable programming problem whose objective function contains the square root of a quadratic form. Since the Kuhn-Tucker stationary point conditions for nondifferentiable programming, which are expressed either in terms of subgradient or in terms of directional derivative, cannot be conveniently cast into the form of a complementarity problem, we adopt a different technique to obtain our existence theorem.