CHAPTER III

CONVERGENT POLYNOMIAL EXPANSION AND
SCALING IN INELASTIC CHARGE-EXCHANGE
- SCATTERING PROCESSES
Differential cross-section-ratios of elastic diffraction scattering of hadrons at high energies exhibit scaling in several variables, some of which have been obtained in model independent methods using rigorous foundations based upon Axiomatic Field Theory (AFT)$^{1,2,5,6,8}$. Other variables have also been hypothesized from physical considerations$^{9,11,14-19}$. Generalisation of model independent results on elastic diffraction scattering for inelastic nondiffractive processes does not seem to have been suggested from theoretical$^{1,2,5,6,8}$ grounds. However, a theoretical basis for the average scaling in a weaker sense in suitable scaling variables has been furnished using Arzela-Ascoli theorem, both for elastic diffractive$^3$ and inelastic nondiffractive$^4$ processes. Experimental data have been shown to verify such a scaling$^3,4$. Also a master scaling law has been proposed$^{22}$ for arbitrary distributions in arbitrary hadronic processes. Assumptions leading to model independent$^{1,2,5,6,8}$ results have been criticised$^4$ as not being theoretically justified. Further, since elastic scaling has been derived$^{1,2,5,6,8}$ on the assumption that there exists a diffraction peak and now scaling extends to inelastic nondiffractive processes, it appears that the origin of scaling has not been properly understood. It has been further remarked$^4$ that since the observed scaling is not confined to diffraction scattering alone, the physical inputs for scaling are likely to be analyticity properties$^4$. 
In view of these, it is very important to demonstrate scaling of the differential-cross-section ratio using convergent polynomial expansion (CPE) which exploits Mandelstam analyticity of the $s$ and $\cos \theta$ planes $^{28,29,43}$.

Other important properties of scaling in the context of CPE or optimised polynomial expansion (OPE) $^{24,25}$ have been pointed out in chapter II and earlier $^{28,29,43}$. If the cross-section-ratio data are shown to exhibit scaling in some suitable conformally mapped variable, scaling function can be known from the knowledge of 'partial waves' by fitting the data at any fixed high energy and in a suitable range of $|t|$ in the scaling region. Once the scaling function is known it can be used for the cross-section ratio $f(s,t)$ at other energies in the scaling region. While carrying out partial wave analysis in the conventional method or using the technique of accelerated convergence $^{30}$ by conformal mapping and optimised polynomial expansion (OPE) $^{24,25}$, it is necessary to know a set of parameters at every energy from fits to the differential-cross-section data. Such a method is cumbersome and may be impossible to carry out at high energies. On the other hand, if scaling is shown to be exhibited in a suitably chosen conformally mapped variable $\chi$, it is sufficient if the scaling function is known by fitting the data at any single energy in the scaling region. Thus, the demonstration of scaling in the context of CPE $^{28,29,43}$ or OPE $^{24,25}$ has tremendous importance from the point of economic use of computer time in data analysis $^{28,29,43}$. 
It has been pointed out in chapter II that even without knowing the scaling function by actual data fitting, it is possible to predict \( f(s,t) \) for any high value of \( s \) and as a function of \( |t| \) in the scaling region from the knowledge of the scaling curve, provided that the scaling variable \( \chi \) is a 'simple' function of \( t \). Since the parameters involved in \( \chi \) are those determined by fitting the slope parameter data by means of formulas which exploit the method of conformal mapping and OPE, stability of extrapolations onto unknown regions of \( s \) and \( t \), where data is not yet available, is also guaranteed. Further, using such techniques, asymptotic behaviour of slope parameters and existence of entire functions for \( f(s,t) \) in the asymptotic energy region can also be ascertained.

In ref. 28 the energy dependence of the slope-parameter data and scaling of the data on \( f(s,t) \) for \( pp \) scattering were very well represented by means of CPE and conformal mapping which did not introduce any spurious cut in the mapped plane or require any knowledge of zeros. But as pointed out in chapter II, when this method is applied to processes possessing unsymmetrically cut \( x \) planes of analyticity, it develops spurious cuts in the mapped plane.

In ref. 29 a scaling variable was developed which in addition to explaining observed shrinkage-antishrinkage of forward peaks and providing useful informations on the asymptotic behaviour of slope parameters displays early onset of scaling of the data on \( f(s,t) \) for \( pp, \bar{p}p, K^+p \) and \( \Lambda ^{+}p \) scattering. But such a variable requires knowledge of at least one real
zero on the physical region for every process for its construction, but the experimental informations on real zeros are very meagre at present. For some cases the formula for the slope parameter needed effective shapes of spectral functions for the description of the data. Also the conformal mapping used develops spurious cuts in the mapped planes for all processes, which is definitely an objectionable feature from the point of view of correct analytic representation and accelerated convergence of polynomial expansion. Further, if spurious cuts are present in the mapped plane it is not possible to obtain informations on the possible existence of entire functions for any of the processes.

In chapter II of this thesis a new scaling variable $X$ has been proposed which does not introduce any kind of spurious cuts in the mapped planes for processes possessing asymmetric cut cosθ planes or require any assumption on the existence of real zeros in the physical region. The CPE developed has given reasonably good fits to the slope-parameter data at high energies for all the elastic diffraction scattering processes yielding useful information on the asymptotic behaviours of slope parameters. As in refs. 28 and 29 the variable proposed has potentialities to reproduce some known scaling variables and Regge behaviour at high energies and small $|t|$ , with

$$|t| \ll \frac{4m_N^2}{|t|} \quad \text{But away from forward angles with} \quad |t| \gg \frac{4m_N^2}{|t|}, \quad \text{a condition which is satisfied by a majority of data points, the variable reduces to } b(s)(\ln t)^2, a$$
variable not proposed by any other theory. Because the spurious cuts are absent in the mapped planes, it has been possible to obtain definite conclusions that for \( pp, \Lambda^+p \), and \( K^+p \) scattering, the cross-section ratios are entire functions of the corresponding scaling variables. Scaling of the cross-section-ratio-data at high energies in the variable \( \chi \) has been shown to be exhibited in a remarkable manner for all the processes.

It has been already pointed out in chapter I and II that there does not exist any model independent derivation on scaling for inelastic nondiffractive processes except for the weaker results on average scaling. In view of the speculation that physical inputs for a correct understanding of scaling are likely to be analyticity properties and success of CPE, which exploits Mandelstam analyticity, in representing scaling of the data for elastic diffractive processes in a remarkable manner, it is very important to investigate whether the same method could be generalised to include inelastic nondiffractive processes also. It will be very convenient to employ the method developed in chapter II for inelastic processes since the variable has been especially designed for any general unsymmetrical cut \( x \) plane of analyticity without generating any spurious cut and without requiring any knowledge of zeros. In this chapter, the method developed in chapter II has been applied to six different inelastic charge exchange nondiffractive processes \( \Lambda^0p \rightarrow \Lambda^0n \).
\( p \rightarrow q n, k^+ p \rightarrow k^0 n, k^- n \rightarrow k^0 n, k^+ p \rightarrow k^0 \Lambda^{++} \)

and \( k^- n \rightarrow k^0 \Lambda^- \). Using Mandelstam analyticity of the \( s \) and \( \cos \theta \) planes a variable \( X \) has been constructed which has the potentialities to reproduce known scaling variables for elastic processes, Regge behaviour, and provide information on the asymptotic behaviours of the slope parameter of the type \( \sim (\ln s)^m \), with \( m = 0, 1, 2 \ldots \). The first term in the CPE in \( X \) yields a very good fit to the available high energy data on slope parameters for all processes. The slope parameter for every process is consistent with asymptotic behaviour of the type \( \sim \ln s \). The CPE developed has one of the limitations that, for finite energies, neither its rate of convergence nor the nature of polynomials are unique. But at asymptotic energies the polynomials are uniquely the Laguerre polynomials and CPE goes over to OPE.

As in other papers\(^{28,29}\) scaling in this variable has not been proved but rather hypothesized basing upon maximal convergence of OPE at asymptotic energies. All the available cross-section-ratio data at high energies for every process exhibit scaling in a remarkable manner. Early onset of scaling of the data are clearly indicated. From the asymptotic behaviours of the slope parameters it is concluded that for every process \( f(s,t) \) is an entire function of the corresponding scaling variable \( X \) at asymptotic energies. Several other limitations of this method, some of which are common to those present in refs. 28, 29 and chapter II\(^{43}\) are also pointed out in course of the development.
of this chapter. Although here no predominance of absorptive part near forward angles has been assumed as in the derivation of model independent results\textsuperscript{1,2,5,6,8}, geometrical models\textsuperscript{9,11,14-19} and earlier papers using CPE\textsuperscript{28,29,43}, it has not been possible to retain the contribution due to poles. In this sense the proposed representation does not conform to correct Mandelstam analyticity.

This chapter is organised in the following manner:

In Sec. 2 a conformal mappings of the $s$ and $\cot \theta$ planes which have been discussed in sufficient detail in chapter II has been described. In this section also arguments in favour of our hypothesis on scaling using CPE have been put forward. Analysis of the experimental data for all the six processes are carried out in Sec. 3, where results on the asymptotic behaviours, existence of entire function, and representation of scaling by the data are reported. In Sec. 4, the results of this chapter have been summarized and discussed and the limitations of the method pointed out. In this section the application of the present method in predicting $f(s,t)$ as a function of $|t|$ has also been pointed out.

Most of the contents of this chapter have already been published\textsuperscript{21}.
Conformal mapping of the $s$ plane has been discussed with sufficient detail in refs. 28, 29 and chapter II and that of a general asymmetric $x = \cos \theta$ plane has been discussed in chapter II for elastic diffractive processes, but without introducing any spurious cut. In this section a discussion has been made as to how they can be applied to develop a CPE for inelastic processes. For clarity some features of CPE as described earlier and in chap. II are also discussed.

A. Conformal mapping of the $\cos \theta$ plane without spurious cuts

As discussed in Chap. II, for an unsymmetrically cut $x = \cos \theta$ plane the conformal mapping of the cuts onto a parabola with focus at the origin in the $Z_0$ plane is achieved by the following steps

$$\omega_0 = \frac{x_+ - x}{x_- + x} \frac{x_- + 1}{x_+ - 1} \quad (3.1)$$

$$Z_0 = (\cosh^{-1} \sqrt{\omega_0})^2 \quad (3.2)$$

In (3.1) $x_+(-x_-)$ is the start of the right (left)-hand cut in the $x$ plane where the physical region exists for $+1 \gg |x_0| > -1$. By this mapping the entire $x$ plane, except the cuts, is mapped onto the interior of the parabola in the $Z_0$ plane. Specific advantages of this conformal
transformation over those used earlier have been discussed in the previous chapter. The image of the right-hand cut forms the forward portion of the parabola and surrounds the image of the forward direction \((x = 1)\) like a crescent and the remaining part of the branches of the parabola is the image of the left-hand cut. Thus by mapping the right-hand cut onto a closer vicinity of the image of the forward direction the variable \(Z_0\) gives a stronger emphasis on the influence of the right-hand cut as compared to that of the left-hand cut. Unlike the mappings used earlier this conformal transformation does not introduce any spurious cut in the mapped plane or require any knowledge of zeros for its construction.

In earlier works and in chap. II the start of a cut for an elastic diffractive process has been taken to be that due to the absorptive part of the amplitude, although at high energies it is the same as that of the real part. Such an assumption was prompted by numerous theoretical investigations and experimental results that near forward angles the absorptive part dominates over the real part for diffraction scattering at high energies. On the other hand, inelastic charge exchange processes are found to be nondiffractive and there is no experimental evidences suggesting that absorptive part dominates near forward angles. On the contrary, there are positive evidences to the effect that at least in some cases the real part dominates near forward angles. Thus,
unlike earlier works²⁸,²⁹,³¹-³³,³⁸ and the assumption of chap. II⁴³ while constructing the conformal mapping for non-diffractive processes, we take \( x_+(-x) \) to be the start of the right (left)-hand cut of the total amplitude in the \( x \) plane. For any \( s \)-channel process \( a + b \rightarrow c + d \), \( x_+(x) \) is related to the \( t(u) \)-channel thresholds and their relations at high energies can be written down as

\[
x_+ \simeq 1 + \frac{2t^+}{s}
\]

\[
x_- \simeq 1 + \frac{2t^-}{s} + \frac{2\Sigma}{s}
\]

where \( \Sigma = m_a^2 + m_b^2 + m_c^2 + m_d^2 \). Since an analysis of the slope parameter data and study of scaling of the differential cross-section data will be carried out only at high energies, approximations (3.3a) and (3.3b) will serve the present purpose. Using (3.3a) and (3.3b) in (3.1) one can write for high energies

\[
(\omega_o \simeq \frac{(1 - \frac{t}{t^+})}{(s + t - \Sigma + t)} \frac{(s + t - \Sigma )}{(s + t^+ - \Sigma + t)} )
\]

By the conformal transformation onto the \( Z_0 \) plane, although the cuts are mapped onto the boundary of the parabola with focus at the origin for all energies, the physical region is mapped onto a finite segment of the right-half of the \( \text{Re}Z_0 \) axis. It is well known that the figure of convergence for Laguerre polynomial expansion is the parabola with focus at the origin but the physical region is the entire right half of the \( \text{Re}Z_0 \) axis. Since, as has been pointed out in
Chap. II, for any finite energy the physical region available is only a finite segment of $0 \leq \Re \mathcal{Z}_0 \leq \infty$, the orthogonal polynomials $\{P_n(\mathcal{Z}_0)\}$ determined by it are not Laguerre polynomials $\{L_n(\mathcal{Z}_0)\}$. Further, if a Laguerre polynomial expansion is written at finite energies, the coefficients in the expansion can not be determined because of lack of correct physical region. Thus, there are ambiguities in using a Laguerre polynomial expansion in $\mathcal{Z}_0$ at finite energies. However, for finite energies a CPE can be written in terms of $\{P_n(\mathcal{Z}_0)\}$ for the differential cross section

$$\frac{d\sigma}{dt} = e^{-\alpha \mathcal{Z}_0} \sum_n a_n(s) P_n(2 \alpha \mathcal{Z}_0) \quad (3.5)$$

where $\alpha$ is, in general, an energy dependent parameter.

Since the length of the image of the physical region varies with energy and spreads the entire right-half of the $\Re \mathcal{Z}_0$ axis like $\sim (\ln s)^2$ for $s \to \infty$, at finite energies the nature of $\{P_n(\mathcal{Z}_0)\}$ also varies. The domains of convergence of $\{P_n(\mathcal{Z}_0)\}$ for finite energies may not be the whole interior of the parabola in the $\mathcal{Z}_0$ planes but may contain only a part of it. Thus, at finite energies the convergence of (3.5) may not be maximum. Since the nature of $\{P_n(\mathcal{Z}_0)\}$ varies with energy, their domain of convergence containing parts of the interior of the parabola also varies. Thus, the rate of convergence of (3.5) also varies with energy. Such an expansion possessing nonoptimal convergence at finite energies has been termed as convergent polynomial expansion (CPE). However, for $s \to \infty$ the correct physical region for Laguerre polynomial expansion
is available in the $Z_0$ plane and the domain of convergence of $\{L_n(Z_0)\}$ contains the image of the domain of analyticity of the entire $x$ plane making the rate of convergence of polynomial expansion maximum.\(^{24,25}\) Thus, as $s \to \infty$, $\{P_n(Z_0)\} \to \{L_n(Z_0)\}$ and CPE goes over to OPE.

B. Conformal mapping of the $s$ plane

As observed in Chap. II\(^{43}\), the formula for the slope of the forward peak computed from eqn. (3.5) will be proportional to $\alpha$. If $\alpha$ is taken to be independent of energy, as in some earlier works\(^{32,33,38}\), the slope parameter will be constant for $s \to \infty$. Experimentally, the slope parameter for various charge exchange scattering processes shows definite evidences of increasing with energy\(^{59-62}\). Such an energy dependence, in the present context, can come from the possible energy dependence of $\alpha$. The need for exploiting the $s$ plane analyticity along with that of the $\cos \alpha$ plane, from the point of view of $S$-matrix theory, has been discussed in refs. 28 and 29. The most general method of exploiting the $s$ plane analyticity by OPE is to expand the function depending upon $s$ in polynomials of a suitably chosen conformally mapped variable. Cutkosky\(^{63}\) has suggested the use of separate conformal mappings and corresponding polynomial expansions for the individual factors in a function, each of which is a function of an independent variable. As an analytic function of $s$, let the total forward amplitude possess the left-hand cut in the region...
the physical right-hand cut in the region \( s_{\text{th}} \leq s < \infty \), where \( s_1 = \sum - t_\omega \) and \( s_{\text{th}} \) is the s-channel threshold. As discussed in chap. II the following conformal transformations:

\[
\omega_s = \frac{s - s_{\text{th}}}{s_{\text{th}} - s_1} \quad (3.6a)
\]

\[
\eta(s) = \sinh^{-1} \sqrt{\omega_s} \quad (3.6b)
\]

\[
\zeta(s) = \eta^2(s) \quad (3.6c)
\]

map the left-hand cut onto a strip (parabola) in the \( \eta \) plane. Entire domain of analyticity in the \( s \) plane excluding the cuts is mapped onto the interior of the strip (parabola) the image of the left-hand cut forming its boundary, but the physical cut is mapped onto the right-half of the \( \Re \eta \) (\( \Re \zeta \)) axis. In chap. II, types of transformations have been found to be very useful in taking account into energy dependence of slope parameters and representing scaling of differential cross section for elastic scattering. A convergent Taylor series expansion for \( \chi(s) \) can be written as

\[
\chi(s) = \left[ \sum_{n=0}^\infty c_n \zeta^n \right] \quad (3.7a)
\]

\[
\sum_{m=0}^\infty d_m \eta^m \quad (3.7b)
\]

In elastic diffraction scattering processes there exist exact results which limit the maximum rate of growth of slope parameter at \( (\ln s)^2 \) for \( s \to \infty \). Since for
s → ∞, \( \eta(s) \sim \ln s \) and \( \varsigma(s) \sim (\ln s)^2 \), the maximum allowed value of \( n(m) \) has been restricted at (2) in earlier works and Chap. II. But since there does not seem to exist analogous results for inelastic processes the Taylor series expansions (3.7a) and (3.7b) have not been truncated. The formula (3.7b) has the potentialities to satisfy \( (\ln s)^m \), with \( m = 0, 1, 2, \ldots \), types of asymptotic behaviours whereas (3.7a) has the potentialities to satisfy \( (\ln s)^{2n} \), with \( n = 0, 1, 2 \ldots \) types of asymptotic behaviours. Since (3.7b) has the capacity to yield more general types of asymptotic behaviours (3.7b) has been preferred for data analysis, as described in the next section.

C. Scaling by convergent polynomial expansion

Following Chapt. II let us also define in this case

\[
\chi(s,t) = \alpha(s) Z_0(s,t) \quad (3.8)
\]

and denote

\[
f(s,t) = \frac{d\sigma}{dt}(s,t) / \frac{d\sigma}{dt}(s,0) \quad (3.9)
\]

where \( \frac{d\sigma}{dt}(s,0) \) is the extrapolated peak value of the differential cross section. Then using (3.5) one can write

\[
f(s,t) = \sum_n a_n(s) P_n(2\chi) \quad (3.10)
\]

where

\[
a_n(s) = \frac{a_n(s)}{\sum_n a_n(s) P_n(o)} \quad (3.11)
\]
which may be, in general, energy dependent. At asymptotic energies the correct physical region in the $\chi$ plane for Laguerre polynomials is achieved and $\left\{ P_n(2\chi) \right\} \rightarrow \left\{ L_n(2\chi) \right\}$ the asymptotic OPE can be written as

$$f(s,t) = \alpha \sum_n e_n(s)L_n(2\chi)$$

In (3.10) and (3.12), besides the accelerated convergence being achieved by conformal mapping of the cut $x$ plane, convergence is further accelerated for small values of $|t|$ for which $|Z_0| \ll 1$. This corresponds to most of the values of $|t|$ lying inside and some $|t|$ outside the peak region. It may be possible to take into account the energy dependence of the 'partial wave' $e_n$ by suitable conformal mapping, but this would require the knowledge of the analytic structure of the 'partial wave' amplitude in the $s$ plane which is not the same as that of the total amplitude. It will be demonstrated in the next section that experimental data on $f(s,t)$ for high energies for all the charge exchange processes considered here scale in a remarkable manner which supports the fact that $e_n(s)$ are energy independent parameters at least in the scaling region. Although OPE (3.10) is valid at finite energies and OPE (3.12) is valid at asymptotic energies, both lead to the same expression for the slope parameter of the forward peak

$$b(s) = \frac{\alpha(s)}{t^*} \left[ 1 + \frac{t^*}{s - \sum t_-} \right]$$

which for high energies reduces to the form

$$b(s) \approx \frac{\alpha(s)}{t^*}$$
This formula is a consequence of the assumption, as adopted in Chap. II and earlier for elastic processes, that only the first term is sufficient to account for the data on differential cross section in a small $|t|$ interval in the small $|t|$ region of the forward peak. For inelastic charge-exchange processes considered here usually the measured slope-parameter corresponds to the slope of the right-hand side of the forward peak observed in a $\frac{d\sigma}{d|t|}$ vs $|t|$ plot. The present assumption regarding the first term is meant to apply for the small $|t|$ region of the peak extrapolated to the forward direction from the right-hand side of the experimentally observed peak. The formula (3.13) with the definition of $\alpha(s)$ given in (3.7b) for fitting the slope-parameter data as a function of $s$ for various processes has been used in the next section.

It may be checked that similar properties of $\chi$ noted for diffraction scattering processes in Chap. II, also apply for the present case. For high energies and all angles

$$\chi \sim b(s) z_0(s, t)$$

(3.15)

If the values of $s$ and $t$ are such that

$$(s + t - \Sigma) \gg |t|$$

(3.16)

(3.4) will yield

$$\omega_0 \approx 1 - \frac{t}{t_+}$$

(3.17)

and

$$z_0(s, t) \approx \left[ \sinh^{-1} \left( \frac{-1}{t_+|t|^{1/2}} \right) \right]^2$$

(3.18)
It is to be noted that the condition (3.16) can always be satisfied by a large majority of data points lying in the peak region even for medium energies and even for data points outside the peak region with values of \( |t| \ll s \), for high energies. For very small values of \( |t| \ll t_+ \) and at high energies

\[
\chi(s,t) \rightarrow tb(s) \rightarrow t(\ln s)^m \quad (3.19)
\]

where the value of \( m \) is decided by the asymptotic behaviour of the slope parameter \( b(s) \). For \( m = 1 \), (3.19) corresponds to Regge type asymptotic behaviour and for \( m = 2 \), (3.19) reproduces the scaling variable of Auberson, Kinoshita and Martin for elastic diffractive processes. The scaling variable \( tb(s) \) has been derived by model independent methods and suggested from geometrical models for elastic scattering. Recently, it has been shown that the data on \( f(s,t) \) for \( \Lambda^- p \rightarrow n^0 n \) and \( \pi^- p \rightarrow \eta^0 n \), when plotted against the variable \( tb(s) \) for small values of \( |t| \) and \( P_{Lab} \gg 20.8 \text{ GeV/C} \), exhibit scaling. For high energies and values of \( |t| \gg t_+ \), a condition which is satisfied by a majority of available data points for all the inelastic processes considered here

\[
\chi(s,t) \sim b(s)(\ln t)^2 \rightarrow (\ln s)^m(\ln t)^2 \quad (3.20)
\]

Such a behaviour has not been predicted in any of the existing scaling variables.

Since the present hypothesis on scaling in \( \chi \),
as in elastic scattering processes, will also be based upon CPE it is useful to examine into the convergence properties of (3.10) and (3.12) in the $\chi$ plane. Since asymptotically $\alpha(s) \sim (\ln s)^m$, the image of the physical region of the $x$ plane spreads the right-half of the real axis in the $\chi$ plane like $\sim (\ln s)^{m+2}$ for $s \to \infty$, although it spreads the right-half of the $\Re Z_0$ axis like $(\ln s)^2$.

Also, since $\Re \chi$ and $\Im \chi$ each is proportional to $\alpha(s)$, for physical values of $s \to \infty$, the images of the $x$-plane cuts in the $\chi$ plane move to infinity like $(\ln s)^m$. The images of the $x$-plane poles which might have limited the convergence of CPE in the $Z_0$ plane are now pushed onto infinity like $(\ln s)^m$ for $s \to \infty$, in the $\chi$ plane. Since the value of $m$ is decided by the asymptotic behaviour of the slope parameter, the realization of the correct physical region for Laguerre-polynomial expansion, the approach from CPE to OPE, and the movement of all the images of the singularities onto infinity in the $\chi$ plane, are faster, for the more is the rate of shrinking of the forward peak $\chi s \to \infty$.

If $m \geq 0$, all the images of the singularities are pushed onto infinity as $s \to \infty$ and the entire $\chi$ plane is available as the domain of analyticity of $f(s,t)$. Hence $f(s,t)$ becomes an entire function of $\chi$ at asymptotic energies. Auberson, Kinoshita and Martin have shown that $f(s,t)$ is an entire function of their scaling variable $z = t(\ln s)^2$ for $s \to \infty$ for elastic diffractive processes, but there does not exist similar model independent results for inelastic processes.
As discussed above, $\mathcal{X}$ has many attractive features to be a scaling variable. It has been also observed that although the rate of convergence of the CPE (3.10) and the nature of the polynomials $F_n(2\mathcal{X})$ occurring in it vary with energy for finite energies, at asymptotic energies the rate of convergence of (3.12) is maximum and the polynomials occurring in it are uniquely the Laguerre polynomials. Because of the maximal rate of convergence of (3.12) it may be possible that in the asymptotic energy region the same set of $e_n$'s may represent the data on $f(s,t)$ for all asymptotic values of $s$ and different values of $|t|$. Thus, if there is any scaling of $f(s,t)$ in $\mathcal{X}$, it must be in the asymptotic energy region. In the next section it has been shown that experimental data on $f(s,t)$ for different inelastic charge exchange scattering processes strongly support the view that $\mathcal{X}$ is a very good scaling variable at high energies.

Before concluding this section, it is necessary to point out other limitations of the CPE used here. The theory of analytic approximation by conformal mapping and OPE emphasizes upon correct analyticity. Although the CPE has been developed with contribution due to cuts, no possible contribution due to poles in the $s$ and $\cos\theta$ planes has been included. There is no reason to believe that for inelastic nondiffractive processes pole contributions are negligible as in elastic diffraction scattering processes. But it may be noted that for the inelastic processes considered here although there are present $s$ and/or $u$ channel poles,
there does not exist any $t$-channel pole closer to the right-half of the physical region ($0 < \chi < 1$) than the two-pion cut and the effect of this cut has been already taken into account by conformal mapping. For $s \to \infty$ these $s$ and/or $u$ channel pole contributions vanish without spoiling the picture of scaling as hypothesized here. But for finite and high energies, for which experimental data are even observed to scale, this simple picture of scaling is spoiled, if pole contributions are retained explicitly. If pole contribution is not included its image in the mapped plane may limit the rate of convergence of polynomial expansion in $Z_\omega$ and $\chi$ for nonasymptotic energies, but as already discussed, for $s \to \infty$, there is no problem of maximal convergence in the $\chi$ plane if $m > \omega$. The present representation does not satisfy correct analyticity at finite energies since no pole-contribution has been included. Another limitation of a more serious nature is that although the present argument in favour of scaling is supported by analyticity and the criterion of maximal convergence, scaling has not been derived here, rather hypothesized. In the geometrical models scaling has been hypothesized for elastic diffraction scattering processes $^8,11,14-19$. 
In this section, analysis of the experimental data for six different inelastic charge exchange processes, \( \Lambda^- p \rightarrow \Lambda^0 n \), \( \Lambda^- p \rightarrow \eta n \), \( K^- p \rightarrow K^0 n \), \( K^- p \rightarrow K^0 \Delta^+ \), and \( K^- n \rightarrow K^0 \Delta^- \) has been carried out. The processes have been chosen because experimental data on slope parameters and differential cross section for these nondiffractive processes are adequately available to carry out analysis by the present method and obtain definite conclusions. From the definition of \( \chi(s) \) and \( \chi \) given in (3.7) and (3.8), respectively, it may be noted that the variable \( \chi \) involves unknown parameters which can be determined by fitting the slope parameter data by the formula (3.13). In this section such fits for various processes are reported and the values of the unknown parameters are obtained. Further, these fits supply information on the asymptotic behaviour of the slope parameter. From the knowledge of the asymptotic behaviour of the slope parameter it is concluded whether in a given process \( f(s,t) \) is an entire function of \( \chi \). To test how good scaling is in \( \chi \), one plots all the available data on \( f(s,t) \) at high energies against \( \chi \).

### A. \( \Lambda^- p \rightarrow \Lambda^0 n \)

For this process
$\pi^- p \rightarrow \pi^0 n$

$\log_{10} f(s,t)$

- $5.9$ GeV/C
- $9.5$
- $13.3$
- $18.2$
- $20.8$
- $40.8$
- $60.4$
- $100.7$
- $150.2$
- $199.3$

$\chi(s,t)$

**FIG. 20(b)**
where \( m = \text{nucleon mass} \). Using these in the formulas (3.6) and (3.13) the available slope parameter data\(^{60,61}\) for \( P_{\text{Lab}} > 5.9 \) GeV/C are fitted. Formula (3.13) gave a very good fit to the data\(^{61,62}\) only with first two terms of \( \alpha(s) \) as defined by (3.7b). A value \( \chi^2/\text{DF} = 1.29 \) was obtained with

\[
d_0 = 0.357, \quad d_1 = 0.212
\]

for 10 data points. This fit has been shown by fig.19 and is consistent with an asymptotic behaviour of the type \( \sim \ln s \).

With this asymptotic behaviour it can definitely be concluded that as \( s \to \infty \), \( f(s,t) \) becomes an entire function of \( \chi \).

With the knowledge of the unknown parameters in \( \chi \) as given by (3.22), available cross-section-ratio data\(^{60,61}\) for all values of \( s \) and \( t \) are plotted against \( \chi \). Fig.20(a) shows such a plot for all the data with \( P_{\text{Lab}} = 5.9 \) to 199.3 GeV/C and for values of \( |t| \) in the peak region. It is very clear that all the data lie on the same curve and scaling is exhibited in a very remarkable manner. As compared with the empirical observation on scaling\(^{20}\) in the variable \( \ln b(s) \) and the average scaling\(^4\), it is found that in the present case scaling is exhibited in a much better fashion encompassing the cross-section-ratio data over a larger range of \( |t| \) and \( P_{\text{Lab}} \). Whereas in earlier works\(^4,20\) scaling has been shown to be exhibited by the data for \( P_{\text{Lab}} > 20.3 \) GeV/C, in the present case scaling is exhibited earlier in the energy scale starting from \( P_{\text{Lab}} = 5.9 \) GeV/C. In Fig.20(b), all the available
cross-section-ratio data for still larger values of $|t|$ and $P_{Lab} \approx 5.9 \text{ GeV/C}$ have been plotted. In this figure although there appears deviations from the scaling curve caused by the data points coming from the dip regions$^{60}$, tendency for scaling is discernible.

**B. $\Lambda^- p \rightarrow \eta n$**

For this process

$$t_+ = 4m^2, t_- = (m + m_n)^2 \quad (3.23)$$

Using these in the formulas (3.6) and (3.13) the available slope parameter data$^{62}$ have been fitted at high energies with $P_{Lab} \approx 20.8 \text{ GeV/C}$. A very good fit to all the available data$^{62}$ only with first two terms of $\alpha(s)$ defined by (3.7b) is obtained with

$$d_0 = 0.115 \quad (3.24)$$

$$d_1 = 0.156$$

This fit yields a $\chi^2/\text{DOF} = 0.55$ for 6 data points. The fit has been shown in Fig. 21. Thus, the data on the slope parameter are consistent with an asymptotic behaviour of the type $\sim \ln s$. This supplies the information that as $s\rightarrow \infty$, $f(s,t)$ for this process is an entire function of $\chi$.

Having known the values of unknown parameters in $\chi$, all the available data$^{62}$ on $f(s,t)$ against $\chi$ are plotted as shown in Fig. 22. Remarkably enough scaling is exhibited by all the available data with $P_{Lab} \approx 20.8 \text{ GeV/C}$ and all values of $|t|$ . Demonstration of scaling of the data in such
FIG. 22

\[ \log_{10} f(s,t) \]

\[ \pi^+ p \rightarrow \eta n \]

- ○ 20.8 GeV/C
- △ 40.8
- □ 64.4
- ● 100.7
- ▲ 150.2
- ■ 199.3

FIG. 22
a remarkable fashion covering such a larger range of $|t|$ has not been observed in other variables $^{4,20}$.

$K^p \rightarrow \bar{K}n$

For this process

$$t_+ = 4m_{\Lambda}^2, \quad t_- = (m_{\Lambda} + m_{\pi})^2 \quad (3.25)$$

Using these in formulas (3.6) and (3.13) the available slope-parameter data $^{59,64,65}$ at high energies with $P_{Lab} > 4$ GeV/C were fitted. With the first two terms of the parameters in (3.7b) having the following values

$$d_0 = 0.082 \quad (3.26)$$

a good fit with $\chi^2/DOF = 3.20$ for 12 data points were obtained. Excluding the point at relatively lower $P_{Lab} = 4$ GeV/C a $\chi^2/DOF = 1.21$ is obtained with the same parameters. This fit has been shown in Fig. 23 and is consistent with the asymptotic behaviour of the type $\sim \ln s$. From the asymptotic behaviour of the slope parameter it is concluded that for this process also $f(s,t)$ is an entire function of $\chi$ for $s \rightarrow \infty$. Having known the values of unknown parameters as given in (3.25), all the available data $^{59,64,65}$ on cross-section-ratio were plotted against $\chi$ as shown in Fig. 24. It is very clear that scaling is exhibited by the data in a remarkable manner in spite of large experimental errors.
FIG. 23

\[ \bar{K}p \rightarrow \bar{K}^0 n \]
$K^- p \rightarrow \bar{K}^0 n$

$\log_{10} f(s, t)$

- $6.0 \text{ GeV}/c$
- $8.36$
- $10.7$
- $12.8$
- $13.0$
- $15.7$
- $24.8$
- $25.0$
- $34.6$
- $40.0$

$\chi(s, t)$

FIG. 24
D. $K^+ n \rightarrow K^0 p$

For this process
\[ t_+ = 4m_A^2, \quad t_- = (m_\Lambda + m_\pi)^2 \]  \hspace{1cm} (3.27)

Using these in formulas (3.6) and (3.13) available high energy data on slope parameters \[^{59,66}p_{\text{Lab}} > 3.8 \text{ GeV/C}\] have been fitted. A very good fit to 6 data points is obtained with the following parameters
\begin{align*}
  d_0 &= -0.027 \\
  d_1 &= 0.159 \hspace{1cm} (3.28)
\end{align*}

yielding \( \chi^2/\text{DOF} = 0.245 \). This fit has been shown in Fig. 25 and is consistent with asymptotic behaviour of the type \( \sim 1/s \). From the knowledge of the asymptotic behaviour it can be concluded that \( f(s,t) \), for this process, is also an entire function of \( X \) for \( s \rightarrow \infty \). Having known the parameters occurring in \( X \), the cross-section-ratio data \[^{59,66}p_{\text{Lab}} > 3.8 \text{ GeV/C}\] on \( f(s,t) \) are plotted against \( X \) for high energies with \( p_{\text{Lab}} \gg 3.8 \text{ GeV/C} \) and all available values of \( |t| \). This is shown in Fig. 26. It is very clear that scaling is exhibited by all the data points in a remarkable manner.

E. $K^+ p \rightarrow K^0 \Delta^{++}$

For this process
\[ t_+ = 4m_A^2, \quad t_- = (m_\Lambda + m_\pi)^2 \]  \hspace{1cm} (3.29)

Using these in formulas (3.6) and (3.13) all available high
FIG. 26

$K^+ n \rightarrow K^0 p$

$\log_{10} f(s,t)$

○ 3.8 GeV/c
△ 6.0
○ 8.36
● 12.80

$\chi(s,t)$

FIG. 26
energy data on slope parameters $^{59,65,67,68}$ with $p_{\text{Lab}} \gg 3$ GeV/C have been fitted. A good fit, as shown in Fig. 27, has been obtained for 10 data points with the following values of the parameters

$$d_0 = -0.240$$
$$d_1 = 0.280$$

yielding a $\chi^2/$DOF = 4.553. However, although two data points at $p_{\text{Lab}} = 12.8$ and 15.7 GeV/C remain away from the solid curve, the fit almost passes through their mean position as can be seen from Fig. 27. Excluding these two points the same fit gives $\chi^2/$DOF = 0.760. This fit is consistent with the asymptotic behaviour of the type $\sim \ln s$ and suggests that for this process also $f(s, t)$ is an entire function of $\chi$ for $s \to \infty$. Having known the values of the unknown parameters in $\chi$ as given by eqn. (3.30) all the available high energy data $^{59,65,67,68}$ on $f(s, t)$ for $p_{\text{Lab}} \geq 5$ GeV/C and all values of $|t|$ are plotted against $\chi$ as shown in Fig. 28. In spite of large experimental errors in the data, the tendency for scaling has been very clearly exhibited in this figure.

For this process also

$$t_+ = \frac{4m_{\pi}}{m_\Lambda}^2, \quad t_- = (m_\Lambda + m_\pi)^2$$

Using these in formulas (3.6) and (3.13) available high energy data on slope parameters $^{59,67,69}$ with $p_{\text{Lab}} \gg 4.48$ GeV/C.
$K^+ p \rightarrow K^0 \Lambda^{++}$

FIG. 27
FIG. 28

$K^+ p \rightarrow K^0 \Delta^{++}$

$\log_{10} f(s,t)$

$\chi(s,t)$

- $5 \text{ GeV/c}$
- $8.36$
- $9.0$
- $12.8$
- $13.0$
- $15.7$
a very good fit, as shown in Fig. 29, has been obtained for 7 data points with the following values of parameters

\[
\begin{align*}
    d_0 &= -0.179 \\
    d_1 &= 0.176
\end{align*}
\]

yielding a \( \chi^2/\text{DOF} = 0.2 \). This fit is consistent with the asymptotic behaviour of the type \( \sim \ln s \) and suggests that for this case also \( f(s,t) \) is an entire function of \( \chi \) for \( s \to \infty \). Having known the values of the unknown parameters in \( \chi \) as given by eqn. (3.32), all the available high energy data are plotted on \( f(s,t) \) against \( \chi \) for \( P_{\text{lab}} \geq 4.48 \text{ GeV/C} \) and all values of \( |t| \). This has been shown in Fig. 30 where the tendency of the data for scaling in \( \chi \) is clearly exhibited, in spite of large experimental errors.

In summarizing the contents of this section it is observed that using analyticity of the \( \chi \) plane along with that of the \( \cos \theta \) plane has given good fits to the available data on slope parameters for all the six processes considered here. The formula developed for the slope parameter needs only two parameters for each process. The conformal mapping used does not generate any kind of spurious cut or require any knowledge of zeros. Also, no effective domains of analyticity \( 32,33 \) are needed for the fit. The asymptotic behaviour of the slope parameter for every case is \( \ln s \). This suggests that for every process as \( s \to \infty \), \( f(s,t) \) becomes an entire function of \( \chi \). All the available data on \( f(s,t) \) for high energies and all values of \( |t| \) for the six processes considered here exhibit scaling in the new
$b(s) (\text{GeV}^2)$

$K^- n \rightarrow K^0 \Delta^-$

FIG. 29
FIG. 30

\[ \kappa^- n \rightarrow K^0 \Delta^- \]

- \( \circ \) 4.48 GeV/c
- \( \triangle \) 5.5
- \( \odot \) 8.36
- \( \bullet \) 12.8

\[ \log_{10} f(s,t) \]

\[ \chi(s,t) \]

FIG. 30
variable $X$ in a very remarkable fashion. Although there has been some empirical evidence\textsuperscript{20} that small $|t|$ data on $f(s,t)$ for values of $P_{\text{lab}} > 20.8$ GeV/C scale in a different variable $tb(s)$ and average scaling in a completely different variable has been shown to work\textsuperscript{4} for the processes $\Lambda^{-}p \rightarrow \Lambda^{0}n$ and $\Lambda^{-}p \rightarrow \eta n$, there has been no other evidence of scaling for the other four processes considered here, prior to this work.

It may be argued that the present CPE has been developed for near forward angles and need not work in representing scaling of the data for larger values of $|t|$ away from the peak region. Among the inelastic processes considered here although there are some data points remaining clearly away from the peak region for $5.9 < P_{\text{lab}} < 18.2$ GeV/C for the process $\Lambda^{-}p \rightarrow \Lambda^{0}n$, these data points are not very much away from the peak region as in $pp$ scattering\textsuperscript{28}. Also, for other processes at high energies, no available data exists very much away from the peak region. Future high energy data for still larger values of $|t|$ will decide whether really there exists scaling in that kinematical domain. In any case there is some positive indication for scaling of the data existing outside the peak region, although it is not sufficient. Some heuristic plausibility arguments can be put forward to support why the present representation should be valid also for larger $|t|$ data. Although no possible pole-contribution has been included, the cut structure of the whole amplitude has been taken into account.
by the conformal mapping of the s and \( \cos \theta \) planes.

Thus if the poles do not contribute and the distant parts of the cuts are unimportant the present representation may be approximately valid for larger \(|t|\) data. Further, an alternative explanation might be that by bringing the distant parts of the right- and left-hand cut to the closer vicinity of the image of the physical region in the mapped plane their effects on scattering for larger \(|t|\) values are possibly being taken into account indirectly, by the CPE, in a crude manner.

It is clear from eqns. (3.17) and (3.18) that the mapped variable \( Z_0 \) reduces to the conformal mapping of the right-hand two-pion cut alone in the \( t \) plane for high energies. The energy dependence of \( \mathcal{O}(s) \) has been effectively taken into account by conformal mapping of the left-hand cut. Thus according to the present approach the left-hand cut of the \( s \) plane and the right-hand cut in the \( x \) or \( t \) plane arising out of two-pion exchange in the \( t \)-channel are responsible for scaling in inelastic nondiffractive processes. Similar observation has been made for diffraction scattering processes in Chap. II\(^{43}\).
III. 4 **SUMMARY AND DISCUSSION, LIMITATION AND APPLICATION**

In this section, the results obtained in this chapter will be summarized and discussed. The application of this method in predicting $f(s,t)$ for higher values of $s$ as a function of $|t|$ in the scaling region is pointed out. Some limitations of this method discussed at various stages of development of this chapter are also pointed out.

A. **Summary and discussion of results**

The conformal mapping used in refs. 29, 32, 33, 38 introduces spurious cuts in the mapped planes when it is applied for processes possessing unsymmetrically cut $x$ plane of analyticity. However, in the previous chapter the conformal mapping used for the unsymmetrically cut $x$ plane does not introduce any spurious cut in the mapped plane for diffraction scattering processes, but when combined with that of the $s$ plane, the CPS proposed gives a very good description of the slope-parameter data at high energies. Such a prescription has been found to yield useful information on the asymptotic behaviour of slope parameters and existence of entire functions for the differential cross-section-ratio. It has been also found that the cross-section-ratio data at high energies exhibit scaling in a remarkable manner in the variable $\chi$. The same method is used here to test whether energy dependence of the slope-parameter and scaling of the differential-cross-section-ratio data can be represented for inelastic nondiffractive
processes. Unlike chapter II\textsuperscript{43}, where the domain of analyticity of the absorptive part in the $x$ plane was used, in this chapter\textsuperscript{21} the cut structure of the whole amplitude has been used for conformal mapping. Using Mandelstam analyticity of the $x$ plane along with that of the $s$ plane in a suitable manner a CPE for inelastic processes has been proposed. But the rate of convergence of the series and the nature of the polynomials are not uniquely fixed for finite energies, but vary with energy. However, for asymptotic energies, the polynomials are uniquely the Laguerre polynomials and CPE goes over to OPE. Use of CPE in $X$ has another important advantage that the approach from CPE to OPE is faster in the $X$ plane, if the slope parameter has the asymptotic behaviour of the type $(\ln s)^m$, with $m > 0$. It is argued that scaling of the differential cross-section-ratio, $f(s,t)$, in the variable $X$ may occur at asymptotic energies. At high energies the scaling variable $X$ has the potentialities to reduce to some known scaling variables proposed for elastic diffraction scattering processes\textsuperscript{1,5,6,8} and reproduce Fadde behaviour for values of $|t| \ll t_+$, but for larger values of $|t| \gg t_+$ the variable is $\sim b(s)(\ln t)^2$. The formula developed for the slope parameter has the potentiality of yielding asymptotic behaviours of the type $\sim (\ln s)^m$, $m = 0, 1, 2, \ldots$ and gives a very good account of the slope parameter data at high energies for the six inelastic processes considered here. Asymptotic behaviour of the slope parameter for every process is an entire function of the corresponding
\( \chi \) for \( s \to \infty \). As remarked in Chap. II and earlier in this chapter, it has been possible to obtain information on the existence of the entire function because of the absence of spurious cuts in the \( \chi \) plane.

All the available data on \( f(s,t) \) at high energies for every process exhibit scaling in \( \chi \) in a remarkable manner. Average scaling in a different variable has been suggested and experimental data for \( \Lambda^- p \to \Lambda^0 n \) and \( \Xi^- p \to \eta n \) obey such a scaling law. Empirically data on \( f(s,t) \) for these two processes are also seen to exhibit scaling in the variable \( tb(s) \). But the range of \( |t| \) covered here is higher than that at ref. 20 and the range of \( P_{\text{Lab}} \) used is also higher than that used in refs. 4 and 20 for the process \( \Lambda^- p \to \Lambda^0 n \). The present scaling variable is entirely new in the kinematical region for \( |t| \gg 4m^2_{\Lambda} \) where the large majority of data points are available. In the variable \( \chi \) scaling of the data on \( f(s,t) \) is seen to be started even for \( P_{\text{Lab}} \gg 5.9 \text{ GeV/C} \) for \( \Lambda^- p \to \Lambda^0 n \).

For the larger \( |t| \) data although there is some spread in the scaling curve, the tendency of scaling is clearly exhibited for this process. For \( \Lambda^- p \to \eta n \) all the available data on \( f(s,t) \) for \( P_{\text{Lab}} \gg 20.8 \text{ GeV/C} \) scale in a better manner. For the first time scaling for the other four inelastic processes, \( K^- p \to \bar{K}^0 n, K^+ n \to K^0 p, K^+ p \to K^0 \Delta^+ \) and \( K^- n \to \bar{K}^0 \Delta^- \) has been noted. Although experimental data for the last four processes are not available for such high values of \( P_{\text{Lab}} \) as in the case of the first two, all the
available data show scaling in spite of large experimental errors. Early onset of scaling in the energy scale in the variable $\chi$ is clearly demonstrated by the data for the five processes $\Lambda^- p \rightarrow \Lambda^0 n$, $K^- p \rightarrow \bar{K}^0 n$, $K^+ n \rightarrow K^0 p$, $K^+ p \rightarrow K^0 \Delta^{++}$ and $K^- n \rightarrow K^0 \Delta^-$. Although scaling for $\Lambda^- p \rightarrow \eta n$ has been shown for the data with $p_{\text{Lab}} > 20.8$ GeV/C, it is speculated from the results of other five processes that in this case also scaling starts earlier in the energy scale.

Plotting $f(s,t)$ vs $|t|$ it has been checked that there is much energy dependence in such plots for inelastic processes, especially the energy dependence is very much prominent for $\Lambda^- p \rightarrow \Lambda^0 n$ and $\Lambda^- p \rightarrow \eta n$. But such energy dependence has been removed for almost all inelastic processes when $f(s,t)$ is plotted against $\chi$ for data points residing inside, near and somewhat away from the peak region. As remarked in Sec. 5 of Chap. II, apparently there is not present very much marked energy dependence in the experimental data on $f(s,t)$ for larger $|t|$ region in the $f(s,t)$ vs $|t|$ plot for $\Lambda^- p$ and $K^+ p$ scattering processes. But the fact that the diffraction peak shrinks continuously for these processes shows that the energy dependence exists at least inside the peak region. For $\bar{p} p$ scattering energy dependence is present for the data points inside and outside the peak region at high energies, although such energy dependence is not large. For $\bar{p} p$ and $K^- p$ scattering the energy dependence in the $f(s,t)$ vs $|t|$ plot is insignificant. In spite of
such wide degree of variation in the $f(s,t)$ vs $|t|$ plot for different elastic diffractive and inelastic nondiffractive processes it is remarkable that the same pattern of scaling is observed for all the processes studied in this chapter and Chap. II, when $f(s,t)$ is plotted against $\chi$. Such a universal feature strongly favours our scaling hypothesis.

It has been suggested that the input for understanding the phenomena of scaling observed in elastic diffractive and inelastic nondiffractive processes is likely to be the analyticity. Although scaling law has not been derived by rigorous theory, the demonstration of scaling for elastic processes in ref. 28 and Chap. II and for inelastic processes in the present chapter, where the conformal mapping used does not generate any spurious cut, leads strongly to the belief that analyticity may be the input towards a rigorous understanding of the scaling phenomena.

All the processes for which the technique of CPE has been applied so far and considered in this and earlier chapter, whether diffractive elastic or nondiffractive inelastic, the right-hand two-point cut in the $t$ plane happens to be the nearest right-hand cut which is supposed to have maximum influence on the scattering in the forward hemisphere. The present analysis suggests that this cut and the left-hand cut in the $s$ plane play a crucial role for explaining the observed scaling phenomenon.
For elastic diffraction scattering processes, Comille has defined a class of scaling functions in which are included series in orthogonal polynomials including Laguerre but excluding Hermite polynomials. As in earlier works and the works of Chap. II, the scaling function in the present case is a series in Laguerre polynomials in the variable $\chi$. Scaling functions for various processes can be known by fitting the data in the $f(s,t)$ vs $\chi$ plots of Figs. 20, 22, 24, 28, 28 and 30. Computation of scaling functions and test of convergence of the proposed series for the processes $\Lambda^p \rightarrow \Lambda^q \eta$ and $\Lambda^p \rightarrow \eta \eta$ have been carried out in Chap. V. An alternative way for obtaining the scaling function is to fit the data on $f(s,t)$ for any high value of $s$ and in a sufficiently large range of $|t|$ in the scaling region. Importance of scaling function, relating to the economic use of computer time for data analysis in the context of CPE, has been pointed out earlier. Even without knowing the scaling function, it is possible to predict $f(s,t)$ as a function of $|t|$ for any high value of $s$ in the scaling region. This important application, which will be discussed later in this section, will be attempted in the next chapter.

B. Limitations of the method

Here several limitations of this approach have been discussed, some of which have been pointed out at several stages of development of this chapter. The first limitation
of the proposed representation is that the representation is not strictly analytic since the possible contribution due to poles in the $s$ and $\cos \theta$ planes has not been included. For all the six processes considered here, it may be noted that the nearest singularity closest to the forward segment, $0 \leq x \leq 1$, is the two-pion cut. For every inelastic process considered the possible pole nearer to this segment than the left-hand cut is that arising out of $u$-channel exchange, whose contribution will vanish in the region where scaling is being observed as $s \to \infty$. Obviously possible poles in the $s$ plane have vanishing contribution for $s \to \infty$. Thus the poles cause no problem for scaling for the inelastic processes considered in this chapter at asymptotic energies. But at nonasymptotic energies, even for which the data appear to scale in $\mathcal{X}$, scaling will be spoiled if pole contributions are included explicitly. Further, since the contribution due to the $u$-channel pole has not been explicitly retained, the presence of the image of the pole in the $Z_0$ and the $\mathcal{X}$ planes at finite energies will limit the convergence of polynomial expansion. But as pointed out in Sec. II, the convergence of polynomial expansion in the $\mathcal{X}$ plane at asymptotic energies will not be affected for any of the processes considered here, since $m = 1$ for every process.
The second limitation of this approach is that, neither the rate of convergence nor the nature of the polynomials in the CPE, are uniquely fixed for finite energies. In particular, there is ambiguity in using the Laguerre polynomial expansion for finite energies as in earlier works and Chap. II. But at asymptotic energies, the rate of convergence is maximum, CPE goes over to OPE and the polynomials are uniquely the Laguerre polynomials. Even then, the slope parameter has the same formula for all energies because of the choice of the same exponential weight function for the construction of orthogonal polynomials at finite energies, and the assumption that only the first exponential term is sufficient for the description of the extrapolated peak of the differential cross section in the small \(|t|\) region in a small \(|t|\) interval. The third limitation is that no explanation has been provided as to why there should be scaling for the larger \(|t|\) data outside the forward peak region as observed in the process \(\Lambda^- p \rightarrow \Lambda^0 n\), although the present model has been developed for scattering near forward angles. Although no precise explanation has been furnished for this phenomenon, one of the plausibility arguments may be that the effect of short range forces represented by the distant parts of the cuts are taken into account indirectly in the CPE by means of conformal mapping, which brings the image of the distant parts of the cuts to the closer vicinity of the image of the forward segment \((0 \leq x \leq 1)\) in the mapped plane.
The fourth limitation is attributed to the fact that in all such representations used so far in this thesis scaling has not been derived from rigorous theoretical basis, but rather hypothesized, although the hypothesis is based upon maximal convergence and uniqueness of polynomials in OPE at asymptotic energies. For a correct understanding of scaling a model-independent derivation using analyticity properties is essential.

**C. Prediction of the differential cross-section-ratio**

The method used here and the scaling curves obtained in the $f(s,t)$ vs $\chi$ plots can be used to predict the function $f(s,t)$ for any higher value of $s$ and as a function of $|t|$ in the scaling region. Such predictions are possible because of simplicity of $\chi$ and can be verified from the results of future experiments. Looking to the scaling curve for any process, the value of the ordinate $f(s,t)$ and the corresponding value of $\chi$ can be read from the graph. Since for high energies and values of $|t|$ within which scaling is observed, one can write using (3.4)

$$\omega \approx 1 - \frac{t}{t^+}$$

(3.33)

and using (3.2) and (3.8) one can write

$$\omega \approx (\cosh \left[ \frac{\chi}{\alpha(s)} \right]^\frac{1}{2} )^2$$

(3.34)

one gets using (3.33) and (3.34)
\[ |t| \approx t_+ \left( \sinh \left[ \frac{\chi}{\alpha(s)} \right] \right)^2 \]  

(3.35)

For any of the processes considered, since \( \chi \) has been noted from the scaling graph corresponding to a known value of \( f(s,t) \), \( \alpha(s) \) is known from the formula (3.7b) and the fit to the slope parameter data as reported in this chapter, the value of \(|t|\) corresponding to a given value of \( f(s,t) \) is known from the eqn. (3.35) for any higher value of \( s \) for which experiments have not yet been performed.

In this way, for a high value of \( s \) different values of \( f(s,t) \) can be noted from the scaling curve and the corresponding \(|t|\) values computed. Predictions of \( f(s,t) \) by this method have been reported in the next chapter for the processes \( \wedge^p \rightarrow \wedge^0n \) and \( \wedge^p \rightarrow \gamma n \) for which the data in the \( f(s,t) \) vs \( \chi \) plots are sufficient to draw scaling curves.