REFERENCES


The scaling variable proposed by Orear basing upon empirical observation is $p_1$ which worked well for the larger angle data at high energies up to $P_{lab} = 24$ GeV/C. A variable combining the variables of the Krisch and Orear type was proposed by Narayan and Sama as early as the year 1964: see for instance D.S. Narayan and K.V.L. Sama, Phys. Lett. 5, 335 (1964).

Section III of this paper has been devoted to the claim by the authors that they observe scaling of the differential cross-section-ratio for $\Lambda^-$p $\rightarrow \Lambda^0$n and $\Xi^-$p $\rightarrow \eta$n in the variable $\zeta = tb(s)$. At first the authors compute approximation to the data points on b(s) by approximating the fits given in Refs. 61 and 62 in the form

$$\frac{d\sigma}{dt}(s,t) = \frac{d\sigma}{dt}(s,0) \cdot b(s)t$$

and compute the following formulas for b(s) using the parameters given in Refs. 61 and 62,

$$b(s) = \begin{cases} 5.2 + 1.85 \ln E_{Lab}, & \text{for } \Lambda^-$p $\rightarrow \Lambda^0$n \\ 3.8 + 1.58 \ln E_{Lab}, & \text{for } \Xi^-$p $\rightarrow \eta$n \end{cases}$$
The authors then plot data on $f(s,t)$ against $z$ using (ii) and show scaling. Scaling of $f(s,t)$ in the variable $Z$ is obvious from (i) which is already known to fit the data for small $|t|$ region. Scaling in the variable $Z$ is also true for all the processes for which the data are known to be approximated by the formula (i) for small $|t|$ values. Almost all the diffraction scattering processes and some of the inelastic diffractive or nondiffractive processes are known to exhibit such a behaviour.


When the interior of the figure of convergence of a series expansion in a mapped plane contains only a part of the mapped image of the original cut plane the expansion has been termed as convergent polynomial expansion (CPE), but when it contains the entire image of the original cut plane the expansion is an optimised polynomial expansion (OPE). For the definitions and
use of CPE see also references 21, 29, 33, and 38.

37. E. Barrelet, 0. Chamberlain, S. Schannon, G. Shapiro, and H. Steiner, Phys. Rev. D 15, 2435 (1977); See also G. Hohler and I. Sabba-stefanescu Zetfurf.
The presence of branch points can be easily checked by noting that the transformation (2.9) yields
\[ \frac{du(w)}{dw} = \frac{1}{\left[(1-w^2)(1-k^2w^2)\right]^{1/2}}.\]

Presence of branch points in such conformal transformations has been discussed in textbooks. See, for instance, J. Mathews and R.L. Walker, "Mathematical Methods of Physics" (W.A. Benjamin Inc. New York, 1970) Chp.7, p.206; F. Bowman, "Introduction to Elliptic Functions with Applications" (Dover, New York, 1961).

The expansion in parabolic variable has a distinct advantage over conventional expansion in Legendre polynomials in x and also over expansion in terms of elliptic variable of Cutkosky and Deo (ref. 24) and is especially suitable for high energies. For detailed discussions see refs. 31-33.


50. Let \( \Gamma(s) \) be the length of the physical region for a given \( s \) in the \( Z_0 \) plane and \( \gamma(s) = 2 \alpha \Gamma(s) \). If we know \( \alpha \), the orthogonal polynomials \( \{P_n(x)\} \) occurring in (2.39), with \( x = 2 \alpha Z_0 \) can be constructed by using the orthogonality relation

\[
\frac{1}{2} \int_0^\infty f(t) \exp(-x) P_n(x) P_m(x) \, dx = \delta_{nm}
\]

Since the length of the physical region varies with energy the nature of the polynomials also varies. However, for \( s \to \infty \), \( \Gamma(s) \to \infty \) and \( \{P_n(x)\} \to \{L_n(x)\} \), the Laguerre polynomials in terms of which the expansion (2.39) converges within the entire parabola in the \( Z_0 \) plane.


52. F. Halzen, 'Model independent features of diffraction', Lectures presented at the 1973 Summer Institute on Particle Interactions at Very High Energies (Louvatin, Belgium).

53. For high energies the formula for the slope parameter obtained in ref. 28 is the same as that given by (2.55). As it has been observed in ref. 28 no effective shapes of spectral functions are needed to fit the high energy data on forward slopes for pp scattering with \( s > 35 \) \( \text{GeV}^2 \). Also for large \( s \) and for values of \( |t| \) such that \( 4t^2 + t_R(s) \gg |t| \), which inequality is satisfied by almost all the available data for pp scattering, the mapping variable \( Z \) in (2.2) which has been used in ref. 28.
is very nearly the same as \( Z_0 \). Therefore, the variable \( \chi(s,t) \), as would be obtained using the formula (2.55) and \( Z_0 \), would be very nearly the same as the scaling variable of ref. 28 at high energies. This would yield very nearly the same scaling curves for \( pp \) scattering as those obtained in ref. 28, when the present approach is applied.

54. T. Lassinski et al., Nucl. Phys. B 37, 1 (1972);
   P. Jenni et al., Nucl. Phys. B 129, 232 (1977);
   C.W. Akerlof et al., Phys. Rev. D 14, 2364 (1976);
   D.S. Ayres et al., Phys. Rev. D 15, 3105 (1977);
   L. Ambats et al., Phys. Rev. D 3, 1179 (1974);
   K.J. Foley et al., Phys. Rev. Letters 11, 503 (1963);
   K.J. Foley et al., Phys. Rev. 181, 1775 (1969);
   Y. Antipov et al., Nucl. Phys. B 37, 333 (1973);

In all those cases (e.g. Ayres et al. and Akerlof et al.) where the slope parameter data \( B \) at \( |t| = 0.2 \text{ GeV}^2 \) and the curvature \( C \) have been reported using parameterization of the type

\[
\frac{d\sigma}{dt} = A e^{bt + ct^2}
\]

with

\[
B = b - 2C \quad |t| \quad \text{av.}
\]

the values of \( b \) at \( |t| = 0 \) were computed from the reported data on \( B \) and \( C \) using

\[
b = B + 2C \quad |t| \quad \text{av}
\]


57. From eqns. (2.61) and (2.62)
\[ d_1 \bigg| n^+_{\text{p}} = 0.100, \quad d_1 \bigg| n^-_{\text{p}} = 0.81 \]

But from eqns. (32) and (33) of Ref. 29
\[ d_1 \bigg| n^+_{\text{p}} = 0.1102, \quad d_1 \bigg| n^-_{\text{p}} = 0.0627 \]

Although asymptotically the same rate of growth is indicated from this analysis and from Ref. 29, quantitatively they do not appear to be approaching the same limit in Ref. 29. However, from the present analysis it is clear that both the slopes may approach the same limit quantitatively, if the error in the value of \( d_1 \) is at least 0.01 for both the scattering processes. In view of the large errors of the data for \( s \ll 40 \text{ GeV}^2 \) such an error is likely to occur in the parameters.

The difference between the results of this analysis and that of Ref. 29 arises because of inclusion of data at low energies in Ref. 29 might be obscuring informations on the asymptotic behaviour.

In this paper slope parameter data have not been reported. Taking the differential cross section data of this paper we have fitted them with the formula

$$\frac{d\sigma}{dt} = A e^{bt}$$

and obtained values of b with errors for each value of laboratory momentum in the range 20.8 < P_{Lab} < 199.3 GeV/C as shown in Fig. 19. However, the data on the slope parameter in the range 5.9 < P_{Lab} < 18.2 GeV/C has been reported, using this type of fit, in ref. 60.

62. O.I. Dahl et al., Phys. Rev. Lett. 37, 80 (1976). In this case the same method as mentioned in Ref. 61 was adopted to obtain the slope parameter data as reported in Fig. 21.

64. I. Ambats et al., Nucl. Phys. B 77, 269 (1974);
V. Blousov et al., Phys. Lett. 43 B, 76 (1973);
66. W. Moninger et al., Phys. Rev. D 8, 38 (1973);
These authors have parametrized the differential-cross-section data by using the formula

$$\frac{d\sigma}{dt} = \left( \frac{d\sigma}{dt} \right)_{t=0} (1-get) \exp(ct)$$

The slope parameters for small $|t|$ at different energies have been computed from this formula and the values of the parameters given by the authors.

For an improved value of total $\chi^2$, the errors in the data on $b(s)$ from ref. 60-62, reduced and the total $\chi^2$ minimized yielding the values of the parameters
given by the eqs. (4.7) were obtained. With the original
errors and these values of the parameters $\chi^2$/DOF
values were the calculated.

ibid. 1545 (1979).