Interconnection Network Topology

2.0 Introduction

With the recent advances in VLSI (Very Large Scale Integration) technology the parallel computers have become increasingly popular among scientists and designers. The parallel computers can have two broad categories: loosely coupled and tightly coupled. The loosely coupled parallel computers also called as multicomputers. In recent days the use of multicomputers in various scientific, engineering and general purpose applications has increased to a very great extent. The continuous demand for research in the area of VLSI has enhanced the importance of multicomputers to a great extent [29]. To cope with the voluminous computation, large scale parallel systems needs to be designed to considerably reduce the communication overhead between the processors. So, the design of an efficient interconnection system becomes a vital issue. The interconnecting network must be scalable, efficient, reliable and cost effective. Several aspects of interconnection networks have been investigated in the past [24, 43, 49, 61]. Most of the early literatures deal with their topological and functional properties. With the emergence of higher packing density systems, research on many other aspects of an interconnection system such as performance, fault tolerance, routing and cost effectiveness has become equally important [48, 62, 179].

It is extremely important and also difficult to determine the best network owing to their differences in performance criteria, application domains, operating environments, cost constraints and implementation technologies. The cube based networks are most suitable for any large scale computing system. Due to their splendid interconnection structures with large bandwidth, logarithmic diameter and high degree of fault tolerance, the cube based networks have received much attention over the past few years [61]. Some other important interconnection network topologies such as trees and multidimensional meshes can also be embedded into the cubes [127]. In recent years, several modifications have been proposed to enhance the performance of cube based networks. Extensive research has resulted in several variations of cubes: such as Hypercube (HC) [78,196,111], Folded hypercube (FHC) [11], Crossed cube (CC)
Chapter II Some Studies on Parallel Computer Interconnection Network Topology

[51], Twisted cubes (TC) [52], Cube connected cycles (CCC) [182] and Hierarchical cubic network (HCN) [69]. These new networks are designed to reduce the diameter of the hypercube.

One of the most important properties of Hypercube networks (HC) is that there exists one edge between two nodes if the hamming distance between the binary node address is one. Due to this basic property design of routing and broadcasting algorithms becomes simple. But all the variations of the hypercube topology do not possess this basic property due to alterations in their network structure. Also there is a demand for increasing the packing density of the interconnection networks. In case of hypercube with higher packing density the link complexity also increases.

In spite of its admitted advantages and merits, the Hypercube and its variants have a common drawback that is the packing density which must be a power of two. Some researchers have suggested a few improvements to overcome this drawback [191,222], but their approaches still suffer from certain disadvantages. For example, the Incomplete Hypercube (IH)[107] which is an important variation of the Hypercube suffers from the problem of fault tolerance because failure of a single node can cause the entire network to be disconnected. The Supercube [15,144, 198] becomes irregular at higher packing density. The routing in the Crossed cube is quite complicated for higher dimensions.

The Dual cube and the Meta cube are two new networks which have very high packing density [131-137]. These networks can connect to millions of nodes with a small node degree. These networks possess the basic property that there exist an edge between two nodes with hamming distance one. Thus the routing can be simple similar to hypercube. However these networks are very large scale networks and therefore their diameter need to be reduced so that the cost of message passing can be reduced to some extent. In communication intensive applications the prediction of message traffic becomes a bottleneck for the parallel computer interconnection networks [29, 62, 72, 173]. The multicomputer systems, which are also termed as loosely coupled parallel systems involve intensive interprocessor communications. As a result the communication overhead increases to a large extent. For this reason, there is always a demand for reducing the diameter, the average distance and hence the message density of a network. Therefore there arises a need to further improve the degree, diameter, average node distance, message traffic density, as a result the interconnection networks can provide faster inter process communication. This can be achieved by adopting the principle of folding to the large scale networks [11].
In this chapter three new parallel computer interconnection network topologies are proposed to overcome the above said limitations of the existing networks. The three new network topologies proposed here are based on the cubic structures. By the application folding technique to the networks further reduction of diameter and traffic congestion becomes possible. The three new topologies proposed in this chapter are: Folded crossed cube, Folded dual cube and Folded meta cube.

This chapter is organized as follows: The Section 2.1 provides the necessary background and discussions. Next three new parallel interconnection topologies are proposed using the concept of folding in subsequent sections. The results and discussions are presented in Section 2.5. Finally the Section 2.6 presents the concluding remarks.

2.1 Background

This current section describes the basic topological structures of some of the existing cube-based networks. The networks considered are: Hypercube HC [196], Crossed cube [51], Dual cube [131] and the Metacube [132]. Without any loss of generality, the terms network and graph has been used interchangeably throughout this Chapter and the Thesis. The different notations used in this Chapter are presented below in the next subsection.

2.1.1 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Connection between a pair of nodes</td>
</tr>
<tr>
<td>D(G)</td>
<td>Diameter of network G</td>
</tr>
<tr>
<td>(\bar{d})</td>
<td>Average inter node distance</td>
</tr>
<tr>
<td>(d_g)</td>
<td>Node degree</td>
</tr>
<tr>
<td>E</td>
<td>Total number of edges</td>
</tr>
<tr>
<td>(f_d)</td>
<td>Fault diameter</td>
</tr>
<tr>
<td>(p)</td>
<td>Total number of nodes</td>
</tr>
<tr>
<td>(R_p)</td>
<td>Reliability of processor</td>
</tr>
<tr>
<td>(R_l)</td>
<td>Reliability of link</td>
</tr>
<tr>
<td>(s)</td>
<td>Source node</td>
</tr>
<tr>
<td>TR or (R(t))</td>
<td>Terminal reliability</td>
</tr>
<tr>
<td>(t)</td>
<td>Destination</td>
</tr>
<tr>
<td>(h)</td>
<td>Message traffic density</td>
</tr>
</tbody>
</table>
Chapter II Some Studies on Parallel Computer Interconnection Network Topology

\[ \lambda \] Link failure rate
\[ \xi \] Cost of the network
\[ \rho \] Ratio of Link cost to processor cost
\[ \sigma \] Ratio of cost of penalty to processor cost
\[ \gamma_a \] Mean inter node distance rate
\[ \gamma_r \] Relative inter node distance rate

The basics of Hypercube, Crossed cube, Dual cube and Meta cube are described in the subsections 2.1.2, 2.1.3, 2.1.4 and 2.1.5 respectively.

2.1.2 Hypercube

The Hypercube is the most popular loosely coupled multicomputer system. It is based on the binary \( n \)-cube network topology [8]. It consists of \( 2^n \) number of parallel processors and each processor is interconnected with \( n \) processors. Thus, both the degree and diameter of the Hypercube network are \( n \). Each of the nodes of the hypercube network has \( n \) bit binary addresses. There exists one edge between two nodes if their node addresses differ by exactly one bit. In other words two nodes are adjacent if the hamming distance between them is one. The Hypercube of dimension three and four are shown in Fig.2.1 (a) and (b) respectively for the purpose of illustration. The \( n \) dimensional Hypercube can be splitted into two \((n-1)\) dimensional Hypercubes.

![Hypercube of dimension 3 and 4](image)

Figure 2.1: Hypercube of dimension 3 and 4 (a) HC3 and (b) HC4
In spite of its admitted advantages, the HC suffers from certain major limitations which are improved in subsequent research papers [191, 222]. As the size of the hypercube multicomputer grows the probability of node and/or link failures become high. In HC, the network size is such quantized that, for adding a single node to the network the size must be duplicated.

2.1.3 Crossed cube

The n-dimensional crossed cube (CCn), is an n dimensional regular graph of $2^n$ nodes [51]. Every node in CCn is identified by a unique binary string of length n. The works reported in [51] define the CCn as follows:

**Definition 2.1:** Two binary strings $X=X_0X_1 \ldots X_n$ and $Y=Y_0Y_1 \ldots Y_n$ of length two are said to be pair related if and only if $xy \in \{(00,00), (10,10), (01,11), (11,01)\}$.

**Definition 2.2:** The n-dimensional crossed cube CCn is recursively defined as follows.

- CC1 is a complete graph on two vertices with labels 0 and 1. For $n>1$, the CCn contains CCn−1 and CCn−1 as sub graphs joined according to the following rule: the vertex $u=0u_{n-2} \ldots u_0$ from CCn−1 and the vertex $v=lv_{n-1} \ldots v_0$ from CCn−1 are adjacent in CCn if and only if
  
  (i) $u_{n-2}=v_{n-2}$ if $n$ is even, and
  
  (ii) for $0 \leq i \leq \lfloor (n - 1)/2 \rfloor$, $u_{2i}v_{2i} \sim v_{2i+1}v_{2i}$

Every vertex in CCn with a leading 0 bit has exactly one neighbour with a leading 1 bit and vice versa. The network structure of crossed cubes of dimension 3 and 4 are depicted in Fig.2.2 (a) and (b) respectively. In the Figure the node addresses are specified using decimal digits for simplicity.
Due to the improved diameter the crossed cube has gained popularity over the Hypercube and a lot of research is being done relating to its properties [53], routing [27], embedding of cycle, tree, mesh [9, 47, 247], Hamiltonicity [93-95], Panconnectivity [55].

2.1.4 Dual cube

The Dual cube topology uses Hypercube as its basic platform [131]. In a Dual cube there are two classes and each class consists of $2^m$ clusters. So, the total number of nodes is $2^m \times 2^m \times 2 = 2^{2m+1}$. Each node in a Dual cube has $(m+1)$ links. The $m$ links are used within the cluster to construct an $m$-cube and the single link is used to connect a node in the cluster of the other class. There is no link between the clusters of the same class. The Dual cube DC(1,2) is shown in Fig. 2.3.

As compared to the hypercube, the Dual cube contains more nodes at the same node degree. At degree three it contains 32 numbers of nodes whereas the hypercube contains only 8 nodes. Though the dual cube is a large scale network, but the routing is simple and uses hamming distance to find the shortest path. The issues regarding communication and fault tolerance of Dual cube are discussed in [136]. The Dual cube is extended and generalized to design the Metacube network [135]. In the following subsection, the detail structure of the Metacube network is presented.
2.1.5 Meta cube

The Metacube network (MC) is an extension of the Dual cube network. It has a two layered structure [132]. The lower level network is a cluster of cubes and in the higher level the clusters are connected in cube fashion. The Metacube network topology has two basic parameters $k$ and $m$. The Metacube network $MC(k,m)$ can be modeled as a graph $G_r(V,E)$ where $V$ represents the set of nodes and $E$ represents the set of edges. A detailed description of this network is found in [135] and is described as follows:

$$|V| = 2^{mh+k},$$

and

$$|E| = r2^{mh+k-1},$$

where $h = 2^k$ and $r = m+k$.

Each node $u \in V$, $a(u)$ denotes the $(mh+k)$ bit binary address which is divided into three parts: a $k$-bit class address, an $(m(h-1))$ bit cluster address and $m$-bit node address. So, the $G_r$ contains $2^k$ number of classes. Each class contains $2^{mh-1}$ clusters and each cluster contains $2^m$ number of nodes. Next, $||a(u)||$ denotes the number of 1's that is the hamming weight in the binary bit representation of $a(u)$. Then $e=(u,v) \in E$, iff $||a(u) \oplus a(v)|| = 1$, where $\oplus$ is the

---

Figure 2.3: Dual cube of dimension 3, DC(1,2)
Figure 2.4: The Metacube of dimension 4, MC(2,2)

XOR operator giving the hamming distance between nodes u and v. So degree(u) = r for every node \( u \in V \). The general topology of MC network of dimension 4 is shown in Fig. 2.4. In MC(2,2), each cluster is a 2-cube contained in 4 classes. The clusters in a square belong to a single class and clusters of same class are not connected.
The following two sections propose three new parallel interconnection network topologies. It also gives a detailed account of their properties and performance analysis. In section 2.2 a new topology called Folded crossed cube is proposed. Two more interconnection topologies Folded dual cube and Folded meta cube are proposed in Sections 2.3 and 2.4 respectively.

2.2 Proposed Topology: Folded Crossed Cube (FCC)

The Folded crossed cube is derived from the Crossed cube by connecting each node to a node farthest from it. The Figure 2.5 depicts the structure of the Folded Crossed cube of dimension 3. The folded crossed cube is a graph \( FCC(n) \) with the same set of vertices as in \( CC_n \) and with the edge set \( E' \) that is a super set of \( E \). Here

\[
E' = |E| + \frac{(\text{Total no of nodes})}{2} = n.2^{2n-1} + 2^{n/2} = (n+1)2^{2n-2}
\]

Now \( CC_n \) is a spanning sub graph of \( FCC(n) \) as the node set of \( CC_n \) is exactly equal to that of FCC as well as the edge set \( E \). Then \( e(u,v) \in E' \), if \( u \) and \( v \) are pair related. Also the hamming distance between \( u \) and \( v \) is either 1 or \( n \) that is, \( \|a(u) \oplus a(v)\| = 1 \) or \( n \). So every vertex in FCC with a leading 0 bit has exactly one neighbour with a leading 1 bit and vice versa.

\[
\text{Figure 2.5: Folded crossed cube } FCC_n, n=3
\]

2.2.1 Topological Properties of FCC

The topological properties of the FCC network are described here.

**Lemma 2.1:** For all \( n \geq 1 \), \((u_{n-1}..u_0, v_{n-1}..v_0)\) is an edge in FCC if and only if there exist an \( l \) with

(i) \( u_{n-1}..u_l=v_{n-1}..v_l \).
(ii) \( u_{i+1} \neq v_{i+1} \)

(iii) \( u \) and \( v \) are pair related Or \( u_{n-1} \ldots u_0 \neq v_{n-1} \ldots v_0 \) that is all bits are complemented.

So \( \forall i, v_i = u_i' \), \( \ldots v_{n-1} = u_{n-1}' \)

**Lemma 2.2:** Let \( (u,v) \) be an edge in FCC. When \( u \) and \( v \) have a left most differing bit at position \( d \) then \( v \) is said to be \( d \) neighbor of \( u \) and edge \( (u,v) \) is an edge of dimension \( d \). Here the farthest node is an adjacent node by complementary link so it will be the \((d+1)th\) dimension edge.

**Node Connectivity:** Network connectivity measures the resiliency of a network and its ability to continue operation despite disabled components i.e. connectivity is the minimum number of nodes or links that must fail to partition the network into two or more disjoint networks. The larger the connectivity of a network the better the ability of the network to cope with failures.

**Theorem 2.1:** The node connectivity of the \( n \) dimensional Folded crossed cube (FCC) is \((n+1)\).

\[ \text{Proof:} \quad \text{Every node with } n \text{ bit address } a(u) \text{ in FCC is connected to } n \text{ nodes at hamming distance } 1 \text{ and one node at hamming distance } n. \text{ The address of the latter node is } a(u)' \text{ that is complement of all bits in } a(u). \text{ So, the degree of FCC is } d_g(u) = n+1 \text{ and FCC is a regular graph of degree } (n+1). \]

**Theorem 2.2:** The number of node disjoint paths between any two nodes of FCC is \((n+1)\).

\[ \text{Proof:} \quad \text{Since every node has } (n+1) \text{ neighbours so it is necessary to remove at least } (n+1) \text{ nodes to disconnect FCC. Hence the result.} \]

**Diameter:** It is defined as the maximum distance between the nodes of the network.

**Theorem 2.3:** The Diameter of the Folded crossed cube (FCC) is \( D(G) = \left\lfloor \frac{n}{2} \right\rfloor \).

\[ \text{Proof:} \quad \text{Consider any two nodes } u,v \in V, \text{ in the node set of the FCC with } ||a(u) \oplus a(v)|| = \left\lfloor \frac{n}{2} \right\rfloor + i, \text{ where } \left\lfloor \frac{n}{2} \right\rfloor < i \leq \left\lfloor \frac{n}{2} \right\rfloor. \text{ Both the nodes } u \text{ and } v \text{ can communicate in at most } \left\lfloor \frac{n}{2} \right\rfloor \text{ hops by correcting the differing bits in their } n \text{ bit node address one at a time. When the hamming distance is less than } n, \text{ a path can always be established between } u \text{ and } v \text{ by using the complementary edge which connects } u \text{ to its farthest node } u'. \text{ The hamming distance between } u' \text{ and } v \text{ is clearly } \left\lfloor \frac{n}{2} \right\rfloor - i. \text{ Therefore the distance } D(u,v) = 1 + \left\lfloor \frac{n}{2} \right\rfloor - i. \text{ Hence the diameter is } \left\lfloor \frac{n}{2} \right\rfloor. \]
**Bisection Width:** It is defined as the number of edges whose removal will result in two distinct sub networks.

**Theorem 2.4:** The bisection width of FCC($n$) is $2^n$.  

(2.4)  

*Proof:* From the construction of FCC($n$), it is clear that two identical $(n-1)$-dimensional crossed cubes $CC^0_{n-1}$ and $CC^1_{n-1}$ are connected by dimension $(n-1)$ edges of $CC_n$. So to separate these two sub graphs we need to remove $2^{n-1}$ number of edges. Next, number of edges called complementary links are introduced to build FCC($n$). So, now removal of $2^{n-1}$ and $\frac{|V|}{2}$ number of edges will disconnect FCC($n$) into two equal halves. So the bisection width becomes $2^{n-1} + 2^{n-1} = 2^n$ as $|V| = 2^n$.

**Cost:** For a symmetric network the cost factor is defined as the product of the diameter and the degree of the node. This factor is widely used in performance evaluation.

**Theorem 2.5:** Cost of FCC($n$) is $\xi = (n + 1) \left( \frac{n!}{\left(\frac{n}{2}\right)!} \right)$.  

(2.5)  

*Proof:* The Cost = degree x diameter.  

The degree of FCC is $(n+1)$ and the diameter is $\left\lceil \frac{n}{2} \right\rceil$. Hence, the cost is given by  

$$\xi = (n + 1) \left( \frac{n!}{\left(\frac{n}{2}\right)!} \right).$$

**Mean Inter node Distance:** The mean inter node distance ($d$) in a regular network is defined as the ratio of the sum of distances between a node and all other nodes to the total number of nodes.

The mean inter node distance for FCC($n$) is derived as follows:

**Lemma 2.3:** The number of nodes at distance $i$ from any node in FCC($n$) is $\left( \binom{n+1}{i} \right)$, for $0 < i < \left\lceil \frac{n}{2} \right\rceil$. For $i = \left\lceil \frac{n}{2} \right\rceil$, this is $\left( \frac{n+1}{2} + 1 \right)$, for even $n$ and is $\left( \frac{n}{2} \right)$ for odd $n$.

*Proof:* For $0 < i < \left\lceil \frac{n}{2} \right\rceil$ and any node $u$ in FCC, there are $\left( \binom{n}{i} \right)$ nodes at Hamming distance $i$ from $u$ according to hypercube properties. Next, $u$ is connected to the node $u'$ by the complementary link. Then there exist $\left( \binom{n}{i-1} \right)$ nodes at distance $(i-1)$ from $u'$. The distance of such nodes from $u$ via $u'$ is $i+l = i$. Then the number of nodes is given by
Chapter II: Some Studies on Parallel Computer Interconnection Network Topology

For there are two cases:

Case (i) When $n$ is even

$$|Z_{n/2}| = \binom{n}{n/2} + \binom{n}{n/2 + 1} = \left(\frac{n+1}{2}\right)$$

Case (ii) When $n$ is odd then

$$|Z_{[n/2]}| = \left(\frac{n}{[n/2]}\right)$$

**Theorem 2.6:** The mean inter node distance $\bar{d}$ of FCC($n$) is given by $\bar{d} = \left(\frac{\sum |Z_i|}{p}\right)$ (2.6)

**Proof:** In FCC network, the number of nodes at a distance $i$ from a given node is given by $Z_i$ will be calculated as discussed in Lemma 2.3. The total number of nodes is given by $p = 2^n$.

So the average distance is given by $\bar{d} = \left(\frac{\sum |Z_i|}{p}\right)$

**Message Traffic Density ($\eta$):** This factor is defined as $\eta = \frac{\bar{d}}{E}$, where $p$ is the total number of nodes $\bar{d}$ is the average node distance and $E$ is the total number of links. It is assumed that each node is sending one message to a node at distance $d$ on the average. $\eta$ is a good measure to estimate the traffic in the network.

**Theorem 2.7:** In the FCC network, the message traffic density is given by $2 \frac{(\sum |Z_i|)}{n+1}$, where $Z_i$ is the number of nodes at distance $i$.

**Proof:** As per the definitions, $\eta = \frac{\bar{d}}{E}$

For FCC$_n$, the mean inter node distance is $\bar{d} = \left(\frac{\sum |Z_i|}{p}\right)$ and $E = (n+1)p/2$.

So, the message traffic density $\eta = \frac{\sum |Z_i|}{n+1}$

**Mean Inter node Distance Rate:**

The absolute mean inter node distance rate is denoted by $\gamma_a$ and similarly the relative one is denoted by $\gamma_r$ for any network $X$ is defined as

$$\gamma_a(X) = \frac{\bar{d}(X)}{\bar{d}(HC_n)}$$ and $\gamma_r = \frac{\bar{d}(HC_n) - \bar{d}(X)}{\bar{d}(HC_n)}$

For the FCC network these two parameters are derived as follows:

$$\gamma_a(FCC) = \frac{\sum_{i=1}^{n} |Z_i| / (2^{n-1})}{n^{3/2}(2^{n-1})}$$
Chapter II

Some Studies on Parallel Computer Interconnection Network Topology

\[ \gamma_r(\text{FCC}) = \frac{n z_r - \Sigma_{i=1}^{n-1} |Z_i|}{2(2^n-1) 2^{n-1}} \], where \( Z_i \) denotes the number of nodes at distance \( i \).

The Table 2.1 shows the comparison of topological properties of FCC with the other networks.

<table>
<thead>
<tr>
<th>Networks</th>
<th>Degree</th>
<th>Diameter</th>
<th>Cost</th>
<th>Bisection width</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC</td>
<td>( n )</td>
<td>( n )</td>
<td>( n^2 )</td>
<td>( 2^n )</td>
</tr>
<tr>
<td>CC</td>
<td>( n )</td>
<td>( \left\lfloor \frac{n+1}{2} \right\rfloor )</td>
<td>( n \left\lfloor \frac{n+1}{2} \right\rfloor )</td>
<td>( 2^n )</td>
</tr>
<tr>
<td>FCC</td>
<td>( n+1 )</td>
<td>( \left\lfloor \frac{n}{2} \right\rfloor )</td>
<td>( \left\lfloor \frac{n}{2} \right\rfloor (n+1) )</td>
<td>( 2^n )</td>
</tr>
</tbody>
</table>

2.2.2 Routing and Broadcasting in FCC

This sub-section proposes a routing algorithm for the FCC(n) network. In the cube based networks, the routing process depends upon the shortest path, the Hamming distance. In FCC, the hamming distance is 1 or \( n \). An algorithm for one-to-one communication in FCC is proposed below.

This algorithm performs the routing between any pair of nodes namely \( u(u_r, u_{r-1}, \ldots, u_0) \), \( v(v_r, v_{r-1}, \ldots, v_0) \) \( \in V \) of FCC(n).

**One-to-one Routing:**

Algorithm One-to-one \((a(u), a(v), r)\)

Begin

\[ a(w) = (a(u) \oplus a(v)) \]

If \( ||a(w)|| = \left\lfloor \frac{n}{2} \right\rfloor \)

Route the message sent from \( u \) via a path composed of links with labels corresponding to bit position which are 1's in \( a(w) \)

Else

send the message to \( u' \) via the complementary link, route the message via a path composed of links with labels corresponding to bit positions that are 0's in \( a(w) \).

end;

Here, \( a(w) = (w_r, w_{r-1}, \ldots, w_0) \). So, \( ||a(w)|| = r \) and bits \( w_i \) are all 1. If \( r \leq \left\lfloor \frac{n}{2} \right\rfloor \), then the path formed between \( u \) and \( v \) will be composed of any one of \( r! \) permutations of \( w_i \)'s. If \( r > \left\lfloor \frac{n}{2} \right\rfloor \),
then a complementary edge must be used somewhere along the path. So, the length of the shortest path in the Folded Crossed cube of dimension \( n \) is at most \( \lceil \frac{n}{2} \rceil \).

**One-to-all Broadcast:**

Generally in the cube based networks, the problem of broadcasting is solved by constructing a spanning tree using the nodes of the network. From the source node all other neighbours are determined using the ‘XOR’ operation on the binary bits of the corresponding node addresses. In the proposed FCC network, for the one-to-all broadcast it is required to construct a spanning tree from any arbitrary node. Due to folding in FCC network, the adjacent neighbours are chosen with hamming distance either \( l \) or \( n \).

Let \( s \) be the source node in \( n \)-dimensional FCC with node address \( (s_{n-1}s_{n-2} \ldots s_0) \) in binary. The node \( s \) can have \((n+1)\) neighbours out of which \( n \) neighbours are at distance 1 and one neighbour at distance \( n \). Then the spanning tree rooted at node \( s \) is constructed as follows:

**Procedure One-to-All broadcast** \((s, u, n)\)

\[
\text{Begin} \\
\text{For } j = 0 \text{ to } 2^n - 1 \text{ begin} \\
\text{If } u \oplus s = n \text{ then } s \text{ sends message to } u(s'_{n-1}s'_{n-2} \ldots s'_0) \text{ endif} \\
\text{Elseif } u \oplus s = 1 \text{ then} \\
\text{For } i = 0 \text{ to } n-1 \\
\text{For } \text{send message to } u(s_{n-1}s_{n-2} \ldots s_i, s_0) \text{ endif} \\
\text{End} \\
\text{End}
\]

**2.2.3 Performance Analysis of FCC**

Among the various measures of performance of interconnection systems, the fault tolerance, fault diameter, cost effectiveness, time cost effectiveness and reliability appear to be the prominent ones. The following discussion concentrates on those aspects of FCC.

**Fault Tolerance:**

In the parallel computing environment fault tolerance of a network is an important characteristic. It can be defined as the maximum number of vertices that can be removed from it provided that the graph is still connected. Hence the fault tolerance of a graph is...
Chapter II Some Studies on Parallel Computer Interconnection Network Topology

defined to be one less than its connectivity. A system is said to be \( k \)-fault tolerant if it can sustain up to \( k \) number of edge faults without disturbing the network [9].

For a symmetric interconnection network, the connectivity is equal to the node degree. For FCC, the node degree is \((n+1)\). So FCC can tolerate up to \( n \) faults.

**Fault Diameter** (\( f_d \)):

Fault diameter estimates the impact on diameter when fault occurs, that is removal of nodes from the network [10]. The fault diameter \( f_d \) of the graph \( G \) with fault tolerance \( f \) is defined as the maximum diameter of the graph obtained from \( G \) by deleting at most \( f \) vertices. For better functioning the fault diameter should be close to the original diameter.

**Theorem 2.8:** The fault diameter of FCC\((n)\) is given by \( f_d = \left\lfloor \frac{n}{2} \right\rfloor + 1 \). \hspace{1cm} (2.8)

**Proof:** In FCC\((n)\), a message originating at any node can travel through \((n+1)\) paths. In case a link failure occurs, then the message travels through one more node. This results in increase in diameter by unity.

So, the diameter of the fault network \( = \) original diameter \( +1 = \left\lfloor \frac{n}{2} \right\rfloor + 1 \).

**Cost Effectiveness Factor (CEF):**

While calculating the cost of a parallel interconnection network, the cost of the processing elements along with the cost of the communication links is considered [197]. In cube based networks, the number of links is a function of the number of processors. The cost effectiveness factor takes this into account and gives more insight to the performance of the system.

**Theorem 2.9:** The cost effectiveness factor of FCC\((n)\) is \( CEF(n) = \frac{1}{1+\rho \left( \frac{n+1}{2} \right)} ; \hspace{1cm} (2.9) \)

where \( \rho \) is the ratio of link to processor cost.

**Proof:** The total number of processors in FCC is \( p = 2^n \). The total number of edges is

\[
E = (n+1) \cdot 2^{n-1}
\]

The total number of edges can be expressed as a function of total number of nodes. Thus,

\[
E = (n+1)p/2 = f(p)
\]

So, \( CEF(n) = \frac{1}{1+\rho \left( \frac{n+1}{2} \right)} \)

Hence, \( CEF(n) = \frac{1}{1+\rho \left( \frac{n+1}{2} \right)} \)
Time Cost Effectiveness Factor (TCEF):

The Time cost effectiveness factor considers the time for solution of a problem as a parameter for evaluating the performance. This factor considers the situations where a faster solution is more rewarding than the slower solutions. When speedup of parallel algorithms is known, the above two factors characterize the profitable use of parallel systems.

For FCC(n), the TCEF is decided by

\[ TCEF(p,T_p) = \frac{1 + \sigma T_1^{a-1}}{1 + \rho g(p) + \frac{T_1^{a-1} \sigma}{p}} \]

Where \( T_1 \) is the time required to solve the problem by a single processor using the fastest sequential algorithm, \( T_p \) is the time required to solve the problem by a parallel algorithm using a multiprocessor system having \( p \) processors and \( \rho \) is the cost of penalty / cost of processors.

**Theorem 2.10:** The TCEF of the FCC network is given by

\[ TCEF(p,T_p) = \frac{1 + \sigma}{1 + \rho (\frac{n+1}{2})^2 + \frac{\sigma}{2}}. \quad (2.10) \]

**Proof:** The total number of processors in FCC is \( p = 2^n \) and the total number of edges is

\[ E = (n+1) 2^{n-1} \]

From Theorem 2.9,

\[ g(p) = \frac{(n+1)}{2} \]

Hence, the Time cost effectiveness factor of FCC is given by

\[ TCEF(p,FCC) = \frac{1 + \sigma T_1^{a-1}}{1 + \rho g(p) + \frac{T_1^{a-1} \sigma}{p}} \]

\[ = \frac{1 + \sigma}{1 + \rho (\frac{n+1}{2})^{2} + \frac{\sigma}{2}} \]

For linear time penalty \( \alpha \) is chosen to be 1 and thus, the TCEF completely depends on the number of processors.

Reliability Analysis in FCC:

The reliability measures of particular interest are the: Terminal Reliability (TR) and Broadcast Reliability [262]. Terminal reliability is generally used as a measure of the robustness of a communication network, is the probability of the existence of at least one fault free path between a given source and destination nodes. For the purpose of analysis, any two nodes are considered. It has been already shown in Theorem 2.2 that the number of node disjoint paths lying between them is \( (n+1) \). Let \( r_i \) be the number of links involved in path \( i \), where \( 1 \leq i \leq n \). Thus, there are \( r_i - 1 \) number of nodes in path \( i \).

Let \( P(E_i) \) be the probability of successful route through the \( i^{th} \) path.
Then $R_l$ be the link reliability with link failure rate is 0.0001 and $R_p$ is the node reliability with processor failure rate is 0.001.

$$R_l = e^{-\lambda t}, \text{ where the constant failure rate } \lambda = 0.0001 \text{ and mission time } t=1000 \text{ and }$$

$$R_p = e^{-\lambda t}, \text{ where } \lambda = 0.001 \text{ and } t=1000.$$  

**Theorem 2.11:** For FCC network the two terminal reliability (TR) is given by

$$TR = 1 - \prod_{i=1}^{n}(1 - R_l^i R_p^{r_i-1}) \quad (2.11)$$

**Proof:** All nodes and links are considered to be identical with their failure rates statistically independent and exponentially distributed.

Now the probability of existence of a successful connection (C) between the source and destination and hence the Terminal Reliability can be given as

$$P(C) = R_l^1 R_p^{r_1-1}$$

So

$$TR = P(C_1 \cup C_2 \cup C_3 \ldots \cup C_n)$$

$$= 1 - \prod_{i=1}^{n+1}(1 - R_l^i R_p^{r_i-1})$$

The following two sections are devoted towards the development of new folded cube based topologies: (i) Folded Dual cube and (ii) Folded Meta cube.

### 2.3 Proposed Topology: Folded Dual Cube (FDC)

The Folded dual cube (FDC) is constructed from the Dual cube DC(1,m) by connecting each node to another node which is farthest from it. The FDC(1,2) is shown in Fig.2.6. The links are represented by solid lines and the complementary links are shown as dashed lines in the Fig. 2.6.

#### 2.3.1 Construction

The Folded dual cube is a two layered structure. Here $r = m + 1$. The Folded dual cube is a graph FDC $(V, E')$ as shown in Fig. 2.6, with the same set of vertices as in DC and with the edge set $E'$ that is a super set of $E$, the edge set of DC. Hence
Chapter II

Some Studies on Parallel Computer Interconnection Network Topology

Figure 2.6: Folded dual cube, FDC(1,2)

\[ E' = |E| + \left( \text{Total no of nodes} \right) / 2 \]
\[ = r \cdot 2^{2r-2} + 2^{2r-1} / 2 \]
\[ = (r+1) \cdot 2^{2r-2} \]

Now DC is a spanning sub graph of FDC and \( e(u, v) \in E' \) iff \( \| a(u) \oplus a(v) \| = 1 \) or \( 2r-1 \) where \( \oplus \) is the XOR operator giving the hamming distance.

In the next sub section the different topological properties of the FDC are derived.

2.3.2 Topological Properties of FDC

**Theorem 2.12:** The node connectivity of FDC is \( (r+1) \).

**Proof:** Every node with \( (2r-1) \) bit address \( a(u) \) in FDC is connected to \( r \) nodes at hamming distance 1 and one node at hamming distance \( 2r-1 \). So degree of FDC is \( d_g(u) = r+1 \) and FDC is a regular graph of degree \( (r+1) \).

**Theorem 2.13:** The number of node disjoint paths between any two nodes of FDC is \( (r+1) \).

**Proof:** Since every node has \( (r+1) \) neighbours so it is necessary to remove at least \( (r+1) \) nodes to disconnect FDC.
Chapter II  
Some Studies on Parallel Computer Interconnection Network Topology

**Theorem 2.14:** The diameter of FDC is \( D(FDC) = 2r - 2 \).

**Proof:** As discussed in [131], suppose \( s \in V \), the node set of FDC differ in \( q \) bit positions. Then in Folded dual cube, if \( s \) and \( t \) are in same cluster, then \( d'(s, t) = \min(q, (2r-1)-q+1) \). If \( s \) and \( t \) are of different class, then \( d'(s, t) = \min(q, (2r-1)-q+1) \). If \( s \) and \( t \) are in different clusters of same class, then \( d'(s, t) = \min(q+2, (2r-1)-q+1) \), when \( q = 2r - 2 \), maximum distance in DC is \( 2r \), but maximum distance in FDC, is \( (2r-2) \). Hence, the result.

**Theorem 2.15:** Bisection width of FDC is \( 2^{2r - 1}/2 \).

**Proof:** For a Dual cube the bisection width is \( 2^{2r - 1}/4 \). In Folded dual cube number of augmented edges = \( 2^{2r - 1}/2 = 2^{2r - 2} \). So, bisection width is \( 2^{2r - 1}/4 + 2^{2r - 1}/4 = 2^{2r - 1}/2 \).

As the Dual cube \( DC(k, m) \) is a special case of the Meta cube network with \( k=1 \) and \( m=2,3,4 \) and so on. The network is generalized for all values of \( k \) and \( m \) and the results are extended to suggest the Meta cube [135]. In a similar fashion the Folded dual cube is generalized and the parameters derived above are extended to propose the Folded Meta cube topology for large scale parallel system in the next section.

### 2.4 Proposed Topology: Folded Metacube Network (FMC)

The proposed topology Folded Metacube, \( FMC(k, m) \) is derived by folding the existing topology of Meta cube \( MC(V, E) \) as shown in Fig.2.4, by connecting each node to a node farthest from it similar to FDC as shown in Fig.2.7. Here \( r = m+k \). The FMC network contains \( 2^k \) classes and each class contains \( 2^m(2^k-1) \) clusters and each cluster is a cube of dimension ‘\( m \)’ then

\[
|V| = 2^{mh+k}, \quad |E| = r 2^{mh+k+1}, \quad \text{where} \quad h = 2^k.
\]

The Folded Metacube can be represented as a graph with \( (V, E') \) as shown in Fig. 2.8, with the same set of vertices as in MC and with the edge set \( E' \) that is a super set of \( E \). In the figure some of the complementary links are shown for better clarity. Now

\[
E' = |E| + \frac{\text{(Total no of nodes)}}{2} = r 2^{mh+k+1} + \frac{|V|/2}{r+1} 2^{mh+k+1}.
\]

The MC is a spanning sub graph of FMC and \( e(u, v) \in E' \), iff \( ||a(u) \oplus a(v)|| = 1 \) or \( (mh+k) \) where \( \oplus \) is the XOR operator giving the hamming distance between the nodes \( u \) and \( v \). Since, each node is connected to its farthest node using the dotted links, \( ||a(u) \oplus a(v)|| = mh+k \). In other words every node \( u \) in FMC is connected to \( r \) nodes with hamming
distance 1 and one node at hamming distance (mh+k). However in the proposed network the clusters of the same class are not connected.

The following sub section presents the topological properties of the FMC(k,m) network.

2.4.1 Topological Properties of FMC

This section describes the various important topological properties of the Folded Metacube network FMC(k,m).

**Theorem 2.16:** The degree of FMC is \((r+1)\), where \(r = k+m\). \(\quad (2.14)\)

**Proof:** In the MC network the node degree is \(r\), where \(r = k+m\). But in FMC, as new complementary edges are introduced to connect each node to its farthest node, so the node degree is increased by 1. Hence the degree of FMC is \(d_g(\text{FMC}) = (r+1)\).

**Theorem 2.17:** The total number of nodes in FMC is \(p = 2^{mh+k}\). \(\quad (2.15)\)

**Proof:** In FMC network the number of nodes is same as that of MC network. It contains \(2^k\) number of classes. In each class there are \(2^m(2^k-1)\) number of clusters. Each cluster in turn contains \(2^m\) number of nodes as each cluster is a cube. Hence the total number of nodes in FMC is given by

\[
p = 2^k \times 2^m(2^k-1) \times 2^m = 2^{mh+k}, \text{ where } h=2^k
\]

**Theorem 2.18:** The total number of edges in FMC is given by

\[
E' = (r+1)2^{mh+k-1}
\] \(\quad (2.16)\)

**Proof:** In MC the total number of edges is given by

\[
E = (m + k)2^{mh+k-1}
\]

In the proposed network the number of complementary edges added is

\[
\left| \frac{V}{2} \right| = \frac{2^{mh+k}}{2}
\]

So, the total number of edges is given by

\[
E' = E + \left| \frac{V}{2} \right| = (m + k)2^{mh+k-1} + \frac{2^{mh+k}}{2} = (m+k+1)2^{mh+k-1}
\]

Hence,

\[
E' = (r+1)2^{mh+k-1}
\]

**Theorem 2.19:** The node connectivity of FMC is \((r+1)\). \(\quad (2.17)\)

38
Chapter II  

Some Studies on Parallel Computer Interconnection Network Topology

Proof: Every node with \((mh+k)\) bit address \(a(u)\) in FMC is connected to \(r\) nodes at a hamming distance 1 and one node at hamming distance \((mh+k)\). So the degree of FMC is \(d_g(u) = r+1\) and FMC is a regular network of degree \((r+1)\).

**Theorem 2.20:** The number of node disjoint paths between any two nodes of FMC is \((r+1)\).

Proof: Since every node in FMC has \((r+1)\) neighbours so it is necessary to remove at least \((r+1)\) nodes to disconnect the FMC.

Hence, the number of node disjoint paths in FMC is \((r+1)\).

**Theorem 2.21:** Diameter of FMC is \(D(FMC)=2r-2\). (2.18)

Proof: Let two nodes, 's' & 't' \(\in V\), the set of nodes in FMC. Their node addresses differ in \(q\) bit positions. In the Folded Metacube, if s and t are in the same cluster, then the maximum distance is given by \(d'(s, t) = \text{min}(q, (mh+k)-q+1)\). If s and t are of different classes, then \(d'(s, t) = \text{min}(q, (mh+k)-q+1)\). If s and t are in different clusters of same class, then \(d'(s, t) = \text{min}(q+2, (mh+k)-q+1)\).

The maximum distance in MC occurs when the node addresses differ in cluster address bits and node address bits, but the class address bits remain the same, i.e, \(2^k (m+1)\). The maximum distance in FMC for the same two nodes is \(2r-2\), where \(r=m+k\).

**Theorem 2.22:** The cost of the FMC network is \(\xi = (r+1)(2r-2)\). (2.19)

Proof: In general, the cost of an interconnection network is defined as the product of degree and diameter. In FMC, the node degree is \((r+1)\) and the diameter is \((2r-2)\).

Hence, the cost of FMC is given by

\[
\text{Cost} = \text{degree} \times \text{diameter} = (r+1)(2r-2)
\]

Hence proved.

**Theorem 2.23:** The Bisection width of FMC is \(2^{mh-1}(1 + 2^{k-1})\) (2.20)

Proof: For a Meta cube the bisection width is \(2^{mh-1}\)

In Folded Meta cube number of augmented edges is equal to \(2^{mh+k-1}\)

So, the bisection width is \(2^{mh-1} + 2^{mh+k-1}/2\)

\[
= 2^{mh-1}(1 + 2^{k-1})
\]

Hence the theorem is proved.

The topological properties of Folded Meta cube is compared with those of other topologies including the parent networks in Table 2.2.
Figure 2.7: Folded meta cube FMC(1,3)
### Table 2.2: Comparison of topological properties of FMC

<table>
<thead>
<tr>
<th>Network</th>
<th>Degree</th>
<th>Diameter</th>
<th>Cost</th>
<th>No. of edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>HC</td>
<td>$n$</td>
<td>$n$</td>
<td>$n^2$</td>
<td>$n \cdot 2^{n-1}$</td>
</tr>
<tr>
<td>FHC</td>
<td>$n+1$</td>
<td>$\left\lceil \frac{n}{2} \right\rceil$</td>
<td>$\left\lceil \frac{n}{2} \right\rceil \times (n+1)$</td>
<td>$(n+1) \cdot 2^{n-1}$</td>
</tr>
<tr>
<td>DC</td>
<td>$(n+1)/2$</td>
<td>$n+1$</td>
<td>$(n+1)^2/2$</td>
<td>$(n+1) \cdot 2^{n-2}$</td>
</tr>
<tr>
<td>MC</td>
<td>$r$</td>
<td>$(m+1)2^k$</td>
<td>$r(m+1)2^k$</td>
<td>$r \cdot 2^{m+k-1}$</td>
</tr>
<tr>
<td>FMC</td>
<td>$r+1$</td>
<td>$2r-2$</td>
<td>$(r+1)(2r-2)$</td>
<td>$(r+1) \cdot 2^{m+k-1}$</td>
</tr>
</tbody>
</table>

#### 2.4.2 Routing and Broadcasting in FMC

In this sub section two distinct algorithms for one-to-one and one-to-all communication in the FMC are proposed followed by the illustration.

**One-to-one Routing:**

This algorithm performs the routing between any pair of nodes namely $u, v \in V$ of FMC.

**Algorithm One-to-one $(a(u), a(v), r)$**

**Begin**

$a(w) = (a(u) \oplus a(v))$

If $||a(w)|| < 2r-2$

\[ \text{Route the message sent from } u \text{ via a path composed of links with labels corresponding to bit position which are 1's in } a(w) \]

Else

\[ \text{send the message to } u' \text{ via the complementary link, route the message via a path composed of links with labels corresponding to bit positions that are 0's in } a(w). \]

**End**;

So the length of the shortest path in Folded Metacube is at most $(2r-2)$, the diameter of the network.

**One-to-All Broadcasting:**

The proposed broadcasting process satisfies the following desirable properties.

1. A node should not send (receive) the message to (from) more than one of its neighbours.
   (i.e. one port communication).
2. A node receives the message exactly once for the whole duration of the broadcasting (no duplication of the message).

Let \( s \) be source node in class0. Then \((r+1)\) neighbours of \( s \) are \( s^i, 0 \leq i \leq r+1 \).

\[ s = (c_{k-1}, c_0, m_{b-1}, \ldots, m_0) \]

Then \( s^i = (c_{k-1}, c_0, m_{i-1}, m_i, \ldots, m_0) \) where \( 0 \leq i \leq r-1 \).

\[ s^i = (1, \ldots, 1, m_{b-1}, \ldots, m_0) \text{ and } s^{r+1} = (c_{k-1}, \ldots, c_0, n_{b-1}, \ldots, m_0) \]

**Algorithm: One-to-All**

1. Source \( s \) sends a message to its neighbour \( s' \) through cross edge.
2. Then \( s \) and \( s' \) broadcast simultaneously in their respective clusters using binomial trees.
3. Next a spanning broadcast tree (SBT) [11] can be constructed for FMC, where each node can be connected by cube edge if the node belongs to the same cluster.
4. If the next node belongs to a cluster of another class, then a cross edge is used. Next, if the next node belongs to different cluster of the same class then a complementary edge is used.

In SBT each node is connected by cube edge or a cross edge, if the hamming distance is less than \( 2r-2 \), otherwise a complementary edge is used. So the height of the spanning broadcast tree is at most \( 2r-2 \). Hence, the broadcasting is done in \( 2(m+k)-2 \) time in contrast to \( 2^k(m+1)+k-1 \) in Metacube. The algorithm is illustrated below through a simple example.

**Illustration of Routing in FMC**

Let us consider any two nodes \( s \) and \( t \) in FMC(2,2). Then, \( mh+k=10 \) bit node address of \( s \) be \((00,0000,0000)\). Let \( t \) be the farthest node then its node address is \((00,1111,1111)\). Both the nodes \( s \) and \( t \) belong to class 0 but of different clusters. The node \( s \) has five immediate neighbors. They are \((00,0000,0001), (00,0000,0010), (10,0000,0000), (01,0000,0000), (11,1111,1111)\)

So the path from \( s \) to \( t \) will be

\( (00,0000,0000) \rightarrow (11,1111,1111) \rightarrow (10,1111,1111) \rightarrow (00,1111,1111) \)

as shown in Fig.2.8.

Further distance is 3.

The next sub section is devoted towards the performance analysis of FMC network.

**2.4.3 Performance Analysis of FMC**

The performance analysis of a parallel computer interconnection network reflects many important aspects of the system including the total cost of the system. To make the parallel interconnection network more attractive, more emphasis is given to its fault tolerance cost effectiveness and reliability. An in depth analysis of the cost effectiveness factor, time
cost effectiveness factor and reliability of the proposed FMC network is made in the subsequent paragraphs.

**Cost Effectiveness Factor:**

*Theorem 2.24:* The cost effectiveness factor of $\text{FMC}(k,m)$ is

$$\text{CEF}(p) = \frac{1}{1 + \rho \left( \frac{m+k+1}{2} \right)}; \quad (2.21)$$

Where $\rho$ is the ratio of link to processor cost.

*Proof:* The total number of processors in FMC is given by $p = 2^{m+k}$

The total number of edges in FMC is $E = (m+k+1)2^{m+k-1}$

$$= (m+k+1)p/2 = f(p)$$

$f(p)$ represents the number of edges in terms of $p$.

So,

$$g(p) = f(p)/p = (m+k+1)/2.$$ 

Hence, the cost effectiveness factor is given by

$$\text{CEF}(p) = \frac{1}{1 + \rho \left( \frac{m+k+1}{2} \right)}$$

Hence, the theorem is proved.

**Time-cost-effectiveness Factor (TCEF):**

*Theorem 2.25:* The time cost effectiveness factor for $\text{FMC}(k,m)$ network is given by

$$\text{TCEF}(p, \text{FMC}) = \frac{1+\sigma}{1+\rho \left( \frac{m+k+1}{2} \right) + \left( \frac{\sigma}{2^{m+k}} \right)} \quad (2.22)$$

*Proof:* For any cube based network the $\text{TCEF}$ is given by,
Chapter II Some Studies on Parallel Computer Interconnection Network Topology

\[ TCEF(p, T_p) = \frac{1 + \sigma T_p^{d-1}}{1 + \rho g(p) + \frac{T_p}{p}} \]

Where \( T_j \) is the time required to solve the problem by a single processor using the fastest sequential algorithm, \( T_p \) is the time required to solve the problem by a parallel algorithm using a multiprocessor system having \( p \) processors and \( \sigma \) is the ratio of the cost of penalty to cost of processors. For linear time penalty in \( T_p \), \( \alpha \) is chosen as 1.

For FMC(k,m), \( p = 2^{mh+k} \), and
\[ g(p) = (m + k + 1)/2 \] (Ref: Theorem 2.24)

Hence, \( TCEF(p, FMC) = \frac{1 + \sigma}{1 + \rho g(p) + \frac{\sigma}{p}} = \frac{1 + \sigma}{1 + \rho \left(\frac{m + k + 1}{2}\right) + \sigma \left(\frac{m + k + 1}{2}\right)} \)

The \( TCEF \) is computed against dimension for the FMC network and also for the MC Network (Ref Table 2.4 and 2.5). It is observed that for the same number of processors, the \( TCEF(p) \) of the Folded Metacube is higher than that of the Metacube. Thus, solving a problem using FMC network will be financially profitable.

Reliability Analysis:

The reliability analysis is an important criterion to evaluate the robustness of a parallel interconnection network [212 and 262]. For the current network, two terminal reliability measures is evaluated and compared. Terminal reliability is the probability of the existence of at least one fault free path between a designated pair of input and output terminals. The FMC being a directed graph, the vertices and edges are weighted with reliabilities of the components they represent. Two nodes A and B are considered with \( n \) number of node disjoint paths lying between them. Let \( r_i \) be the number of links involved in path \( i \), where \( 1 \leq i \leq n \). Thus there are \( r_i - 1 \) number of nodes in path \( i \).

Let \( P(C_i) \) be the probability of successful route through the \( i^{th} \) path.

Then \( R_l \) be the link reliability with link failure rate (\( \lambda \)) is 0.0001 and \( R_p \) is the node reliability with processor failure rate (\( \lambda \)) is 0.001.

So \( R_l = e^{-\lambda t} \), where \( \lambda = 0.0001 \) and \( t=1000 \) and \( R_p = e^{-\lambda t} \), where \( \lambda = 0.001 \) and \( t=1000 \).

\[ \text{Theorem 2.26:} \] For FMC(k,m) network, the two terminal reliability is given by
\[ TR = 1 - \prod_{i=1}^{nm+k} R_l^r_i R_p^{r_i-1} \]

\[ \text{Proof:} \] All nodes and links are considered to be identical with their failure rates statistically independent and exponentially distributed. Now the probability of existence of a successful connection between the source and destination can be given by
Normally there are three possible cases. Namely i) only paths are reliable, ii) only nodes are reliable, iii) both nodes and paths are unreliable. For the current work case (iii) is considered for FMC and MC networks. The same reliability evaluation is also done keeping dimension (r) fixed at 3 with different values of mission time \( t \).

The next section relates the topological properties of the proposed topologies to the respective parameters of the basis networks.

### 2.5 Results and Discussions

In this section, the different topological and performance parameters of the two proposed networks FCC and FMC are evaluated and the results are compared. The comparisons of results bring out the advantages and limitations of the proposed systems.

#### 2.5.1 Discussions on FCC Network

This section evaluates different parameters of FCC and a comparison is made with other networks. Different topological properties are compared in Table 2.1 in subsection 2.2.1 previously.

The comparison of node degree Vs. dimension is depicted in Fig. 2.9. The degree of the network increases with dimension due to addition of complementary links. The node degree of the hypercube and the crossed cube are same. So they have a common curve.

The diameter of a network is the maximum number of links that must be traversed to send a message to any node along a shortest path. The lower the diameter of a network, the shorter is the time to send a message from one node to the node farthest away from it. The Fig. 2.10 depicts the variation of diameter with increase in dimension for the various networks. The diameter of FCC gets remarkably reduced as compared to hypercube.

Next the comparison of cost is shown in Fig. 2.11. The diameter of the FCC network is reduced as compared the HC and CC. Hence, the cost of the FCC network is also reduced being a product of degree and diameter.
Figure 2.9: Comparison of degree Vs. dimension of FCC network

Figure 2.10: Comparison of diameter of FCC network
Chapter II  
Some Studies on Parallel Computer Interconnection Network Topology

Figure 2.11: Comparison of cost of FCC network

Table 2.3: Comparison of average node distance of FCC

<table>
<thead>
<tr>
<th>$n$</th>
<th>HC</th>
<th>CC</th>
<th>FCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>1.25</td>
<td>1.37</td>
<td>1.25</td>
</tr>
<tr>
<td>4</td>
<td>2.00</td>
<td>1.90</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
<td>2.15</td>
<td>2.062</td>
</tr>
<tr>
<td>6</td>
<td>3.00</td>
<td>2.75</td>
<td>2.406</td>
</tr>
<tr>
<td>7</td>
<td>3.50</td>
<td>3.25</td>
<td>2.906</td>
</tr>
<tr>
<td>8</td>
<td>4.00</td>
<td>3.75</td>
<td>3.269</td>
</tr>
<tr>
<td>9</td>
<td>4.50</td>
<td>4.12</td>
<td>3.760</td>
</tr>
<tr>
<td>10</td>
<td>5.00</td>
<td>4.62</td>
<td>4.146</td>
</tr>
<tr>
<td>11</td>
<td>5.50</td>
<td>5.12</td>
<td>4.640</td>
</tr>
<tr>
<td>12</td>
<td>6.00</td>
<td>5.50</td>
<td>5.030</td>
</tr>
<tr>
<td>13</td>
<td>6.50</td>
<td>6.00</td>
<td>5.530</td>
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<td>15</td>
<td>7.50</td>
<td>6.87</td>
<td>3.780</td>
</tr>
<tr>
<td>20</td>
<td>10.0</td>
<td>7.75</td>
<td>5.350</td>
</tr>
<tr>
<td>30</td>
<td>15.0</td>
<td>13.75</td>
<td>13.26</td>
</tr>
</tbody>
</table>
Then the average node distance of FCC network is evaluated and compared against that of the parent networks. The computed values are shown in Table 2.3. The comparison is shown in Fig. 2.12. The average node distance of n-dimensional HC is $\frac{n}{2}$. In CC it is reduced due to changed connections. The FCC network has much reduced average distance than HC and CC due to folding as the farthest nodes are one step ahead. Hence the message density is also reduced a lot. The computed values are shown in Table 2.4. For FCC it is close to 0.75. In case of higher dimensions also it never exceeds 0.8. However, the traffic density of Hypercube is 1. The variation of message density is shown in Fig. 2.13. In Table 2.5 the computed values of mean inter node distance rates both absolute and relative are shown.

Table 2.4: Comparison of message traffic density of FCC

<table>
<thead>
<tr>
<th>n</th>
<th>HC</th>
<th>CC</th>
<th>FCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.00</td>
<td>1.00</td>
<td>0.466</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.913</td>
<td>0.625</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>0.951</td>
<td>0.600</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>0.860</td>
<td>0.687</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.916</td>
<td>0.687</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>0.928</td>
<td>0.726</td>
</tr>
<tr>
<td>8</td>
<td>1.00</td>
<td>0.937</td>
<td>0.726</td>
</tr>
<tr>
<td>9</td>
<td>1.00</td>
<td>0.916</td>
<td>0.752</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>0.925</td>
<td>0.753</td>
</tr>
<tr>
<td>11</td>
<td>1.00</td>
<td>0.931</td>
<td>0.773</td>
</tr>
<tr>
<td>12</td>
<td>1.00</td>
<td>0.916</td>
<td>0.773</td>
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<tr>
<td>13</td>
<td>1.00</td>
<td>0.923</td>
<td>0.790</td>
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</table>
Chapter II

Some Studies on Parallel Computer Interconnection Network Topology

Figure 2.12: Comparison of average node distance of FCC

Figure 2.13: Comparison of message traffic density Vs. dimension of FCC
Table 2.5: Comparison of mean inter node distance rates of FCC

<table>
<thead>
<tr>
<th>n</th>
<th>$\gamma_a$</th>
<th>$\gamma_r$</th>
</tr>
</thead>
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<td>2</td>
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<td>0.300</td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>1.333</td>
<td>0.250</td>
</tr>
<tr>
<td>5</td>
<td>1.212</td>
<td>0.175</td>
</tr>
<tr>
<td>6</td>
<td>1.246</td>
<td>0.198</td>
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<td>7</td>
<td>1.204</td>
<td>0.169</td>
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<tr>
<td>8</td>
<td>1.223</td>
<td>0.182</td>
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<td>1.196</td>
<td>0.164</td>
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<tr>
<td>10</td>
<td>1.205</td>
<td>0.170</td>
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<tr>
<td>11</td>
<td>1.185</td>
<td>0.156</td>
</tr>
<tr>
<td>12</td>
<td>1.192</td>
<td>0.161</td>
</tr>
<tr>
<td>13</td>
<td>1.175</td>
<td>0.149</td>
</tr>
<tr>
<td>15</td>
<td>1.984</td>
<td>0.496</td>
</tr>
<tr>
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<td>1.869</td>
<td>0.465</td>
</tr>
<tr>
<td>30</td>
<td>1.131</td>
<td>0.116</td>
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</tbody>
</table>

Table 2.6 and 2.7 present the computed values of cost effectiveness and time cost effectiveness of FCC(n). In both the cases FCC exhibits better cost effectiveness and time cost effectiveness in comparison to hypercube and crossed cube as evaluated in [196].

Table 2.6: Cost effectiveness factor of FCC(n) ($\alpha = 1$)

<table>
<thead>
<tr>
<th>n</th>
<th>$\rho = 0.1$</th>
<th>$\rho = 0.2$</th>
<th>$\rho = 0.3$</th>
<th>$\rho = 0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.833</td>
<td>0.714</td>
<td>0.625</td>
<td>0.555</td>
</tr>
<tr>
<td>4</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td>5</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
<td>6</td>
<td>0.740</td>
<td>0.588</td>
<td>0.487</td>
<td>0.416</td>
</tr>
<tr>
<td>7</td>
<td>0.714</td>
<td>0.555</td>
<td>0.454</td>
<td>0.384</td>
</tr>
<tr>
<td>8</td>
<td>0.689</td>
<td>0.526</td>
<td>0.425</td>
<td>0.357</td>
</tr>
<tr>
<td>9</td>
<td>0.66</td>
<td>0.500</td>
<td>0.400</td>
<td>0.333</td>
</tr>
<tr>
<td>10</td>
<td>0.645</td>
<td>0.476</td>
<td>0.377</td>
<td>0.312</td>
</tr>
</tbody>
</table>
Chapter II Some Studies on Parallel Computer Interconnection Network Topology

Table 2.7: Time cost effectiveness factor of FCC(n), \( \alpha = 1, \sigma = 1 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \rho = 0.1 )</th>
<th>( \rho = 0.2 )</th>
<th>( \rho = 0.3 )</th>
<th>( \rho = 0.4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.509</td>
<td>1.311</td>
<td>1.311</td>
<td>1.038</td>
</tr>
<tr>
<td>4</td>
<td>1.523</td>
<td>1.280</td>
<td>1.103</td>
<td>0.969</td>
</tr>
<tr>
<td>5</td>
<td>1.502</td>
<td>1.226</td>
<td>1.035</td>
<td>0.896</td>
</tr>
<tr>
<td>6</td>
<td>1.464</td>
<td>1.165</td>
<td>0.968</td>
<td>0.827</td>
</tr>
<tr>
<td>7</td>
<td>1.420</td>
<td>1.106</td>
<td>0.905</td>
<td>0.766</td>
</tr>
<tr>
<td>8</td>
<td>1.375</td>
<td>1.050</td>
<td>0.849</td>
<td>0.713</td>
</tr>
<tr>
<td>9</td>
<td>1.331</td>
<td>0.999</td>
<td>0.799</td>
<td>0.666</td>
</tr>
<tr>
<td>10</td>
<td>1.289</td>
<td>0.951</td>
<td>0.7544</td>
<td>0.6248</td>
</tr>
</tbody>
</table>

The Fig. 2.14 and 2.15 show the variation of Cost-effectiveness and Time-cost effectiveness factor with respect to dimension. The Time-cost effectiveness of FCC network attains its maximum when \( p = 1.6\sigma/\rho \). Thus the FCC network is financially most profitable when it has \( p = 1.6\sigma/\rho \).

Next the terminal reliability of the FCC network is compared with the parent networks. Table 2.8 and 2.9 show the computed values for reliability of FCC network. The

Figure 2.14: Cost effectiveness factor for FCC network (\( \alpha = 1 \))
The proposed network is more reliable than the parent networks as it contains more node disjoint paths with increasing values of ‘n’ as shown in Figure 2.16. The Figure 2.17 depicts the superiority of the proposed network in terms of reliability while keeping n the node degree fixed at 10 but varying mission time t from 1000 to 10,000. For evaluation the processor failure rate and link failure rate are assumed to be 0.001 and 0.0001 respectively. At mission time 1000 the networks are equally reliable but latter the proposed FCC network is more reliable than the crossed cube and the hypercube networks.

Table 2.8: Comparison of Reliability of FCC ($\lambda_l = 0.0001, \lambda_p = 0.001$, t=1000Hrs)

<table>
<thead>
<tr>
<th>n</th>
<th>HC</th>
<th>CC</th>
<th>FCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.029853</td>
<td>0.108906</td>
<td>0.824142</td>
</tr>
<tr>
<td>4</td>
<td>0.039605</td>
<td>0.117863</td>
<td>0.825910</td>
</tr>
<tr>
<td>5</td>
<td>0.049259</td>
<td>0.12673</td>
<td>0.827660</td>
</tr>
<tr>
<td>6</td>
<td>0.058816</td>
<td>0.135508</td>
<td>0.829392</td>
</tr>
<tr>
<td>7</td>
<td>0.068276</td>
<td>0.144198</td>
<td>0.831107</td>
</tr>
<tr>
<td>8</td>
<td>0.077642</td>
<td>0.152800</td>
<td>0.832805</td>
</tr>
<tr>
<td>9</td>
<td>0.086913</td>
<td>0.161316</td>
<td>0.834485</td>
</tr>
<tr>
<td>10</td>
<td>0.096091</td>
<td>0.169747</td>
<td>0.836149</td>
</tr>
<tr>
<td>11</td>
<td>0.105177</td>
<td>0.178092</td>
<td>0.837796</td>
</tr>
<tr>
<td>12</td>
<td>0.114172</td>
<td>0.186354</td>
<td>0.839427</td>
</tr>
</tbody>
</table>
Table 2.9: Reliability comparison with time for FCC

\( (\lambda_t = 0.0001, \lambda_p = 0.001, n = 10) \)

<table>
<thead>
<tr>
<th>Time (Hrs)</th>
<th>HC</th>
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<th>FCC</th>
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</thead>
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<tr>
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<td>0.652323</td>
<td>0.729968</td>
<td>0.966914</td>
</tr>
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<td>2000</td>
<td>0.096091</td>
<td>0.169747</td>
<td>0.836149</td>
</tr>
<tr>
<td>3000</td>
<td>0.010032</td>
<td>0.036111</td>
<td>0.743418</td>
</tr>
<tr>
<td>4000</td>
<td>0.00101</td>
<td>0.009131</td>
<td>0.670653</td>
</tr>
<tr>
<td>5000</td>
<td>0.000101</td>
<td>0.00257</td>
<td>0.606571</td>
</tr>
<tr>
<td>6000</td>
<td>1.02E-05</td>
<td>0.000756</td>
<td>0.548816</td>
</tr>
<tr>
<td>7000</td>
<td>1.02E-06</td>
<td>0.000226</td>
<td>0.496586</td>
</tr>
<tr>
<td>8000</td>
<td>1.02E-07</td>
<td>6.78E-05</td>
<td>0.449329</td>
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<tr>
<td>9000</td>
<td>1.02E-08</td>
<td>2.04E-05</td>
<td>0.40657</td>
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<tr>
<td>10000</td>
<td>1.03E-09</td>
<td>6.15E-06</td>
<td>0.367879</td>
</tr>
</tbody>
</table>

Figure 2.16: Comparison of two terminal reliability for FCC network

\( (\lambda_t = 0.0001, \lambda_p = 0.001, t=1000\text{Hrs}) \)
Figure 2.17: Variation of reliability with time for FCC network (n=10)

2.5.2 Discussions on FMC Network

In this sub section different performance parameters of the FMC are evaluated and compared against those of other networks. The Fig 2.18 depicts the variation of degree with network dimension. Due to the augmentation of complementary links, the degree of the FMC is observed to be slightly greater than that of Metacube but quite less than HC and FHC. For comparison the HC and FHC networks are considered with equal number of nodes.

Figure 2.19 shows the comparison of diameter with respect to dimension. It proves the superiority of the Folded Meta cube over MC due to sufficient reduction in diameter. The diameter of FMC is appreciably reduced at higher dimensions. As compared to hypercube the diameter of FMC is quite less. However, it is approximately equal to that of FHC.

The Figure 2.20 presents comparison of cost among the four networks with respect to dimension. The cost of FMC is found to be smaller than that of Metacube. The cost is sufficiently reduced when number of nodes in the system is increased. The new network is found to have reduced cost while retaining higher packing density as compared to Hypercube and Folded Hypercube.
Figure 2.18: Comparison of degree of FMC network

Figure 2.19: Comparison of diameter of FMC network
Since the Folded Dual cube is a special case of Folded Metacube, the broadcast time comparison is first done in FDC network keeping $k$ as 1. It is found that in the FDC network
the broadcast is done faster than the other networks. The broadcast time is less for FDC upto dimension 7 and beyond that its value is slightly greater than that of FHC as FDC contains more number of nodes. The result is shown in Fig. 2.21.

The same result is now extended for all values of k. In the next comparison only FMC and MC networks are taken into consideration as both them have equal number of nodes. As with degree 3 the Hypercube and Folded hypercube both contain only 8 nodes and Metacube contains 32 nodes. The Fig. 2.22 shows the broadcast time comparison of FMC and MC network. In FMC network broadcasting is done in less number of steps.

![Graph showing broadcast time vs. dimension in FMC network](image)

Figure 2.22: Broadcast time Vs. dimension in FMC network

Thus the FMC network exhibits quite a good improvement in broadcast time over its parent networks while connecting to millions of nodes.

Also, it is observed that for the same number of processors, the CEF(p) of FMC network is quite less than that of the MC network as shown in Table 2.10 and 2.11. The CEF values are tabulated for all values of k and m. The Fig. 2.23 shows the variation of CEF of FMC(2,m) with respect to dimension for different values of ρ. The monotonically
decreasing nature of the curves show that the FMC is cost effective and tabulated values show the superiority over the MC network.

Similarly, the TCEF values in Table 2.12 and 2.13 show that the FMC network will give faster solution to a problem. The Fig. 2.24 shows the variation of TCEF for FMC(k,m) against dimension. Here keeping \( \rho \) fixed the graphs are plotted for different values of \( k \) and \( m \). The comparison of TCEF for FMC and MC with dimension is shown in Fig. 2.25. The Figure depicts the superiority of FMC over MC as it will give faster solution. In the figure the first four values are for FMC(1,m). Then the next values are evaluated for \( k=2 \). The TCEF for FMC attains its maximum value with \( p = 1024 \) that is the total number of processors.

![Figure 2.23: Cost effectiveness factor of FMC network](image)

Figure 2.23: Cost effectiveness factor of FMC network
### Table 2.10: Cost effectiveness factor of FMC ($\sigma=1$)

<table>
<thead>
<tr>
<th>$k$</th>
<th>$m$</th>
<th>$\rho =0.1$</th>
<th>$\rho =0.2$</th>
<th>$\rho =0.3$</th>
<th>$\rho =0.4$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.833</td>
<td>0.714</td>
<td>0.625</td>
<td>0.555</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
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<td>4</td>
<td>0.740</td>
<td>0.588</td>
<td>0.487</td>
<td>0.416</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.740</td>
<td>0.588</td>
<td>0.487</td>
<td>0.416</td>
</tr>
<tr>
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<td>4</td>
<td>0.714</td>
<td>0.555</td>
<td>0.454</td>
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</tbody>
</table>

### Table 2.11: Cost effectiveness factor of MC ($\sigma=1$)

<table>
<thead>
<tr>
<th>$k$</th>
<th>$m$</th>
<th>$\rho =0.1$</th>
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<tr>
<td>1</td>
<td>2</td>
<td>0.869</td>
<td>0.689</td>
<td>0.689</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.833</td>
<td>0.714</td>
<td>0.625</td>
<td>0.555</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.833</td>
<td>0.714</td>
<td>0.625</td>
<td>0.555</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.800</td>
<td>0.666</td>
<td>0.571</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.769</td>
<td>0.625</td>
<td>0.526</td>
<td>0.454</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.740</td>
<td>0.588</td>
<td>0.487</td>
<td>0.416</td>
</tr>
</tbody>
</table>
Chapter II  
Some Studies on Parallel Computer Interconnection Network Topology

Table 2.12: Time Cost Effectiveness Factor of FMC with $\sigma = 1$

<table>
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<tr>
<th>$k$</th>
<th>$m$</th>
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<th>$\rho = 0.3$</th>
<th>$\rho = 0.4$</th>
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<td>1.693</td>
<td>1.502</td>
<td>1.350</td>
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<td>3</td>
<td>1.655</td>
<td>1.426</td>
<td>1.243</td>
<td>1.106</td>
</tr>
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<td></td>
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<td>1.248</td>
<td>1.051</td>
<td>0.908</td>
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<td>1.567</td>
<td>1.285</td>
<td>1.095</td>
<td>0.951</td>
</tr>
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<td>0.930</td>
<td>0.733</td>
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</table>

Table 2.13: Time cost effectiveness factor of MC with $\sigma = 1$

<table>
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<th>$K$</th>
<th>$m$</th>
<th>$\sigma = 0.1$</th>
<th>$\sigma = 0.2$</th>
<th>$\sigma = 0.3$</th>
<th>$\sigma = 0.4$</th>
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<td>1.248</td>
<td>1.051</td>
<td>0.908</td>
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<tr>
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<td>2</td>
<td>1.598</td>
<td>1.332</td>
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<td>1.249</td>
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<td>1.111</td>
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</table>
Figure 2.24: Time cost effectiveness factor of FMC(2,m) network (ρ =0.1)

The Figures 2.26 to 2.28 show the variation of reliability of FMC and MC. The result, is based on different values of m and k with the link failure rate 0.0001 and node failure rate 0.001. It is observed that the reliability of MC network lies within 0.7 to 0.3 for k=1. For k=2 it is within 0.6 to 0.2. When k is 3 the reliability range is reduced to 0.55 to 0.25. However, for FMC network the reliability lies within 0.9 to 0.8 for all values of dimension.

Fig 2.25: Comparison of Time-cost-effectiveness factor of FMC
Figure 2.26: Comparison of reliability with dimension for FMC(1,m) network
\((\lambda_1 = 0.0001, \lambda_p = 0.001, t=1000)\)

Two terminal reliability comparison shows that Folded Metacube network is highly reliable when compared with the Metacube network. The Fig. 2.29 shows the superiority of FMC over MC network. The reliability is measured against mission time from 1000 Hrs to 10,000 Hrs with link failure rate and node failure rate as 0.0001 and 0.001 respectively. With increased degree, the FMC network possesses better node disjoint paths than MC network.

Figure 2.27: Comparison of reliability with dimension for FMC(2,m) network
\((\lambda_1 = 0.0001, \lambda_p = 0.001, t=1000)\)
Figure 2.28: Comparison of reliability with dimension for FMC network

\[ (\lambda_t = 0.0001, \lambda_p = 0.001, t = 1000) \]

Hence the two terminal reliability of FMC network is more than that of MC. For evaluation 4 dimensional networks are considered with the total number of nodes \( p \) assumed to be 1024.

Figure 2.29: Comparison of reliability with time for FMC
2.6 Conclusions

This chapter introduced three new cube-based parallel interconnection networks called Folded crossed cube, Folded dual cube and Folded meta cube. The crossed cube bears an improvement in diameter due to change in the connections between the nodes. By applying the principle of folding the diameter of CC with odd dimensions in substantially reduced, thereby decreasing the cost of communication. Also, the Folded meta cube being a large scale network, its diameter is also reduced for which the reliability of the network is increased. The basic property of the networks are derived and compared with their parent networks. Algorithms for routing and broadcasting are proposed for the new topologies. The cost of the proposed topologies are found to be less than the parent networks and found to be superior in terms of reliability, cost effectiveness and time cost effectiveness. The reduced diameter helps to speed up the overall operation of large scale parallel systems. So the performance improvements pay off the hardware overhead due to increase in the node degree. With improved broadcast time the FMC network is found to be a better choice for large scale parallel systems. It also provides better performance and efficient inter processor communication.