Chapter 7

Conclusion

Fuzzy Fractal Analysis is introduced as a new branch of Fractal Geometry in Mathematics as well as noteworthy applications of Multifractal Dimensions in signal and image analysis, are also concerned widely in this Thesis.

Firstly, Fisher Fractals in the Hausdorff metric spaces are defined by generalizing the Hutchinson-Barnsley theory for an iterated function system of Fisher contractions and by using the Fisher fixed point theorem. The existence and uniqueness theorem of fractals is proved by means of Fisher contractions in the Hausdorff metric space. Fuzzy fractals and multivalued fuzzy fractals are constructed in the fuzzy metric spaces by generalizing the Hutchinson-Barnsley theory for an iterated function system of fuzzy contractions and multivalued fuzzy contractions respectively. The existence and uniqueness theorems of fuzzy fractals and multivalued fuzzy fractals are proved in the standard fuzzy metric spaces by using the fuzzy Banach contraction theorem. Consequent results on fuzzy fractals and multivalued fuzzy fractals are discussed in the standard fuzzy metric spaces with respect to the standard Hausdorff fuzzy metrics. Fuzzy B-contraction properties of the Hutchinson-Barnsley operator on the fuzzy hyperspace of with respect to the Hausdorff fuzzy metrics are presented. Then the relationships between the Hausdorff fuzzy metrics on the fuzzy hyperspaces are investigated.
Besides that, Intuitionistic fuzzy fractals and multivalued intuitionistic fuzzy fractals are framed in the intuitionistic fuzzy metric spaces by generalizing the Hutchinson-Barnsley theory for an intuitionistic fuzzy contractions, multivalued intuitionistic fuzzy contractions, intuitionistic fuzzy B-contractions, multivalued intuitionistic fuzzy B-contractions, intuitionistic fuzzy Edelstein contractions and multivalued intuitionistic fuzzy Edelstein contractions. The existence and uniqueness theorems of intuitionistic fuzzy fractals and multivalued intuitionistic fuzzy fractals are established in the standard intuitionistic fuzzy metric spaces by using the intuitionistic fuzzy Banach contraction Theorem. Also, subsequent results on intuitionistic fuzzy fractals and multivalued intuitionistic fuzzy fractals are studied in the standard intuitionistic fuzzy metric spaces with respect to the Hausdorff intuitionistic fuzzy metrics. Intuitionistic fuzzy B-contraction and intuitionistic fuzzy Edelstein contraction properties of the Hutchinson-Barnsley operator on the intuitionistic fuzzy hyperspace with respect to the Hausdorff intuitionistic fuzzy metrics are analyzed. The relationships between the Hausdorff intuitionistic fuzzy metrics on the intuitionistic fuzzy hyperspaces are analyzed. Many interesting results and remarkable applications of the developed generalizations such as Collage Theorem, Falling Leaves Theorem and so on are explored.

As an application part of the thesis, the Modified form, Improved form and Advanced form of GFD are defined newly for the recognition of Seizure free (Normal) and Epileptic (Interictal & Ictal) EEGs. Fractal Spectra and Range of Fractal Spectra are obtained for the Normal, Interictal & Ictal EEGs using GFD, MGFD, IGFD & AGFD Methods. Fractal Spectra and Range of Fractal Spectra for Normal, Interictal & Ictal EEGs are compared to distinguish the Normal, Interictal and Ictal EEGs through the graphical methods. It is observed that, the designed methods are well organized when compared with classical GFD Method.
Mainly Advanced form of GFD is more reliable method among the proposed methods in the analysis of Epileptic EEG Signals. The statistical tool namely ANOVA Test illustrated that there is significant differences in the values corresponding to the Normal, Interictal and Ictal EEGs in the designed methods than the GFD Method. The proposed scheme demonstrated that the Modified, Improved and Advanced forms of GFD are well proficient mathematical tools in the analysis and classification of Healthy & Epileptic EEG Signals than the GFD Method.

In addition to that, a novel technique based on wavelet denoising to improve the pre-processing of EEG signals is constructed and the classification of healthy and epileptic EEG signals is studied by using multifractal measures such as Generalized Fractal Dimensions. Wavelet decomposition of EEG into sub-bands through DWT proved that approximation and detail coefficients provided the best classification rates by graphically and statistically using GFD as a classification measure. It is shown that the denoising process enhances the differences among the GFD values for the normal, interictal and epileptic ictal EEGs. Biomedical EEG recordings taken from both healthy and epileptic patients during interictal & ictal periods showed the wavelet denoising and GFD classification method is very accurate by graphical and statistical methods. Also the Normal Probability Plot shows that the Epileptic EEG Data has significant nonlinearity but the normal and interictal EEGs are dominated by Gaussian Linear Process.

Furthermore, the fuzzy multifractal theory is developed in order to define the Fuzzy Generalized Fractal Dimensions by introducing fuzzy membership function in the classical Generalized Fractal Dimensions method. It is shown that, the defined Fuzzy GFD method accurately classifies the complexity of the chaotic waveforms such as Weierstrass functions by comparing graphically with the classical GFD method. Also the defined Fuzzy GFD is a generalized method of the classical GFD.
The designed schemes include Multifractal Analysis based on the designed GFD, Fuzzy GFD and the Wavelet Decomposition through DWT, are the strong detectors and indicators of the state of illness of the Epileptic Patients. The proposed epileptic seizure detection techniques based on multifractal dimension and wavelet transform plays a crucial role in signal classification and especially in the area of research about Epilepsy. This scheme will help the medical specialists and physicians as an additional tool to analyze, classify, detect or predict the Epileptic Seizure before the onset time among the Epileptic peoples, so that the patient can be cautious about further disturbances, and the lives of affected people will be protected from the unexpected disasters.

Finally, the multifractal measure such as Generalized Fractal Dimensions for gray scale and RGB color images is presented and invoked in the recognition of noisy and noise free gray scale images as well as RGB color images. The performances of GFD are tested with various natural gray scale and RGB color images. It is proved that GFD discriminates the noisy and denoised images with high accuracy, and illustrated the classification rate through graphical methods by comparing GFD values for Original, Noisy and Denoised gray scale and color images. Further, the fuzzy multifractal dimensions are defined for both gray scale and RGB color images. It is assured that the Multifractal Analysis using GFD plays a significant role in the image analysis to detect the noise level in all type of experimental gray scale and RGB color images such as biomedical images, biometric images, satellite images, agricultural images and so on. Hence the roll of the GFD and fuzzy GFD will be very efficient in the image analysis in order to detect the level of noises.
In future, the fractal and multivalued fractal concepts will be initiated and analyzed in the Fuzzy Topological Spaces, Topological Vector Spaces, b-Metric Spaces and Product Spaces and so on, and the contraction properties of HB operator on these spaces will be studied accordingly. The developed fuzzy multifractal measures will be utilized in biomedical signal analysis as well as biomedical and realistic gray scale and color image analysis. Also the intuitionistic fuzzy metric concepts will be invoked in the fast detection of noisy pixels and peer group concept in the impulsive noise detection of standard and biomedical images. Mostly, fuzzy and intuitionistic fuzzy fractal concepts will be employed in all scientific areas, particularly in signal and image processing.