Fluctuations and Correlations of Conserved Charges in an Excluded Volume Hadron Resonance Gas Model

2.1 Introduction

One of the primary goals of ultra-relativistic heavy-ion collision experiments is to study the thermodynamic aspects of strongly interacting matter at very high-temperature and/or net-baryon density. Whereas at low net-baryon density and high-temperature lattice data seem to indicate a smooth cross over [1, 2] from hadronic to a quark-gluon matter, at high net-baryon density and low temperature, the system is expected to have a first order transition [3–8]. So the first order phase transition at high net-baryon densities and low temperature should end at a critical end point (CEP) which is a second order phase transition point as one moves towards high temperature and low net-baryon density region, in the phase diagram of strongly interacting matter [9–12].
Several experimental programs have been ongoing or planned to study the phase transition of strongly interacting matter under extreme condition. At present, the Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and the Large Hadron Collider (LHC) at CERN are studying the region of high temperature and low net-baryon density (or low chemical potential) in the phase diagram. The low-temperature and high baryonic chemical potential $\mu_B$ region is being studied in the Beam Energy Scan (BES) program of RHIC. This region will also be studied extensively in future fixed target experiments like Compressed Baryonic Matter (CBM) experiments at FAIR and in heavy ion collision experiments at NICA. These experiments would also explore the first order line along with the location of the CEP in the phase diagram.

The unveiling of the nature of phase transition needs a proper understanding of Quantum Chromo Dynamics (QCD), the theory of strong interactions. Unluckily, the non-perturbative nature of the phenomena inhibits the application of first principle QCD for the investigation of strongly interacting matter under extreme condition. In this respect, Lattice Quantum Chromo Dynamics (LQCD) provides the direct approach to study QCD at high temperature \cite{1, 13–18}. However, LQCD has its own restrictions due to the discretization of space-time.

In contrast, effective models \cite{19–30} provide an easier alternative for the investigation of the strongly interacting matter in the non-perturbative region. Some of these models have been quite successful in explaining physics of strongly interacting matter. For instance, Polyakov Nambu-Jona-Lasinio (PNJL) model has been used to examine various aspects of physics of strongly interacting matter at high temperatures and found to reproduce the zero density LQCD data quite successfully \cite{24}. The hadron resonance gas (HRG) model, on the other hand, \cite{31} has been very successful in explaining the hadron yields in central heavy ion collisions from AGS up to RHIC energies \cite{32–39}. HRG model has also been successful in explaining the low-temperature region of the equation of state (EOS) \cite{40–42}.

Different probes have been studied to verify whether the produced medium in its early stage after the heavy-ion collision was indeed in the QGP phase. We have discussed some of them in the Sec. 1.5 of the chapter 1. In this chapter, we will discuss the physics of the phase transition of strongly interacting matter by studying the correlations and fluctuations of conserved charges. Susceptibilities are associated with fluctuations via the fluctuation-dissipation theorem \cite{43}. An estimate of the intrinsic statistical fluctuations in a thermal system is given by the corresponding susceptibilities. At the finite temperature and chemical potential fluctuations of conserved charges are sensitive indicators of the phase transition from hadronic matter to the QGP. Further, the existence of the CEP
can be indicated by the divergent fluctuations. For the vanishing net-baryon density, the transition from hadronic to QGP phase is continuous and the fluctuations are not supposed to lead to any singular behaviour. LQCD calculations have been made for many of these susceptibilities at zero chemical potentials [44–47]. It has been shown that at the small chemical potential the susceptibilities rise rapidly around the transition region [47].

As the prediction of HRG model for the system at chemical freeze-out is in very good agreement with the experimental data, it would be interesting to study the susceptibilities as well as higher order cumulants using this model and its interacting version i.e. the Excluded Volume Hadron resonance gas (EVHRG) model [41, 48–58]. In fact, as the higher cumulants are expected to be more sensitive to the phase transition, any deviation of the experimental observation from the model calculations may be taken as a sign of new physics.

HRG model is based on the Dashen, Ma and Bernstein theorem [59] which explains that a dilute system of strongly interacting matter can be explained by a gas of free resonances. Further, the attractive interaction is taken care of by these resonance particles. On the other hand, both the long range attractive as well as the short-range repulsive interactions are important, especially at high temperature and/or high-density region, for the description of strongly interacting matter [56]. Furthermore, near critical temperature HRG model calculations tends towards Hagedorn divergence which may be due to the absence of the repulsive interaction [41]. This repulsive interaction part is included through the excluded volume effects in the HRG [49] and is commonly known as EVHRG model. EVHRG equations of state have also been used for the hydrodynamical models of Nucleus-nucleus collision [60–62].

In this chapter, we study the temperature \( T \) dependence of susceptibilities of different conserved quantities such as net-baryon, net-strangeness and net-charge up to order four using HRG / EVHRG model. Baryon-strangeness and charge-strangeness correlation functions have been estimated at different \( T \) and \( \mu_B \). Different values of baryon and meson radii have been used to study their effect on susceptibilities and correlations. We have examined experimental observables in the framework of HRG as well as EVHRG models. The ratio of cumulants of distribution of conserved quantities are related to the ratios of different order of susceptibilities [63–65] such as \( \sigma^2/M = \chi^2/\chi^1 \), \( S\sigma = \chi^3/\chi^2 \) and \( \kappa\sigma^2 = \chi^4/\chi^2 \) where \( M \) is the mean, \( \sigma \) is the standard deviation, \( S \) is the skewness, \( \kappa \) is the kurtosis of the distribution of conserved quantities and \( \chi^n \) are the \( n^{th} \) order susceptibilities. In general, higher order susceptibilities are extra sensitive to the large correlation length and hence the critical point [30]. This signifies that any memory of
the large correlation length retained in the thermal system at chemical freeze-out would be reflected in the behaviour of higher order cumulants. We have studied the energy dependence of ratios of cumulants for net-proton, net-kaon and net-charge and compared with the available result for experimental data of fluctuation of net-proton and net-charge within the proper transverse momentum and pseudo-rapidity acceptance.

2.2 Hadron Resonance Gas Model

In Hadron Resonance Gas (HRG) model, the system of thermal fireball consists of all the hadrons and resonances given in the particle data book [66]. There are varieties of HRG models in the literature. Different versions of this model and some of the recent works using these models may be found in Refs. [31–42, 48–53, 56–58, 67–79]. HRG model is not only successful in describing the hadron yields in central heavy ion collisions from AGS up to RHIC energies [32–39] but also successful in describing the bulk properties of hadronic matter in thermal and chemical equilibrium [40, 41, 67].

The grand canonical partition function of a hadron resonance gas [31, 41] can be written as

$$\ln Z_{id} = \sum_i \ln Z_{id}^i,$$

(2.1)

where sum is over all the hadrons. $id$ refers to ideal i.e., non-interacting HRG. For particle $i$,

$$\ln Z_{id}^i = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)],$$

(2.2)

where $V$ is the system-volume, $g_i$ is the degeneracy factor, $T$ is the temperature, $E_i = \sqrt{p^2 + m_i^2}$ is the single particle energy, $m_i$ is the mass and $\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$ is the chemical potential. $B_i$, $S_i$, $Q_i$ are respectively the baryon number, strangeness and charge of the particle, $\mu$s being corresponding chemical potentials. The upper and lower sign corresponds to baryons and mesons respectively. Partition function depends in general on five parameters i.e., $V, T, \mu_B, \mu_S, \mu_Q$. However, only three are independent, since $\mu_Q$ and $\mu_S$ can be found from conservation of different quantum numbers like baryon number, charge and strangeness [31, 41].

The partition function is the basic quantity from which one can determine various thermodynamic quantities of the thermal system created in the heavy ion collisions. The partial pressure $P_i$, the particle density $n_i$, the energy density $\varepsilon_i$, and the entropy density $s_i$ can be calculated using the standard definitions,
\[ P^{id}_i = \frac{T}{V} \ln Z^{id}_i = \pm \frac{g_i T}{2\pi^2} \int_0^\infty p^2 \, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \]

\[ n^{id}_i = \frac{T}{V} \left( \frac{\partial \ln Z^{id}_i}{\partial \mu_i} \right)_{V,T} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp[(E_i - \mu_i)/T] \pm 1}, \]

\[ \varepsilon^{id}_i = \frac{E^{id}_i}{V} = -\frac{1}{V} \left( \frac{\partial \ln Z^{id}_i}{\partial \frac{1}{T}} \right)_{\hat{\mu},\hat{T}} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i, \]

\[ s^{id}_i = \frac{S^{id}_i}{V} = \frac{1}{V} \left( \frac{\partial (T \ln Z^{id}_i)}{\partial T} \right)_{V,\mu} \]

\[ = \pm \frac{g_i}{2\pi^2} \int_0^\infty p^2 \, dp \left[ \ln \left( 1 \pm \exp\left(\frac{-(E_i - \mu_i)}{T}\right) \right) \pm \frac{(E_i - \mu_i)}{T(\exp((E_i - \mu_i)/T) \pm 1)} \right]. \]

Since Eqs. 2.3 - 2.6 determine the thermodynamic properties of the system, those are called equations of state (EOS) of the system.

In ideal HRG model point like particles are considered. Attractive interactions between hadrons are taken care via a presence of resonance but ignores repulsive interactions. This simple model has only a few parameters. Despite its simplicity, this model is very successful in describing the hadron yields in central heavy ion collisions. The repulsive interactions are also needed, especially at very high temperature and / or large chemical potential, to catch the basic qualitative features of strong interactions where ideal gas assumption becomes inadequate. In the EVHRG model [41, 42, 48–51, 53, 56–58, 74–78, 80], hadronic phase is modelled by a gas of interacting hadrons, where the geometrical sizes of the hadrons are explicitly incorporated as the excluded volume correction to approximate the short-range van der Waals type repulsive hadron-hadron interaction.

Excluded volume corrections were first introduced in [48] but it was thermodynamically inconsistent. A thermodynamically consistent excluded volume correction was first proposed in [49]. In EVHRG model pressure can be written as

\[ P(T, \mu_1, \mu_2, \ldots) = \sum_i P^{id}_i (T, \hat{\mu}_1, \hat{\mu}_2, \ldots), \]

where for \( i \)’th particle the chemical potential is

\[ \hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, \ldots) \]
where \( V_{ev,i} = 4 \frac{4}{3} \pi R_i^3 \) is the volume excluded for the \( i \) th hadron with hard core radius \( R_i \). In an iterative method, one can get the total pressure. Pressure \( P(T, \mu_1, \mu_2, \ldots) \) is suppressed compared to the ideal gas pressure \( P^{id} \) because of the smaller value of effective chemical potential. The other thermodynamic quantities like \( n_i, s, \varepsilon \) can be calculated from Eqs. 2.7 - 2.8 as

\[
n_i = n_i(T, \mu_1, \mu_2, \ldots) = \frac{\partial P}{\partial \mu_i} = \frac{n_i^{id}(T, \hat{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}, \tag{2.9}
\]

\[
s = s(T, \mu_1, \mu_2, \ldots) = \left( \frac{\partial P}{\partial T} \right)_{\mu_1, \mu_2, \ldots} = \frac{\sum_i \varepsilon_i^{id}(T, \hat{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}, \tag{2.10}
\]

\[
\varepsilon = \varepsilon(T, \mu_1, \mu_2, \ldots) = \frac{\sum_i \varepsilon_i^{id}(T, \hat{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \hat{\mu}_k)}. \tag{2.11}
\]

This correction scheme is thermodynamically consistent i.e. EOS after corrections obey the relation

\[
s = \varepsilon + P - \sum \mu_i n_i. \tag{2.12}
\]

### 2.3 Equations of state at \( \mu = 0 \)

In this work we have taken all the hadrons listed in the particle data book [66] up to the mass of 3 GeV. Figure 2.1 shows variation of normalised pressure, energy density, entropy density and number density with temperature at \( \mu = 0 \). Similar figures can also be found in Ref. [41, 58]. The pressure, energy density, entropy density and number density are found to increase rapidly with the increase of temperature. In EVHRG we have considered different values of \( R_b \) and \( R_m \). There is essentially no effect of interaction till \( T = 0.13 \) GeV in all these quantities as can be seen from the figure. The reason for this is that the effective degree of freedom does not increase much up to this temperature and therefore correction due to excluded volume is small. It can be seen from this figure that at large \( T \), all these quantities are reduced by almost 30% compared to the non-interacting HRG model if we take radii of all the hadrons to be 0.2 fm. If we increase radii of hadrons further to 0.3 fm, the suppression is even more. From Fig. 2.1(a), where we plot pressure as a function of temperature, one can see that the continuum limit lattice data from [18, 81] agrees within error-bar with HRG and EVHRG model up to \( T \sim 0.17 \) GeV.
Figure 2.1: Variation of equations of state with temperature at $\mu = 0$. Non interacting means ideal HRG model i.e. without excluded volume correction and interacting refers to EVHRG. $R_m$ refers to radius of mesons and $R_b$ refers to radius of baryons. Lattice data for pressure are taken from Bazavov et.al [81] and Borsányi et.al [18].

The difference of the EVHRG model, as compared to HRG model, is governed by the radius parameter. The electromagnetic charge radii of hadrons have been measured by different groups [82–86]. For example, the radii for $p$, $\Sigma^-$, $\pi^-$ and $K^-$ are around 0.8 fm, 0.9 fm, 0.7 fm and 0.6 fm respectively. One can also define a strong interaction radii [87, 88] which comes out to be around the same values. In accordance with these results, a value of 0.8 fm for baryons and 0.62 fm for mesons were proposed earlier [53]. On the other hand, Braun-Munzinger et al. [34] demonstrated that a more realistic approach is to incorporate repulsive behaviour of the NN potential using hard-core radius ($\sim 0.3$ fm) as obtained from nucleon-nucleon scattering [89]. The corresponding meson radius should not exceed that of baryons. A similar hard-core radius of 0.2-0.3 fm has also been proposed in Ref. [90] to explain the proton-proton scattering data. It has also been shown earlier that to justify a hydrodynamic approach to heavy ion collisions within the hadron phase the hard-core radii of hadrons should be $r \geq 0.2$ fm in EVHRG model [54]. In the present study we have taken an approach similar to [34] and have used different baryon ($R_b$) and meson ($R_m$) radii between 0.2-0.3 fm.
2.4 Equations of state at finite $\mu_B$

![Graphs a, b, c, d showing variation of $P$, $\varepsilon$, $s$, and $n$ with $\mu_B$ at fixed temperatures.]

Figure 2.2: Variation of equations of state with $\mu_B$ at constant $T$ keeping $\mu_S = \mu_Q = 0$.

Figure 2.2 shows $P$, $\varepsilon$, $s$, and $n$ as a function of $\mu_B$ at fixed $T$. Two sets of temperatures e.g., $T = 0.1$ GeV and 0.15 GeV, have been considered. Our results show that there is a small effect of interaction till $\mu_B = 0.4$ GeV in the EOS. Beyond $\mu_B = 0.4$ GeV we see quite a substantial change in all these quantities. The change is more pronounced at higher temperatures. One can see from this figure that at large $\mu_B$ the energy, entropy and number density in EVHRG model are suppressed by a factor of 2 or more, compared to HRG if we take the radii of all the hadrons to be 0.2 fm. This is expected as the finite radius acts as the repulsive interaction between hadrons. If we increase the size of the baryons further to 0.3 fm the suppression is even more. However the thermodynamic quantities are less sensitive to the mesonic radii as can be seen from Fig. 2.2. The plot for $R_b = R_m = 0.3$ fm is almost same as that for $R_b = 0.3$ fm, $R_m = 0.2$ fm and is suppressed compared to $R_b = R_m = 0.2$ fm. This is an expected result as the system is dominated by baryons at high $\mu_B$. 
2.5 Fluctuations of different conserved charges

Derivatives of the grand canonical partition function ($Z$) with respect to the chemical potential define susceptibilities which experimentally become available through event-by-event analysis of fluctuations of conserved charges such as baryon number, electric charge, strangeness and others.

The $n^{th}$ order susceptibility is defined as

$$
\chi_x^n = \frac{1}{V T^3} \frac{\partial^n (\ln Z)}{\partial (\mu_x T)^n},
$$

where $\mu_x$ is the chemical potential for conserved charge $x$. For our present purpose $x = B$ (baryon), $S$ (strangeness) and $Q$ (electric charge).

Expressions of susceptibilities, up to order four, in HRG model can be written as follows:

$$
\chi_x^1 = \sum_i \frac{g_i x_i^1}{2\pi^2 T^3} \int_0^\infty f_i p^2 dp,
$$

$$
\chi_x^2 = \sum_i \frac{g_i x_i^2}{2\pi^2 T^3} \int_0^\infty (f_i \mp f_i^2) p^2 dp,
$$

$$
\chi_x^3 = \sum_i \frac{g_i x_i^3}{2\pi^2 T^3} \int_0^\infty (f_i \mp 3f_i^2 + 2f_i^3) p^2 dp,
$$

$$
\chi_x^4 = \sum_i \frac{g_i x_i^4}{2\pi^2 T^3} \int_0^\infty (f_i \mp 7f_i^2 + 12f_i^3 - 6f_i^4) p^2 dp,
$$

where

$$
f_i = \frac{1}{\exp[(E_i - \mu_i)/T] \pm 1}.
$$

One can calculate susceptibilities in EVHRG model following the method described in the previous section.

2.5.1 Fluctuations at $\mu = 0$

In Fig. 2.3 we have shown temperature dependence of second order susceptibilities for various conserved charges at zero chemical potentials ($\mu_B = \mu_S = \mu_Q = 0$). The second order susceptibilities are found to increase rapidly with increasing temperature. Near $T = 0.1$ GeV magnitude of $\chi_Q^2$ is almost double compared to $\chi_S^2$ and the magnitude of $\chi_B^2$ is almost zero at this temperature. At low-temperature fluctuation of a particular charge is dominated by lightest hadrons carrying that charge. The dominant contribution
to $\chi_B^2$ at low temperatures comes from protons / neutrons (lightest baryon), while $\chi_S^2$ receives leading contribution from kaons (lightest strange hadron) and $\chi_Q^2$ from $\pi^\pm$ (lightest charged hadron). Since pions are lighter compared to proton and kaon, magnitude of $\chi_Q^2$ is more than that of $\chi_B^2$ and $\chi_S^2$. In EVHRG we have considered different values of $R_b$ and $R_m$ as shown in Fig. 2.3. It can be seen that there is almost no effect of interaction till $T = 0.13$ GeV in fluctuations. Similar behaviour was observed in the case of equations of state as well (Fig. 2.1). Above $T = 0.13$ GeV we find quite a large change in second order susceptibilities. One can see from this figure that at large $T$, second order fluctuations are decreased by almost 30% compared to the non-interacting hadrons if we take radii of all the hadrons to be 0.2 fm. If we increase radii of hadrons further to 0.3 fm, the suppression is even more. We have compared our result with LQCD data [15, 16]. It can be seen that up to $T = 0.18$ GeV, $\chi_B^2$ is in good agreement with LQCD if we consider radii of all hadrons to be 0.2 fm whereas $\chi_Q^2$ is in good agreement with LQCD for $R_m = 0.2$ fm and $R_b = 0.3$ fm. The meson radius plays a significant role for $\chi_S^2$ and $\chi_Q^2$ but not for $\chi_B^2$ which can be seen from the figure. This is an expected result since in $\chi_S^2$ and $\chi_Q^2$ both the baryons and mesons contribute whereas in $\chi_B^2$ only baryons contribute. One should, however, note that the dependence of $\chi_B^2$ on $R_m$ is not completely negligible.
In ideal HRG model, under Boltzmann approximation, $\chi_B^4 \approx \chi_B^2$ and $\chi_B^1 \approx \chi_B^3$ as only baryons with baryon number one contribute to various susceptibilities [64]. In contrast, in the case of higher order susceptibilities, electric charge and strangeness are expected to show larger values as hadrons with multiple charge or strangeness get larger weight. A similar behaviour is expected for the EVRHG model as well. In Fig. 2.4 we have shown a variation of fourth order susceptibilities with the temperature at $\mu = 0$. The nature of all the fourth order susceptibilities is similar to second order susceptibilities. As anticipated, magnitudes of fourth order susceptibilities are larger compared to that of second order susceptibilities for strangeness and electric charge. However, this is not true for baryon number fluctuations. Although contributions in $\chi_B$ are only from baryons, these quantities also depend on the size of mesons as can be seen from Fig. 2.3(a) and Fig. 2.4(a). This dependence can be understood from Eqs. (2.7) and (2.8) which shows the dependence of chemical potential on hadronic radii through pressure. Hence, even for baryon number susceptibilities, there may be a small difference in magnitudes of susceptibilities at different $R_m$ for EVHRG. We compare our result with LQCD data ($N_\tau = 6, 8$) [91–93]. The LQCD data for $\chi_B^4$ and $\chi_S^4$ are in excellent agreement with HRG model up to $T = 0.16$ GeV. Whereas for $\chi_Q^4$, LQCD data is lower compared to
both HRG and EVHRG.

### 2.5.2 Fluctuations as a function of $\mu_B$

![Figure 2.5: Susceptibilities $\chi^1_B$, $\chi^2_B$, $\chi^3_B$, $\chi^4_B$ as a function of $\mu_B$ keeping $T$ fixed and $\mu_S = \mu_Q = 0$.](image)

In this section, we discuss the results of fluctuations at large $\mu_B$. Figures 2.5-2.7 show $\chi^x_\mathcal{O}$ as functions of $\mu_B$ for different conserved charges ($\mathcal{O}$) at a fixed temperature and keeping $\mu_S = \mu_Q = 0$.

Figure 2.5 shows the plots of susceptibilities for conserved baryon number. One can see that susceptibilities for all the orders increase with increasing $\mu_B$. Here we consider $T = 0.1$ GeV and 0.15 GeV for the purpose of our illustration. For each temperature, we show variation of the quantities with $\mu_B$ in HRG and EVHRG model with $R_b$ and $R_m$. The effect of interaction is found to be more pronounced at high temperatures. It can be seen from these figures that at high $T$ and large $\mu_B$ the magnitude of fluctuations are suppressed by a factor of 2 or more, compared to HRG model if we take the radii of all the hadrons to be 0.2 fm. At high $\mu_B$ fluctuations are very sensitive to $R_b$ but not to $R_m$ as in this region system is dominated by baryons. $\chi^1_S$ and $\chi^3_S$ are proportional to the odd power of the strange quantum number of the particles. Dominant contribution to $\chi_S$.
comes from $Λ$, the lightest strange baryon with strange quantum number $−1$ (at $μ_S = 0$ strange mesons don’t contribute to $χ^3_S$ and $χ^4_S$ because particle and anti-particle terms are equal in magnitude but opposite in sign). As a result, $χ^3_S$ and $χ^4_S$ remain negative (Fig. 2.6(a) and Fig. 2.6(c)).

As discussed earlier, there are contributions from multiple-charged hadrons in the strangeness and electrical charge sector only, baryon number being always one. Hence, at high $μ_B$ region magnitudes of $χ^3_x$ and $χ^4_x$ are found to be almost double compared to that of $χ^1_x$ and $χ^2_x$ respectively for $x = S, Q$ (Fig. 2.6 and Fig. 2.7), though the change in $χ_B$ remains small as shown in Fig. 2.5. At very small $μ_B$ and given $T$ (0.1 or 0.15 GeV) system is dominated by mesons and $χ_B$ is small. With increase in $μ_B$, system will be populated by proton, neutron as well as hyperons and $χ_B$ will increase. This increase will be sharper for HRG due to the nonexistence of repulsive interaction.

2.6 Correlations among different conserved charges

Correlations among different conserved charges may act as probes of the structure of QCD at finite temperature and / or chemical potential. In QGP, as baryon numbers
as well as electric charges are carried by different flavors of quarks, a strong correlation is expected between B-Q, Q-S as well as B-S. In the hadronic sector, on the other hand, presence of baryons and mesons would generate completely different types of correlations between these quantities. Therefore, these correlations are expected to show changes across the phase transition which are characteristics of the changes in the relevant degrees of freedom.

Correlation functions, $\chi_{xx'}^{ij}$, are defined by

$$\chi_{xx'}^{ij} = \frac{1}{VT^3} \frac{\partial^{i+j}(\ln Z)}{\partial (\mu_x^i) \partial (\mu_{x'}^j)}, \quad (2.19)$$

where $x$ and $x'$ correspond to conserved charges $B,S,Q$ and $\mu$’s are chemical potentials of corresponding conserved charges.

The expression of correlation in HRG model is

$$\chi_{xx'}^{11} = \sum_i \frac{g_i x_i x'_i}{2\pi^2 T^3} \int_0^\infty (f_i + f_i^2) p^2 dp. \quad (2.20)$$

In order to study the experimental results, suitable ratios have been suggested to estimate these contributions [94]. Three ratios, namely, coefficient of baryon-strangeness
correlation $C_{BS}$ [94], coefficient of electric charge-strangeness correlation $C_{QS}$ and coefficient of baryon-electric charge correlation $C_{BQ}$ can be written as [15]

$$C_{BS} = -3 \frac{\chi_{BS}^{11}}{\chi_{S}^{2}}, \quad C_{QS} = 3 \frac{\chi_{QS}^{11}}{\chi_{S}^{2}}, \quad C_{BQ} = \frac{\chi_{BQ}^{11}}{\chi_{B}^{2}}.$$ (2.21)

### 2.6.1 Correlations at $\mu = 0$

![Graphs showing variations of $\chi_{BS}^{11}$, $\chi_{QS}^{11}$, and $\chi_{BQ}^{11}$ with temperature at $\mu = 0$.](image)

Figure 2.8: Variation of $\chi_{BS}^{11}$, $\chi_{BS}^{11}$, and $\chi_{QS}^{11}$ with temperature at $\mu = 0$. Lattice data for continuum extrapolation is taken from Ref. [15].

In this section, we show the temperature dependence of various correlation functions at $\mu_B = 0$. We also compare our result with LQCD data [15]. Fig. 2.8 shows the variation of correlations between conserved charges with temperature around $\mu_x = \mu_{x'} = 0$. Since at very low-temperature main contribution comes from pions (baryon number as well as strange quantum number = 0), all the correlations remain zero. The next hadron to be excited is kaon and as a result $\chi_{QS}^{11}$ becomes non-zero around $T = 0.075$ GeV. The leading contribution to $\chi_{QS}^{11}$ in the hadronic sector is due to charge kaons which have same sign for charge and strange quantum number. So it remains positive and increases with temperature. Other correlations pick up non-zero values approximately
above $T = 0.1 \text{ GeV}$ when baryons start populating the system. $\chi_{BS}^{11}$ is proportional to the product of baryon number and strange quantum number. So here most of the contribution is due to lightest baryon $\Lambda$ which has baryon number $+1$ and strange quantum number $-1$. Other contributing particles, such as $\Sigma$, $\Xi$ etc. also have relative negative sign between baryon number and strange quantum number. As a result, $\chi_{BS}^{11}$ remains negative. This negative value increases with $T$ due to the increase in the population of the strange baryons. Again, HRG results show a sharp increase compared to those from EVHRG model. Due to the contribution from mesons (mainly kaons) in $\chi_{QS}^{11}$, meson radius plays a dominating role and an increase in meson radius produces larger suppression in EVHRG model. On the other hand, baryon radius plays an important role for both $\chi_{BS}^{11}$ as well as $\chi_{BQ}^{11}$. The dominant contribution in $\chi_{BQ}^{11}$ at low temperature is due to proton and antiproton and for both of those baryon number as well as charge carry the same sign. So the value of $\chi_{BQ}^{11}$ remains positive and shows a sharp increase with temperature in HRG, EVHRG being suppressed due to the effect of hard-core radius. In the quark sector strange quarks (antiquarks) carry $1/3$ ($-1/3$) baryon number and the relative sign between baryon and strange quantum number is always negative. As a result lattice result for $\chi_{BS}^{11}$ shows a similar trend as that of HRG. On the other hand, $\chi_{QS}^{11}$ is always positive as charge as well as strangeness for the strange quark is negative. Though the lattice values at lower temperature ($0.15 \text{ GeV}$) is close to HRG and EVHRG, $\chi_{BS}^{11}$ as well $\chi_{QS}^{11}$ for lattice increase faster than those for HRG as strange quark mass is much smaller compared to strange baryons. At high-temperature quark masses decrease (which will eventually become zero at Stefan-Boltzmann limit) and lattice results would start saturating whereas HRG results would keep on increasing. As a result lattice values are suppressed compared to HRG results both for $\chi_{QS}^{11}$ and $\chi_{BS}^{11}$. At high temperature LQCD result of $\chi_{BQ}^{11}$ vanishes as the quarks become effectively massless and the weighted sum of the charges of up, down and strange quarks vanishes [15]. In the case of $\chi_{BQ}^{11}$, EVHRG ($R_m=0.2 \text{ fm}$ and $R_b=0.3 \text{ fm}$) results agree well with the lattice data for $T < 0.17 \text{ GeV}$. But at higher temperatures, the weighted sum of charges of massless quarks vanishes so the lattice results would start saturating with the further increase in temperature whereas HRG results would keep on increasing similar like $\chi_{QS}^{11}$ and $\chi_{BS}^{11}$.

Figure 2.9 shows variation of $C_{BS}$, $C_{QS}$ and $C_{BQ}$ with temperature at $\mu = 0$. We compare our result with LQCD data [15]. At low temperature $C_{BS}$ is much lower than one because the denominator comprises mostly kaon whereas contribution to the numerator comes from baryons, mostly $\Lambda$, which is less populated than kaon. It increases with temperature but remains below one due to the larger contribution from mesons. If all the hadronic radii are same, the interaction effect gets cancelled. On the other hand,
for larger baryonic radii, baryon population gets suppressed and $C_{BS}$ value for EVHRG becomes lower than HRG. In the case of simple QGP system, or more generally, for a system where the quark flavors are uncorrelated, this value will be unity. Since lattice $\chi_{BS}^{11}$ is higher, lattice $C_{BS}$ values remain above the HRG (and EVHRG) and saturates at high temperature.

At very low temperature, contribution to numerator of $C_{QS}$ come from charged kaons only while the contributions to denominator come from all the kaons. So $C_{QS}$ comes out to be around 1.5. With the increase of temperature, lambda starts populating the system and $C_{QS}$ becomes close to 1. Except at very high temperatures, there is no effect of interaction in most of the temperature range. Here, a higher baryonic radius (compared to meson radius) induces a small increase in $C_{QS}$. Interaction effects get cancelled if radii are same for all hadrons. $C_{BS}$ and $C_{QS}$ are related through the equation $C_{QS} = 0.5(3 - C_{BS})$ [15]. So in lattice studies [15] at low temperature, where $C_{BS}$ is very small $C_{QS}$ is near 1.5, same compared to HRG (and EVHRG) and at high temperature tends to one.

At low temperatures, contribution to the numerator of $C_{BQ}$ come from protons only while the contributions to denominator come from both protons and neutrons. As a result
\(C_{BQ}\) is close to 0.5. It decreases slowly with increase in \(T\) as at high \(T\) heavier neutral baryons like \(\Lambda\) start contributing to the denominator whereas only charged baryons contribute to the numerator. For both HRG and EVHRG \(C_{BQ}\) is very close to each other due to the cancellation of interaction effects. In the LQCD, \(C_{BQ}\) approaches zero at high \(T\) as the system is in QGP phase where \(\chi^{11}_{BS}\) is zero. However, at the lower temperature, it approaches the HRG / EVHRG values.

### 2.6.2 Correlations as a function of \(\mu_B\)

![Figure 2.10: \(\chi^{11}_{BS}, \chi^{11}_{BQ}, \chi^{11}_{QS}\) as a function of \(\mu_B\) keeping \(\mu_S = \mu_Q = 0\).](image)

At finite \(\mu_B\) (fixed \(T\)) due to the larger contribution from effective chemical potentials, the effect of interaction, compared to zero \(\mu_B\) case, is more pronounced on correlations as shown in Fig. 2.10. The plots are given for different \(R_m\) and \(R_b\). It can be seen that at low \(\mu_B\), \(\chi^{11}_{BS}\) and \(\chi^{11}_{BQ}\) are zero whereas \(\chi^{11}_{QS}\) is non-zero as pions and kaons are main contributors in this range of parameters. At large \(\mu_B\), baryons start populating the system and both B-S and B-Q correlations increase more sharply compared to \(\chi_{QS}\) as kaons remain the main contributor in \(\chi_{QS}\) and charged hyperon population is much smaller. There is no effect of interaction till \(\mu_B = 0.3\) GeV in \(\chi^{11}_{BS}\) and \(\chi^{11}_{BQ}\) as can be seen from this figure. However at high \(T\), \(\chi^{11}_{QS}\) is affected by interaction even at low \(\mu_B\).
CHAPTER 2. FLUCTUATIONS-CORRELATIONS IN EVHRG MODEL

Figure 2.11: Variation of $C_{BS}$, $C_{QS}$ and $C_{BQ}$ with $\mu_B$ keeping $\mu_S = \mu_Q = 0$.

Figure 2.11 shows variation of $C_{BS}$, $C_{QS}$ and $C_{BQ}$ with $\mu_B$ at a fixed $T$ and $\mu_S = \mu_Q = 0$. At low $\mu_B$, kaon population is dominating in the denominator of $C_{BS}$ and it is less than unity. On the other hand increase in $\mu_B$ facilitates the strange baryon population and $C_{BS}$ increases with increasing $\mu_B$. Further, the effect of large $\mu_B$ becomes more pronounced for lower $T$. So that $C_{BS}$ at $T=0.1$ GeV is larger than that at $T = 0.15$ GeV for large $\mu_B (> 0.6$ GeV). Similarly at low $\mu_B$, $C_{QS}$ is around 1.5 as it gets the contribution from mainly charged kaons in the numerator and both charged and neutral kaons in the denominator. With the increase in $\mu_B$, $\Lambda$ starts populating the system so that $C_{QS}$ keeps decreasing. As this effect is more pronounced for larger $T$, $C_{QS}$ is lower for larger $T$ at large $\mu_B (> 0.7$ GeV).

At low $\mu_B$, for small $T$, $C_{BQ}$ gets the contribution from protons and neutrons in the denominator and only protons in the numerator. So it is close to 0.5 as shown earlier in Fig. 2.9(c). At larger $T$ (0.1 or 0.15 GeV), $\Lambda$ starts contributing in the denominator and $C_{BQ}$ becomes less than 0.5. Similarly as $\mu_B$ increases, the population of protons, neutrons and lambda increases. As a result $C_{BQ}$ becomes less than 0.5. At high $\mu_B$, $C_{BQ}$ saturates as the rate of increase becomes same for different species. For HRG and EVHRG $C_{BQ}$ is close to each other due to the cancellation of the interaction effect.
2.7 Experimental scenario

Experimentally net-charges $N_q (= N_q^+ - N_q^-)$ have been measured in a finite acceptance on an event by event basis. The detailed discussions can be found in Sec. 1.5.1. The mean ($M_q$), variance ($\sigma_q^2$), skewness ($S_q$) and kurtosis ($\kappa_q$) (sec: 1.5.1.3.5), of net-charge distribution are related to the different order of susceptibilities by the following relations

\[ M_q = \langle N_q \rangle = VT^3 \chi_q^1, \]  
\[ \sigma_q^2 = \langle (\delta N_q)^2 \rangle = VT^3 \chi_q^2, \]  
\[ S_q = \frac{\langle (\delta N_q)^3 \rangle}{\sigma_q^3} = \frac{VT^3 \chi_q^3}{(VT^3 \chi_q^2)^{3/2}}, \]  
\[ \kappa_q = \frac{\langle (\delta N_q)^4 \rangle}{\sigma_q^4} - 3 = \frac{VT^3 \chi_q^4}{(VT^3 \chi_q^2)^2}, \]

where $\delta N_q = N_q - \langle N_q \rangle$. The mean, variance, skewness and kurtosis are respectively estimations of the most probable value, width, asymmetry and the peakedness of the distribution.

From the above equations, volume independent ratios can be defined by the following relations

\[ \frac{\sigma_q^2}{M_q} = \frac{C_2}{C_1} = \frac{\chi_q^2}{\chi_q^1}, \]  
\[ S_q \sigma_q = \frac{C_3}{C_2} = \frac{\chi_q^3}{\chi_q^2}, \]  
\[ \kappa_q \sigma_q^2 = \frac{C_4}{C_2} = \frac{\chi_q^4}{\chi_q^2}, \]

where $C_n$ is the $n$th order cumulants (sec: 1.5.1.3.3) of the charge distribution.

The advantage of using the above-mentioned ratios of cumulants is that they are expected to be independent of the volume of the system. In nucleus-nucleus collision experiments, beam energy ($\sqrt{s_{NN}}$) is varied to scan the phase plane. At a particular $\sqrt{s_{NN}}$, if CEP is reached, there will be large fluctuations. Hence along the chemical freeze-out line, one would expect a non-monotonic behaviour of the quantities given in Eq. 2.26 [65, 95]. To compare the ratios of cumulants with experiment one has to know chemical freeze-out $T$ and $\mu$ at a particular $\sqrt{s_{NN}}$. From the experimental information about particle yields or particle ratios, chemical freeze-out temperature and baryonic chemical potential can be extracted [36, 38, 64, 96–99]. In our calculations chemical freeze-out parameters are taken from the Ref. [96].
CHAPTER 2. FLUCTUATIONS-CORRELATIONS IN EVHRG MODEL

Chemical freeze-out curve $T(\mu_B)$ can be parametrized by [96]

$$ T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad (2.27) $$

where $a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV$^{-1}$, $c = 0.053 \pm 0.021$ GeV$^{-3}$. The energy dependence of the $\mu_q$ can be parametrized as [64]

$$ \mu_x(\sqrt{s_{NN}}) = \frac{d_x}{1 + e_x\sqrt{s_{NN}}}, \quad (2.28) $$

where $d_x$ and $e_x$ are listed in table 2.1. Variation of $T$ and $\mu_B$ with $\sqrt{s_{NN}}$ has been shown in Fig. 1.3(b).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$d_x$ (GeV)</th>
<th>$e_x$ (GeV$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>1.308 $\pm$ 0.028</td>
<td>0.273 $\pm$ 0.008</td>
</tr>
<tr>
<td>$S$</td>
<td>0.214</td>
<td>0.161</td>
</tr>
<tr>
<td>$Q$</td>
<td>$-0.0211$</td>
<td>0.106</td>
</tr>
</tbody>
</table>

Table 2.1: Parametrization of chemical potentials $\mu_x$ along the freeze-out curve.

The $T$ - $\mu_B$ parametrisation with $\sqrt{s_{NN}}$ is obtained by fitting the different particle ratios obtained experimentally at different $\sqrt{s_{NN}}$. So for EVHRG model, in general, the chemical freeze-out parameters are expected to be different for different hadronic radii. In our study, considering all the particle ratios, we found the chemical freeze-out parameters to be same as [96] for $R_b = R_m$. On the other hand, for $R_B \neq R_m$ case, for the radii used here, the chemical freeze-out parameter are found to be within $\pm 0.005 - 0.008$ GeV of the above fit. Furthermore, phenomenological chemical freeze-out condition of fixed energy per nucleon around 1 GeV, calculated in our model, matches quite well with the chemical freeze-out parametrization mentioned above. Hence we have used the above mentioned chemical freeze-out parametrization for comparison with the experimental results. It may also be noted here that as $\sqrt{s_{NN}}$ increases from 7.7 GeV to 200 GeV, $T$ increases from around 0.140 GeV to 0.166 GeV and corresponding $\mu_B$ varies between 0.421 GeV to 0.023 GeV. Corresponding $\mu_S$ and $\mu_Q$ are expected to vary between 0.095 GeV to 0.006 GeV and $-0.011$ GeV to $-0.001$ GeV [64].

In terms of transverse momentum ($p_T$) and rapidity ($y$) and azimuthal angle ($\phi$) $d^3p$ and $E_i$ can be written as $d^3p = p_T m_{T_i} \cosh y \, dp_T \, dy \, d\phi$ and $E_i = m_{T_i} \cosh y$, where $m_{T_i} = \sqrt{(p_T^2 + m_i^2)}$. For an example, $\chi_1^x$ in HRG model can be written as

$$ \chi_1^x = \sum_i \frac{g_i x_i}{(2\pi)^2 T^3} \int \int \frac{p_T m_{T_i} \cosh y \, dp_T \, dy}{\exp[(m_i \cosh y - \mu_i)/T] \pm 1}. \quad (2.29) $$
Similarly in terms of $p_T$ and $\eta$, $\chi^1_x$ in HRG model can be written as

$$\chi^1_x = \sum_i g_i x_i \frac{1}{(2\pi)^2 T^3} \int \int \frac{p_T \cosh \eta \, dp_T \, d\eta}{\exp[(\sqrt{(p_T \cosh \eta)^2 + m_i^2 - \mu_i})/T] + 1}.$$

These methods have been used to set the momentum and rapidity acceptance range to compare the present results with the experimental data.

In Fig. 2.12 we show energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for net-proton. We compare our result with experimental data of net-proton fluctuations for $(0 - 5)$% central Au-Au collisions measured the STAR [100] collaboration. Experimental data has been measured at mid rapidity ($|y| < 0.5$) and within the transverse momentum range $0.4 < p_T < 0.8$ GeV. Same acceptance has been used for different hadronic radii in the present work. At low energy, $\sigma^2/M$ is almost unity and its value increases with increase the of $\sqrt{s_{NN}}$ as can be seen from the Fig. 2.12(a). Both HRG and EVHRG model give almost same result irrespective of the value of radii. At low energy $S\sigma$ for HRG is almost unity and its value decreases with increase of $\sqrt{s_{NN}}$ as can be seen from the Fig. 2.12(b). At low $\sqrt{s_{NN}}$, $S\sigma$ for EVHRG is less than that of HRG model and suppression increases with the increase of radii. However, at high $\sqrt{s_{NN}}$, HRG as well as EVHRG
model give almost same result. Experimental data of $S\sigma$ can be described well with EVHRG model for radii of hadrons between $0.3 - 0.4$ fm. At low energy $\kappa\sigma^2$ (Fig. 2.12(c)) in HRG model is slightly less than unity and its value reaches to one as we move to high energy. There is prominent suppression of $\kappa\sigma^2$ in EVHRG model at low $\sqrt{s_{NN}}$. The effect of hadronic volume is more prominent for $\kappa\sigma^2$ because n-th order susceptibility carries upto n-th power term of the distribution function as can be seen from Eqs. 2.14-2.17. Therefore ratios of higher order susceptibilities are more sensitive to hadronic volume. The $\kappa\sigma^2$ of net-proton matches within error-bar with EVHRG model at $\sqrt{s_{NN}} \geq 39$ GeV and $\sqrt{s_{NN}} \leq 11.5$ GeV but at intermediate energies EVHRG model overestimates the experimental data. Departure of experimental data of $S\sigma$ and $\kappa\sigma^2$ for net-proton at intermediate energies ($\sqrt{s_{NN}} = 19$ GeV and 27 GeV) may be an hint of the existence of quark degrees of freedom or different physics process which is not included in HRG/EVHRG model.

![Figure 2.13: Energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for net-kaon.](image)

In Fig. 2.13 we show energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for net-kaon. We choose transverse momentum and pseudo-rapidity within the range $0.2 < p_T < 2.0$ GeV and $-0.5 < \eta < 0.5$ in our calculation. At low energy $\sigma^2/M$ for net-kaon is slightly more than unity and its value increases rapidly with increase of $\sqrt{s_{NN}}$ and result is almost same for both HRG and EVHRG model. Radii of hadrons practically have no
effect on the results as shown in Fig. 2.13(a). $S\sigma$ for net-kaon decreases with increase of $\sqrt{s_{NN}}$ as can be seen from Fig. 2.13(b). Value of $S\sigma$ in EVHRG model is suppressed compared to HRG model and suppression increases with increase of radii of hadrons. In Fig. 2.13(c) energy dependence of $\kappa\sigma^2$ for net-kaon is shown. In this case, volume effect is more prominent, as discussed earlier. $\kappa\sigma^2$ for net-kaon increases with increase of $\sqrt{s_{NN}}$ and then saturates. The value of $\kappa\sigma^2$ is suppressed in EVHRG model compared to HRG model and suppression increases with the increase of radii of hadrons. At all energies, $\kappa\sigma^2$ for net-kaon lie between 0.9 to 1.1 in both HRG and EVHRG.

![Figure 2.14](image-url)

Figure 2.14: Energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for net-charge. Experimental data is taken from Ref. [101].

Figure 2.14 shows the energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for net-charge. We have chosen transverse momentum and pseudo-rapidity within the range $0.2 < p_T < 2.0$ GeV and $-0.5 < \eta < 0.5$ in our model calculation. We compare our results with experimental data of net-charge fluctuations for $(0-5)\%$ central Au-Au collisions measured by the STAR [101] collaboration. Figure 2.14(a) shows $\sigma^2/M$, for net-charge, increases rapidly with increase of $\sqrt{s_{NN}}$ and compared to experimental values our results are suppressed at higher energies. $S\sigma$ for net-charge decreases with increase of $\sqrt{s_{NN}}$ as can be seen from Fig. 2.14(b). Experimental data of $S\sigma$ for net-charge matches within error-bar with HRG/EVHRG model at $\sqrt{s_{NN}} \leq 19.6$ GeV and $\sqrt{s_{NN}} \geq 62.4$ GeV but at other
intermediate energies HRG/EVHRG model over estimate the experimental data. Figure 2.14(c) shows $\kappa \sigma^2$ for net-charge slowly decreases with increase of $\sqrt{s_{NN}}$ and then saturates. At all $\sqrt{s_{NN}}$, $\kappa \sigma^2$ for net-charge lie between $1.4$ to $1.8$ in HRG / EVHRG model which is in agreement with experiment within error-bar at $\sqrt{s_{NN}} \geq 11.5$ GeV. Calculations for ratios of higher order fluctuations of electric charge using LQCD can be found in Ref. [102].

2.8 Discussion and Conclusion

We have shown fluctuations ($\chi^x_i; i = 1 - 4; x = B, S, Q$) of various conserved charges like net baryon number, net strangeness and net charge at finite temperatures and chemical potentials using interacting and non-interacting hadron resonance gas model and compared them with the LQCD as well as the available experimental data.

We can draw following inferences from the present study:

- In general HRG results are larger compared to EVHRG, the difference being higher for higher temperatures and densities.

- At high temperatures and $\mu_B = 0$, compared to HRG, second order fluctuations from EVHRG fits better to LQCD continuum data for the radius of hadrons between $0.2 - 0.3$ fm.

- The LQCD data ($N_T = 6, 8$) for $\chi^4_B$ and $\chi^4_S$ are closer to HRG/EVHRG model up to $T = 0.16$ GeV. Whereas, both HRG and EVHRG model overestimates LQCD data for $\chi^4_Q$.

- Correlations at $T \neq 0$ and $\mu_B = 0$ shows much stronger dependence on the degrees of freedom involved. LQCD continuum data for both $\chi^{11}_{BS}$ and $\chi^{11}_{QS}$ are closer to HRG results (higher than EVHRG) at lower $T$ but rises less sharply and becomes less than HRG for higher $T$. On the other hand, $\chi^{11}_{BQ}$ calculated in LQCD is close to EVHRG results at lower $T$. Initially, it increases with $T$ but then decreases beyond $T = 0.165$GeV as discussed earlier.

- The correlation ratios from both HRG and EVHRG disagree with LQCD continuum data though the trends are qualitatively similar. This difference could be attributed to the fact that LQCD incorporates transition from hadronic to QGP medium, whereas only hadronic phase is present in HRG / EVHRG model. At high $\mu_B$, the magnitudes of susceptibilities and correlations are higher correspond to $\mu = 0$ case.
The susceptibilities, as well as correlations, are found to increase with $\mu_B$ at fixed temperatures for both HRG and EVHRG, the effect of interaction being larger for higher $T$.

Since low energy collisions usually correspond to larger $\mu_B$, the effect of repulsive interaction, as present in EVHRG model, is distinguishable for lower energies. Moreover, this difference is more pronounced for higher order susceptibilities.

With increase of $\sqrt{s_{NN}}$, chemical potential decreases sharply whereas temperature increases slowly. Therefore, $\chi^1$ and $\chi^3$ decrease with increase of $\sqrt{s_{NN}}$ whereas $\chi^2$ and $\chi^4$ decrease slowly and then saturate. As a result $\sigma^2/M$ increases with increase of $\sqrt{s_{NN}}$, $S\sigma$ decreases with increase of $\sqrt{s_{NN}}$ whereas $\kappa\sigma^2$ remains almost constant in all energies.

Although the variations of $\sigma^2/M$ and $S\sigma$ with $\sqrt{s_{NN}}$ seem to describe the experimental data well, ratio of higher order cumulants $\kappa\sigma^2$ shows large deviations. It may be an indication of quark degrees of freedom. It would be interesting to look at the other higher order cumulants as they are supposed to have a stronger dependence on correlation length and hence will be more sensitive to the critical fluctuations.

In general, EVHRG seems to have a better agreement with the LQCD continuum data at low $T$ and $\mu_B = 0$. However, the comparison with the experimental data does not provide us with a clear preference between HRG and EVHRG.

It may be noted that in Figs.2.3(b) and 2.8(a) LQCD results are larger than all the HRG results in the crossover temperature range of $0.14 - 0.15$ GeV. The LQCD may become closer to the HRG by the inclusion of non-PDG additional strange baryons which, though not observed experimentally, are predicted by the quark-model and also observed in the LQCD spectrum [103].

**Bibliography**


[15] HotQCD Collaboration, A. Bazavov et al., *Fluctuations and Correlations of net baryon number, electric charge, and strangeness: A comparison of lattice QCD*


