Chapter 6

Conclusions and future directions

The practical realization of non-Gaussian states are easier rather than that of perfect Gaussian states. These non-Gaussian states are more efficient in studying different quantum mechanical properties. In this thesis, nonlocality of non-Gaussian states which is higher compared to the Gaussian states has been studied. In this chapter, we have briefly summarized the important results obtained in this thesis and discussed some possible future directions.

In the second chapter, we have examined the weak equivalence principle of gravity at the quantum level. We have provided two ways of examination using a non-Gaussian wave packet controlled by a tunable parameter. First, the position detection probabilities of particles described by our tunable non-Gaussian wave-packet projected upwards against gravity around the classical turning point and also around the point of initial projection are calculated. These probabilities exhibit mass dependence at both these points, thereby reflecting the quantum violation of the weak equivalence principle [Chowdhury et al., 2012]. Secondly, we have calculated the mean arrival time of freely falling particles using the quantum probability current, which also turns out to be mass dependent [Chowdhury et al., 2012]. Such a mass dependence is shown to be enhanced by increasing the non-Gaussianity parameter of the wave packet, thus signifying a stronger violation of the weak equivalence principle [Chowdhury et al., 2012] through a greater departure from Gaussianity of the initial wave packet. The mass dependence of both the position detection probabilities and the mean arrival time vanish in the limit of large mass. Thus, compatibility between the weak equivalence principle and quantum mechanics is recovered in the macroscopic limit of the latter. We
have also exhibited a selection of Bohm trajectories to illustrate these features in
the free fall case using our non-Gaussian wave packet [Chowdhury et al., 2012].
It should be noted that the validity of this work is confined to non-relativistic
quantum mechanics. The use of our special kind of non-Gaussian wave packet can
facilitate the experimental verification of the violation of WEQ and can also ver-
ify the quantitative departure of the violation of WEQ from that obtained using
Gaussian nature of the wave packet.

In the third chapter, we have provided an example of non-locality in classical op-
tics. Here, we have considered optical beams with topological singularities which
possess Schmidt decomposition and shown that such classical beams share many
features of two mode entanglement in quantum optics. We have demonstrated
the coherence properties of such beams through the violations of Bell inequality
for continuous variables using the Wigner function. This violation is a conse-
quence of correlations between the \((x, p_x)\) and \((y, p_y)\) spaces which mathemati-
cally play the same role as nonlocality in quantum mechanics. The Bell viola-
tion for the Laguerre-Gaussian beams is shown to increase with higher orbital
angular momenta \(l\) of the vortex beam [Chowdhury, Majumdar, and Agarwal,
2013]. This increase is reminiscent of enhancement of nonlocality for many parti-
cle Greenberger-Horne-Zeilinger states or for higher spins. The states with large
\(l\) can be easily produced using spatial light modulators. The increase of nonlo-
cality is again proved by the corresponding increase of the quadrature correlation
function [Chowdhury, Majumdar, and Agarwal, 2013]. It is possible to experi-
mentally realize the predicted values of the correlation function as a function of
the beam parameters. As two-point correlation functions are used in our Bell’s
inequality, shear Sagnac interferometry [Iaconis and Walmsley, 1996; Singh et al.,
2006; Zhang and Mukamel, 2007] may be used to verify experimentally our pre-
dicted Bell violation and its enhancement for vortex beams with higher angular
momentum.

Non-Gaussian states have correlations higher than second order and to detect
that correlation, one needs some detection criterion that contains higher order
terms. As an example of this and in view of the increasing importance of non-
Gaussian entangled states in quantum information protocols like teleportation and
violations of Bell’s inequalities, the steering of continuous-variable non-Gaussian
entangled states is investigated in the fourth chapter. The EPR steering for Gaus-
sian states may be demonstrated through the violation of the Reid inequality
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In the fifth chapter, we have described how a stronger uncertainty relation will provide a correspondingly stronger steering inequality. Here, we have derived a fine-grained uncertainty relation for the measurement of two incompatible observables on a single quantum system of continuous variables using the operational definition of Wigner function, and have shown that continuous-variable systems are more uncertain than discrete-variable systems [Chowdhury, Pramanik, and Majumdar, 2015]. Using the derived fine-grained uncertainty relation, we have formulated a new steering criterion for continuous-variable systems and have given an application of this for NOON states. Entropic steering criterion is able to demonstrate steerability of NOON states for \( N = 1 \) only [Chowdhury et al., 2014]. But using our derived steering criterion, we are able to reveal the steerability of NOON states for all \( N > 1 \) that has hitherto not been possible using other criteria [Chowdhury, Pramanik, and Majumdar, 2015]. Therefore, our fine-grained steering criterion is
stronger than previously existing ones. We further obtain a monogamy relation for our steering inequality which leads to an, in principle, improved lower bound on the secret key rate of a one-sided device independent quantum key distribution protocol for continuous variables under individual attacks. By considering Alice’s knowledge about Bob’s set of observables, we have calculated the lower bound of the secret key rate as unity [Chowdhury, Pramanik, and Majumdar, 2015] for the shared maximally entangled state of continuous variables, whereas the lower bound is just half for discrete variables. We have used the Wigner function as quasiprobability distribution, which can be experimentally reconstructed by homodyne detection techniques that are currently realizable with high efficiency [Wakui et al., 2007]. Recently, experimental Bell violation is also confirmed [D’Angelo et al., 2006; Jeong et al., 2003; Kuzmich, Walmsley, and Mandel, 2000] using similar techniques for continuous-variable systems. Therefore, this should facilitate the experimental verification of our steering criterion. Further analysis of more general security attacks, as well as consideration of decoherence effects would be needed to assess the practical viability of such key generation protocols.