4. TOPOLOGY OPTIMIZATION

4.1 INTRODUCTION

Recently the topology optimization or layout optimization has become a popular topic in the field of optimal design. Number of research papers published by various authors\textsuperscript{120-192} indicates the significance of the topic. It is necessary to apply difficult mathematical and mechanical tools for the solution even in case of simple structures. The mathematical programming tools have some limitations on the number of design variables. Hence it requires an iterative solution technique to be adopted. In this chapter one can see that the problem of optimizing structural topologies when loads are variable and have a nonzero cost and the
fictitious weight of the structure that contains the cost modified weight of
the elements is the overall measure of the problem.

Classical theories of variable force optimization, based on optimality
criteria and adjoint displacement fields were developed by Rozvany and
Mroz\textsuperscript{165}. Topology optimization for variable external forces were first
discussed in terms of the exact optimal truss topologies, taking the cost
of external forces (e.g. at supports) into consideration. Rozvany\textsuperscript{165}
,Logo\textsuperscript{145,146}, Buhl\textsuperscript{127} assumed that the support costs are independent of
the reactions. Pomezanski\textsuperscript{162} introduced a new aspect of the support
optimization in case of truss structures. To obtain the correct optimal
topology some filtering methods were applied by Diz and Sigmund\textsuperscript{131} to
avoid the so-called checker-board patterns.

4.2 THEORY

Topology Optimization is different from shape optimization because
shape optimization methods work in a range of allowable shapes which
have fixed topological properties.

Topology optimization generates the optimal shape of a mechanical
structure. Given a predefined domain in the 2D/3D space with boundary
conditions and external loads, the intention is to distribute a percentage
of the initial mass on the given domain such that a global measure takes
a minimum. Without any further decisions and guidance of the user, the
method will form the structural shape thus providing a first idea of an
efficient geometry. The design space is discretized by the finite element
method to represent the material distribution and at the same time the structural behavior. Therefore lesser deflections are produced by more material. So, the optimization constraint is the volume of the material. Integration of the selection field over the volume can be done to obtain the total utilized material volume.

Topology optimization can be implemented through the use of finite element methods for the analysis and optimization techniques based on Homogenization method, Optimality criteria method, level set, Moving asymptotes, Genetic algorithms. A brief discussion on these methods is given below.

4.2.1 Homogenization Method or Density Method

The main idea of the homogenization method is to replace the difficult layout problem of material distribution by a much easier sizing problem for the density and effective properties of a perforated composite material obtained by cutting small holes in the original homogeneous material.

The power law approach must be combined with perimeter constraints, gradient constraints or filtering techniques to ensure the existence of solutions. Sigmund\textsuperscript{170}, Gea\textsuperscript{136} presented a microstructure-based design domain method, which employs a closed-form expression for the effective Young’s modulus and shear modulus in terms of phase properties and volume fractions.

For this the material of the structure is represented as a porous continuum with certain periodic microstructure or layered composites of
different ranges of densities. In this method material micro structure is assumed to contain periodic voids of any shape. Using a normal formulation, the density of the element with rectangular voids may be determined by

$$\rho = 1.0 - (1.0-a)(1.0-b)$$  \hspace{1cm} (4.1)

Where \((1.0-a)(1.0-b)\) represent total volume of void in an element. If \(a=b=0\) represent state of void for the element and \(a=b=1\) implies that the element is solid representing the real material and intermediate values of \(a\) and \(b\) represent fictitious material.

### 4.2.2 Performance-Based Topology Optimization Method

Liang\(^{12,13}\) developed the performance-based optimization (PBO) for topology design of continuum structures using Performance indices. Performance-based optimality criteria were proposed and incorporated in PBO algorithms to identify the optimum from an optimization process. In this method practical design requirements are taken into consideration to aim at specific performance level.

In PBO design, strength, serviceability and cost performance requirements must be satisfied by the design. Limiting values specified by the design codes govern the strength and serviceability requirements. The weight of a structure is used as the performance objective is the weight of the structure and performance-based constraints are stresses, displacements and mean compliance. The overall stiffness of the
structure is achieved by minimizing the strain energy. The optimization problems can be stated in mathematical forms as follows:

\[
\text{Minimize } W = \sum_{e=1}^{N} W_e(t) \\
\text{Subjected to } \sigma_{\text{max}} \leq \sigma^* \\
\begin{align*}
    u_j &\leq u_j^* \quad (j=1\ldots m) \text{ or} \\
    C &\leq C^*
\end{align*}
\]

where \( W \) is the total weight of the structure,
\( w_e \) is the weight of the \( e \)th element,
\( t \) is the thickness of all elements,
\( t^l \) is the lower bound on the element thickness,
\( t^u \) is upper bound on the element thickness,
\( N \) is the total number of elements,
\( \text{max } s \) is the maximum von Mises stress of an element in the structure under applied loads,
\( * s \) is the maximum allowable stress,
\( j u \) is the absolute value of the \( j \)th constrained displacement,
\( * j u \) is the prescribed limit of \( j u \),
\( m \) is the total number of displacement constraints,
\( C \) is the absolute value of the mean compliance of the structure,
\( * C \) is the prescribed limit of \( C \).

\textbf{4.2.3 Method of moving asymptotes (MMA)}
Method of Moving Asymptotes was developed by Svanberg\textsuperscript{175}. MMA uses a special type of convex approximation. For each step of the iterative process, a strictly convex approximating sub-problem is generated and solved. Moving asymptotes control the generation of sub problems and stabilize and speed up the convergence.

### 4.2.4 Optimality criteria method

This method was proposed by Prager\textsuperscript{88,89} for solving continuous and discrete systems. This is based on finding suitable criteria for specialized design conditions and developing iterative procedure to find optimum design. Berke and Venkayya\textsuperscript{2} and others used this method for solving discrete systems.

### 4.2.5 Sequential Convex programming

The objective function is approximated by a uniformly convex function, inequality constraints by convex functions, and equality constraints by linear functions. Thus, optimization problem is replaced by a separable, convex, and nonlinear sub-problem which is much easier to solve. Numerical results show the advantages of an interior point method for solving the sub-problem. It is possible to reduce the size of the internally generated linear systems, where the major part of the computing is spent to \( m \), which is favorable when \( m \) is small compared to \( n \) as is the case for topology optimization problems. Zilber, Schittowski and Moritzen\textsuperscript{192}
studied very large scale optimization problems by sequential convex programming methods.

### 4.2.6 Level set method

The level-set method is a numerical method for finding the shapes. Numerical computations can be done on grids with curves and surfaces using level set method. This approach is called the Eulerian approach. Also, the level-set method makes it very easy to follow shapes that change topology, for example when a shape splits in two, develops holes, or the inverse of these operations. For modeling time varying objects Level-set method a great tool.

Xianghua Xing, Michael Yu Wang\(^{183}\) studied topology optimization of cantilever beam using level set method based on streamline diffusion finite element method. Wang and Guo\(^{180}\) also studied level set method for topology optimization.

### 4.2.7 Genetic Algorithm

The GA operates on a representation of the geometry and the simplest form of representation is a (binary) bit-array representation which defines the geometry by an array of ‘on’ and ‘off’ bits (i.e. ones and zeroes) that correspondingly maps onto the design space. Wang and Tai\(^{182}\) investigated structural topology optimization using Genetic Algorithms.

### 4.2.8 Morphological Genetic Algorithm

To overcome the shortcomings of checkerboard patterns and the lack of control over structural connectivity, a morphological representation
had been developed. In this method chromosome representation and a set of genetic operators are designed to increase the geometric characteristics of optimally good designs in the process of evolution.

In the morphological representation, the structure is characterized by a set of input/output locations. Typical support points or the load points are the input locations and the points where the structural behavior is of importance are the output locations. More than two input/locations must be defined in every structure. This is because every structure must have parts that interact with its surroundings by way of at least one fixed support region(input location) and one loading region(output location). The morphological representation scheme has been developed and presented by Tai, K. and Chee, T.H178.

4.3. GENERAL TOPOLOGY OPTIMIZATION PROBLEM STATEMENT

Topological optimization is sometimes referred to as layout optimization by the researchers. The goal of topological optimization is to find the best use of material for a body such that an objective criterion (i.e., global stiffness, natural frequency etc.) is achieved subject to given constraints (i.e., volume reduction)(figure 4.1). Topological optimization does not require optimization parameters (i.e., independent variables to be optimized)to be defined. In topological optimization, the material distribution function over a body serves as optimization parameter.
In topological optimization objective function \( (f) \) is minimized or maximized subject to the defined constraints \( (g_j) \). Densities of each finite element\( (i) \) are treated as design variables \( (\eta_i) \) in the topological problem. The pseudo density for each element varies from 0 to 1; where \( \eta_i \approx 0 \) represents material to be removed; and \( \eta_i \approx 1 \) represents material that should be kept. Mathematically the optimization problem expressed as:

\[
f=\text{minimize or maximize w.r.to } \eta_i
\]

Subjected to

\[
0 \leq \eta_i \leq 1 \quad \text{where } i=1,2,3......N
\]

\[
g_{jl} < g_j < g_{ju} \quad \text{where } j=1,2,3........M
\]

\( N=\text{Number of finite elements} \)

\( M=\text{Number of constraints} \)

\( g_j=\text{Computed } j^{th} \text{ constraint value} \)

\( g_{jl}=\text{lower bound for } j^{th} \text{ constraint} \)

\( g_{ju}=\text{upper bound for } j^{th} \text{ constraint} \)

In the present problem ANSYS software which is robust and with built-in topology optimization module is used to model, analyze and perform topology optimization. The topological optimization process consists of
defining objective and constraints

initializing optimization

executing topological optimization

There are two options available in the ANSYS topology optimization module, optimality criteria (OC) approach which is the default choice and sequential convex programming (SCP) approach.

4.4 MAXIMUM STATIC STIFFNESS DESIGN (Subject to Volume Constraint)

In a static topology optimization problem, the purpose is to determine the material distribution, which optimizes a certain objective function (e.g. minimum compliance, maximum force, maximum displacement) for a structure with given loads and supports, subject to a prescribed volume. The distribution of the material is limited to the design domain, $\Omega$, which forms part of a larger domain which can include areas prescribed to be solid or void. The general topology optimization problem is depicted in Figure 4.1.

In the case of “maximum static stiffness” design subject to a volume constraint, which sometimes is referred to as the standard formulation of the layout problem, for example one seeks to minimize the energy of the structural static compliance (UC) for a given load case subject to a given volume reduction. Minimizing the compliance is equivalent to maximizing the global structural static stiffness. Minimum compliance topology
optimization problems impose a constraint on the amount of material which can be utilized. In this case, the optimization problem is formulated as a special case of equation (4.7), (4.8) and (4.9) as

$$U_C = \text{a minimum w.r to } \eta_i$$ \hspace{1cm} (4.10)

Subjected to

$$0 \leq \eta_i \leq 1 \quad \text{where } i=1,2,3\ldots N$$ \hspace{1cm} (4.11)

$$V \leq V_0 - V^*$$ \hspace{1cm} (4.12)

Where

$$V=\text{Computed volume}$$

$$V_0=\text{Original volume}$$

$$V^*=\text{Amount of material to be removed}$$

**4.5 MAXIMUM DYNAMIC STIFFNESS DESIGN** (Subject to Volume Constraint)

In case of the "Maximum Dynamic Stiffness" design subject to a volume constraint one seeks to maximize the $i$th natural frequency ($\bar{\omega}_i > 0$) determined from a modal analysis subject to a given volume reduction. In this case, the optimization problem is formulated as:
\( \bar{\omega}_i = \text{a maximum w.r to } \eta_i \quad (4.13) \)

Subjected to

\( 0 \leq \eta_i \leq 1 \quad \text{where } i=1,2,3,\ldots,N \) \quad (4.14)

\( V \leq V_0 - V^* \) \quad (4.15)

Where

\( \bar{\omega}_i = \text{i}^{\text{th}} \text{ natural frequency computed} \)

\( V = \text{Computed volume} \)

\( V_0 = \text{Original volume} \)

\( V^* = \text{Amount of material to be removed} \)

Maximizing a specific eigen frequency is a typical problem for an eigen frequency topological optimization. However, during the course of the optimization it may happen that eigen modes switch the modal order. For example, at the beginning we may wish to maximize the first eigen frequency. As the first eigen frequency is increased during the optimization it may happen, that second eigen mode eventually has a lower eigen frequency and therefore effectively becomes the first eigen mode. The same may happen if any other eigen frequency is maximized during the optimization. The sensitivities of the objective function
become discontinuous, which may cause oscillation and divergence in the iterative optimization process. In order to overcome this problem, several mean-eigen frequency functions (Ω) are considered. Hence in the present paper instead of maximizing the fundamental frequency minimization of weighted frequency is considered as the objective function in case 2 as mentioned in the following sections.

4.6 WEIGHTED FORMULATION

Given m natural frequencies (ω₁, ...., ωₘ), the following weighted mean function (Ωₘ) is defined:

\[ Ωₘ = \sum_{i=1}^{m} W_i/ω_i \]  (4.16)

where

ωᵢ = iᵗʰ natural frequency

Wᵢ = weight for iᵗʰ natural frequency

The functional maximization equation (4.16) is replaced with

Ωₘ = a maximum w.r to ηᵢ

4.7 ELEMENT CALCULATIONS
While compliance, natural frequency, and total volume are global conditions, certain and critical calculations are performed at the level of individual finite elements. The shell-93 element used for topology optimization in the present thesis. The total volume, for example, is calculated from the sum of the element volumes; that is,

\[ V = \sum_i \eta_i V_i \]  

(4.17)

\( V_i \) = volume for element i

Elasticity tensor for each element is,

\[ [E_i] = [E(\eta_i)] \]  

(4.18)

where the elasticity tensor is used to equate the stress and strain vector, designed in the usual manner for linear elasticity:

\[ \{\sigma_i\} = [E_i] \{\varepsilon_i\} \]  

(4.19)

where

\( \{\sigma_i\} \) = stress vector of element i

\( \{\varepsilon_i\} \) = strain vector of element i

4.8 NUMERICAL EXAMPLES

4.8.1 Example-1 Cantilever beam
4.8.1.1 Problem definition

The example of cantilever beam is considered from the reference of Xianghua Xing\textsuperscript{183}. The cantilever beam is having the dimensions as length of the beam is 2m, height is 1m and a concentrated load of 1k N. Three cases of load location considered are

- case i Load acting on the middle of the right free edge
- case ii Load acting on the top of the right free edge
- case iii Load acting on the bottom of the right free edge

The maximum volume is 0.5 of the volume of the design domain.

4.8.1.2 Initial Geometry

The cantilever is modeled using 4 key points and one area. The area is descritized using shell-93 elements. Initially a uniform mesh with 100-by-50 shell elements is used, and the size of each element is 0.02. Material properties considered are Elastic modulus as 1 k N/m\textsuperscript{2} and the poisson’s ratio as 0.3. Analysis is done and the volume and deflections are calculated. The initial models are shown in figure 4.2, 4.4, 4.5 respectively for all the above mentioned cases.

4.8.1.3 Structural Compliance Minimization as objective

The objective is to minimize the structural compliance with a constraint as volume reduction by 50%. The design variables are material densities ranging from 0-1. The optimization technique used was optimality criterion method.

4.8.1.4 Discussion of Results
case i Load acting on the middle of the right free edge
The volume was reduced by 50 % and the structural compliance was minimized from an initial value of 145.952 to 61.93 showing an improvement of 57.6% in 32 iterations. The final shape obtained matches well with the reference author\textsuperscript{183} done by level set method. The comparison is shown in figure 4.3.

For case ii of loading the beam at the top free end the optimized shape is arrived in 40 iterations and the density variation is as depicted in figure 4.4. The structural compliance values at the beginning and for the optimum design are 164.92 and 68.68 respectively with a percentage reduction of 58.35%.

For the third case the optimized shape obtained is shown in figure 4.5. The structural compliance was reduced from an initial value of 164.92 to 69.53 showing a reduction of 57.84% in 19 iterations and the density plot of optimized beam is shown in figure 4.5.

4.8.2 Example 2- Deep Beam

In tall buildings and foundations usually deep beams are used. Many researchers suggested numerous design models for deep beams. However, in the case of beams with web openings design manuals offer insight into the design of deep beams. A method commonly suggested for the design of deep beams with openings is strut-and-tie model which is the result of topology optimization with high volume reduction.

4.8.2.1 Problem Definition
This example is taken from the work of Kimmich and Ramm. The dimensions of the beam are as shown in the figure 4.7. The beam is subjected to a line load of intensity 1.0kN/m and its thickness is 0.2m. Modulus of Elasticity, E=100000 kN/m\(^2\) and poisson’s ratio is 0.2. The objective is to minimize the structural compliance with a constraint on volume reduction varying from 30%, 50%, 70%.

**4.8.2.2 Initial geometry**

The beam is modeled in ANSYS using 16 key points. Volumes are created using these key points and are descritized using 10-noded solid-92 elements. The total number of elements is 8412. The line load is applied as shown in the figure 4.7. The support conditions are assumed as fixed. The deep beam is analysed for the given loading and the initial volume of the beam is found to be 9.85 m\(^3\) and maximum nodal displacement is with no stress violation.

**4.8.2.3 Structural Compliance Minimization as objective**

The objective is to minimize the structural compliance with a constraint as volume reduction by 30%, 50% and 70%. The design variables are material densities ranging from 0-1.

**4.8.2.4 Discussion of Results**

The structural compliance was minimized from to 0.0125 to 0.00762, 0.0191 to 0.0092 and 0.0541 to 0.02257 for volume reductions of 30%, 50% and 70% respectively in 14, 28, 32 iterations. The reduction in structural compliance is 39.2%, 51.8% and 58.04% respectively for the
above mentioned cases. The density plots and the iteration histories are presented in the figures 4.8 and 4.9 respectively. Summary of structural compliance values are presented in table 4.1. It clearly noted that more increase in volume reduction results truss like structures. This gives an idea of location of web openings in the beam.

**4.8.3 Example-3 Bridge Pier Problem**

**4.8.3.1 Problem Definition**

This example is taken from the reference of Roopesh kumar and Rao NVR\textsuperscript{105} which in turn is taken from the reference of Viswanatha\textsuperscript{112} and the topology optimization was performed. The dimensions of the bridge pier are; pier length 13.5m, width 1.5m, height 7.2m, pier cap length 22.5m, cross section varying from 1.5m x 2.2m above the pier to 1.5m x 0.8m at the free end. The bed blocks are of size 0.75m x 0.575m x 0.3m. Each end bed block is subjected to the reaction intensity of 1.4609 N/m\textsuperscript{2} and the intermediate bed block with an intensity of 1.3913 N/m\textsuperscript{2}. The problem is shown in figure 4.10. The objective is to minimize the structural compliance for various percentage of volume reduction.

**4.8.3.2 Initial Geometry**

The bridge pier is modeled with 96 key points and 12 volumes in ANSYS. The model is discretized using 10-noded solid-92 elements. The total number of elements in the model is 14808. Loads are applied on each end bed block with a pressure intensity of 1.4609 N/m\textsuperscript{2} and on the
intermediate bed block with an intensity of 1.3913 N/m². Static analysis is carried out and the initial volume of the pier is found to be 90.70 m³. Initial geometry modeled in ANSYS is shown in figure 4.10.

### 4.8.3.3 Discussion of the Results

The structural compliance was minimized from 0.4556 x 10⁷ to 0.2727 x 10⁷, 0.7031 x 10⁷ to 0.3078 x 10⁷ for volume reductions of 30%, 50% respectively. Summary of structural compliance values are presented in table 4.2. The reduction in structural compliance is 40.14%, 56.22% respectively for the above mentioned cases. The density plots and the iteration histories are presented in the figures 4.11 and 4.12. It is observed that more volume reduction (70%) in some cases resulted into more unpractical topologies.

### 4.8.4 Example 4-Inverse Models

#### 4.8.4.2 Problem Definition

In the present paper a flat plate with a central concentrated load is considered initially and the deflection profile is inverted by 180° to get the deflection free inverse model for a specified loading. Free vibration analysis is carried out on the inverse model Block Lanczo’s method in ANSYS software. In the present paper the support conditions considered are

- fixed corners
- simply supported corners
- fixed Edges
simply supported edges

4.8.4.3 Objective functions

Topology optimization of the shell has been carried out, under two different objective functions.

Case 1: Maximization of static stiffness can be achieved by minimization of structural compliance, the constraint on the total material volume of the structure should be reduced to 50% of the initial volume. The solution approach used for minimum compliance problem is optimality criteria approach, which is by default in ANSYS topology optimization module.

Case 2: Maximization of Dynamic stiffness can be achieved by maximizing the weighted frequency (for first five frequencies) with a constraint that total material volume of the structure should be reduced to 50% of the initial volume. The solution approach used for maximum weighted frequency problem is sequential convex programming approach (SCP). In the present case as there is a volume reduction the weighted frequency is reduced.

4.8.4.4 Initial Geometry

Taking the symmetry of the structure as an advantage, a quarter of a shell has been modeled in ANSYS applying symmetry boundary conditions. Initially quarter of the flat plate (5m x5m) is modeled using 4 key points. It is discretized into number of finite elements using 4noded
shell93 elements. Shell93 element in ANSYS has the advantage of taking different thicknesses at 4 nodes. Four thickness variables T1,T2,T3,T4 at four corners of the quarter plate are considered as design variables using a thickness function as mentioned in equation 3.22. The thickness of the plate between the nodes is considered to vary smoothly. Initially thickness of the plate is assumed as 10mm uniform throughout the plate area. Material properties are considered as that of isotropic steel. A concentrated load of 10k N is applied at the centre node of the plate.

4.8.4.5 Discussion of Results

4.8.4.5.1 Corners fixed

In the case of inverse model fixed on four corners

Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was 56292.3 and after 11 iterations it was reduced to 14336.1 with a percentage reduction of 74.53%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 0.4110 and it was reduced to 0.40112 (because of 50% volume reduction) for a volume reduction of 50% in 52 iterations. The convergence accuracy adopted was 0.001.

4.8.4.5.2 Corners simply supported

In the case of inverse model simply supported on four corners
Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was $0.219517 \times 10^7$ and after 22 iterations it was reduced to $738258$ with a percentage reduction of $66.37\%$

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was $0.205546$ and it was reduced to $0.189554$. (because of 50% volume reduction).

4.8.4.5.3 Edges fixed

In the case of inverse model fixed on all edges

Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was $99674.6$ and after 12 iterations it was reduced to $33577.1$ with a percentage reduction of $66.31\%$.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was $0.9053$ and it was reduced to $0.7557$ in 25 iterations. (because of 50% volume reduction).

4.8.4.5.4 Edges simply supported

In the case of inverse model simply supported on all edges
Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was 159088 and after 28 iterations it was reduced to 81637.8 with a percentage reduction of 48.68%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 0.637651 and it was reduced to 0.567352 with a percentage reduction of 11.02% in 17 iterations. The values of initial and optimized frequencies are presented in the table 4.3. The density plots and optimization histories are presented in figures 4.13-4.16 for all the boundary conditions.

4.8.5 Example 5-Cylindrical Shells

4.8.5.1 Problem definition

Same numerical example studied in section 3.9 is considered. The study has been extended for free vibration analysis and topology optimization for different boundary conditions. The concrete shell is subjected to its own weight and a vertical uniform load, for different design criteria. The shell thickness is 50 mm and the structure covers a surface of 6m x 12 m. Young's modulus of the material is 30 GPa and Poisson's modulus is 0.2. The structure is subjected to a vertical uniform load of 5 kN/m². The shell can be supported on the right edges, on the curved ones or on them all at the same time (Fig. ). Topology optimization of the shell has been carried out, under two different objective functions.
**Case 1:** Maximization of static stiffness can be achieved by minimization of structural compliance, the constraint on the total material volume of the structure should be reduced to 50% of the initial volume. The solution approach used for minimum compliance problem is optimality criteria approach, which is by default in ANSYS topology optimization module.

**Case 2:** Maximization of Dynamic stiffness can be achieved by maximizing the weighted frequency (for first five frequencies) with a constraint that total material volume of the structure should be reduced to 50% of the initial volume. The solution approach used for maximum weighted frequency problem is sequential convex programming approach (SCP).

### 4.8.5.2 Initial Geometry

In the present analysis, the shell structure is modeled in ANSYS using nine key points, two straight lines for the right edges and the rest eight by segmented cubic splines. Areas are generated and discretized using shell-93 elements. The height of the shell structure considered is 3m in the model. Various boundary conditions considered are (i) Right edges supported (ii) Curved edges supported (iii) Right and Curved Edges supported.

The shell structure is analysed and initial volume is found to be 5.3456m³ for all the cases and the initially fundamental frequencies found from the modal analysis are 0.4269 Hz, 0.9816 Hz, 3.2972Hz for
right edges simply supported, curved edges simply supported and all the four edges simply supported respectively.

4.8.5.3 Results and Discussions

4.8.5.3.1 Shell supported on right edges:

In the case of shell supported on right edges for

Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was 3511.28 and after 31 iterations it was reduced to 1964.03 with a percentage reduction of 44.07%.

Case 2: With an objective function of maximizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 45.2509 and it was reduced to 23.7568 with a percentage reduction of 47.5% in 31 iterations.

4.8.5.3.2 Shell supported on curved edges:

In the case of shell supported on curved edges for

Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was 28562.9 and after 18 iterations it was reduced to 13334.2 with a percentage reduction of 53.3%.

Case 2: With an objective function of minimizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of
weighted frequency was 25.5262 and it was reduced to 14.8309 with a percentage reduction of 49% in 32 iterations.

**4.8.5.3.3 Shell supported on four edges:**

In the case of shell supported on four edges for

Case 1: With an objective function of minimizing the structural compliance with a constraint on volume reduction by 50%, initially the structural compliance was 1534.61 and after 19 iterations it was reduced to 1060.91 with a percentage reduction of 30.87%.

Case 2: With an objective function of minimizing the weighted frequency with a constraint on volume reduction by 50%, initially the value of weighted frequency was 156.12 and it was reduced to 89.58 with a percentage reduction of 42.62% in 32 iterations. Initial and optimized values of first five fundamental frequencies are presented in table 4.1.

The density plots of topology optimization for case 1 and case 2 for all the boundary conditions are presented in figure 4.18 and figure 4.19 respectively. The iteration histories of case 1 and case 2 for objective function and constraint are presented in figure 4.20 and figure 4.21 respectively.

In all the cases the initial volume was 5.334 m$^3$ and was reduced by 50% to 2.6728 m$^3$. 
Table 4.1 Deep beam Problem: Initial and optimum values of Structural Compliance for different percentages of volume reduction

<table>
<thead>
<tr>
<th>Percentage Reduction</th>
<th>Initial</th>
<th>Final</th>
<th>No. of iterations</th>
<th>Percentage reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% Volume Reduction</td>
<td>0.0125</td>
<td>0.00762</td>
<td>14</td>
<td>39.2%</td>
</tr>
<tr>
<td>50% Volume Reduction</td>
<td>0.0191</td>
<td>0.0092</td>
<td>28</td>
<td>51.8%</td>
</tr>
<tr>
<td>80% Volume Reduction</td>
<td>0.0541</td>
<td>0.02257</td>
<td>32</td>
<td>58.04%</td>
</tr>
</tbody>
</table>

Table 4.2 Bridge Pier Problem: Initial and optimum values of Structural Compliance for different percentages of volume reduction

<table>
<thead>
<tr>
<th>Percentage Reduction</th>
<th>Initial</th>
<th>Final</th>
<th>No. of iterations</th>
<th>Percentage reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>30% Volume Reduction</td>
<td>0.4556</td>
<td>0.2727</td>
<td>7</td>
<td>40.14%</td>
</tr>
<tr>
<td>50% Volume Reduction</td>
<td>0.7031</td>
<td>0.3078</td>
<td>14</td>
<td>56.22%</td>
</tr>
<tr>
<td>70% Volume Reduction</td>
<td>0.1291</td>
<td>0.04323</td>
<td>20</td>
<td>66.49%</td>
</tr>
</tbody>
</table>
Table 4.3 Inverse Shell Model Problem: Initial and optimized frequencies of inverse models (with weighted frequency as objective)

<table>
<thead>
<tr>
<th>Support condition</th>
<th>S.no</th>
<th>Initial frequencies (Hz)</th>
<th>Optimized frequencies (Hz) after 50% volume reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corners fixed</td>
<td>1</td>
<td>0.1205</td>
<td>0.1189</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.1739</td>
<td>0.1600</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.2790</td>
<td>0.3324</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.3107</td>
<td>0.3483</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.3807</td>
<td>0.4253</td>
</tr>
<tr>
<td>Corners simply supported</td>
<td>1</td>
<td>0.0162</td>
<td>0.0185</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0283</td>
<td>0.0188</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.13605</td>
<td>0.1251</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.2067</td>
<td>0.1812</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.3637</td>
<td>0.3692</td>
</tr>
<tr>
<td>Edges fixed</td>
<td>1</td>
<td>0.1733</td>
<td>0.1264</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3419</td>
<td>0.2820</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.4305</td>
<td>0.3485</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.4818</td>
<td>0.4700</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.5400</td>
<td>0.4832</td>
</tr>
</tbody>
</table>
Table 4.4 Cylindrical Shell Problem: Initial and optimum Eigen frequencies for all boundary conditions of cylindrical shells for weighted frequency as objective

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>All edges supported</th>
<th>curved edges supported</th>
<th>straight edges supported</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Frequency</td>
<td>Final Frequency</td>
<td>Initial Frequency</td>
</tr>
<tr>
<td>1</td>
<td>3.2928</td>
<td>2.2249</td>
<td>0.97937</td>
</tr>
<tr>
<td>2</td>
<td>4.1838</td>
<td>2.7023</td>
<td>1.1213</td>
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<tr>
<td>3</td>
<td>6.1465</td>
<td>3.2190</td>
<td>2.4098</td>
</tr>
<tr>
<td>4</td>
<td>6.2996</td>
<td>3.9834</td>
<td>2.4181</td>
</tr>
<tr>
<td>5</td>
<td>6.4462</td>
<td>4.0440</td>
<td>3.4521</td>
</tr>
</tbody>
</table>

Figure 4.1 General Topology optimization Problem
Figure 4.2 Cantilever beam problem: Initial ANSYS model with (a) mid end point load (b) Optimum density plot

Figure 4.3 (a) Cantilever beam problem: End load at middle Edge: Optimized model after 32 iterations (b) Cantilever beam problem: Optimized model by the reference author using level set method

Figure 4.4 Cantilever beam problem: End point load at top Edge and Optimized Density plot after 40 iterations
Figure 4.5 Cantilever beam problem: End load at bottom edge: Optimized Density plot after 19 iterations

Figure 4.6 Deep Beam Problem: Geometry

Figure 4.7 Deep Beam problem: ANSYS model

<table>
<thead>
<tr>
<th>Reduction of Volume by</th>
<th>30%</th>
<th>50%</th>
<th>80%</th>
</tr>
</thead>
</table>
Figure 4.8 Deep Beam Problem: Density plots for various cases of volume reduction

<table>
<thead>
<tr>
<th>Set No Vs Structural Compliance</th>
<th>Set No Vs Volume</th>
</tr>
</thead>
</table>

Figure 4.9 Deep Beam Problem: Iteration Histories for objective function and constraint for various % of volume Reductions
Figure 4.10 Bridge Pier Problem: (a) Geometry and (b) ANSYS model using solid-92 elements

![Figure 4.10 Bridge Pier Problem: (a) Geometry and (b) ANSYS model using solid-92 elements](image)

<table>
<thead>
<tr>
<th>Reduction of Volume by 30%</th>
<th>Reduction of Volume by 50%</th>
</tr>
</thead>
</table>

Figure 4.11 Bridge Pier Problem: Density plots for various cases of volume reduction

![Figure 4.11 Bridge Pier Problem: Density plots for various cases of volume reduction](image)

<table>
<thead>
<tr>
<th>For a Volume reduction of 30%</th>
<th>For a Volume reduction of 50%</th>
<th>For a Volume reduction of 70%</th>
</tr>
</thead>
</table>

Figure 4.12 Bridge Pier Problem: Iteration Histories for objective

![Figure 4.12 Bridge Pier Problem: Iteration Histories for objective](image)
function

<table>
<thead>
<tr>
<th>Corners fixed</th>
<th>Corners simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edges fixed</td>
<td>Edges simply supported</td>
</tr>
</tbody>
</table>

Figure 4.13 Inverse Shell Model problem: Density plots for various support conditions for minimizing the structural compliance as objective for inverse models
Corners fixed

Corners simply supported

Edges fixed

Edges simply supported
Figure 4.14 Inverse Shell Model problem: Density plots for various support conditions for minimizing the weighted frequency as objective for inverse models

<table>
<thead>
<tr>
<th>Corners fixed</th>
<th>Corners simply supported</th>
<th>Edges fixed</th>
<th>Edges simply supported</th>
</tr>
</thead>
</table>

Figure 4.15 Inverse Shell Model problem: Optimization History for various support conditions for minimizing the structural compliance as objective Vs iteration number for inverse models

<table>
<thead>
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<th>Corners fixed</th>
<th>Corners simply supported</th>
<th>Edges fixed</th>
<th>Edges simply supported</th>
</tr>
</thead>
</table>

Figure 4.16 Inverse Shell Model problem: Optimization History for various support conditions for minimizing the weighted frequency as objective Vs iteration number for inverse models
Figure 4.17 Cylindrical Shell problem: Figure showing initial shapes of the concrete shell for various edge conditions

<table>
<thead>
<tr>
<th>Density plot</th>
<th>Un averaged Density plot</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Density plot" /></td>
<td><img src="image2.png" alt="Un averaged Density plot" /></td>
</tr>
</tbody>
</table>

**Legend:**
- **Straight edges supported**
- **Curved edges supported**
- **All edges supported**
Both edges supported

Figure 4.18 Cylindrical Shell problem: Density plots for minimum structural compliance case for various support conditions

<table>
<thead>
<tr>
<th>Density plot</th>
<th>Un averaged Density plot</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Density plot" /></td>
<td><img src="image2.png" alt="Un averaged Density plot" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Density plot" /></td>
<td><img src="image4.png" alt="Un averaged Density plot" /></td>
</tr>
<tr>
<td><strong>Straight edges supported</strong></td>
<td><strong>Curved edges supported</strong></td>
</tr>
</tbody>
</table>
both edges supported

Figure 4.19 Cylindrical Shell problem: density plots for minimum weighted frequency case for various support conditions

<table>
<thead>
<tr>
<th>Objective function Vs iteration number</th>
<th>Constraint Vs iteration number</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph 3" /></td>
<td><img src="image4.png" alt="Graph 4" /></td>
</tr>
</tbody>
</table>

Straight edges supported
Curved edges supported

![Graph](image1.png)

both edges supported

Figure 4.20 Cylindrical Shell problem: Iteration history for minimum structural compliance case for various support conditions

<table>
<thead>
<tr>
<th>Objective function Vs iteration number</th>
<th>Constraint Vs iteration number</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Straight edges supported

![Graph](image4.png)

Curved edges supported

![Graph](image5.png)