2. REVIEW OF LITERATURE ON STRUCTURAL OPTIMIZATION

2.1 GENERAL PERSPECTIVE

This second chapter deals with the literature review on Shape and Topology optimization performed separately and also on integrated Thickness/Shape and topology optimization. Today, optimization is one of the most discussed topics of engineering. Structural designing methods usually depend on the formulae and result in a feasible design may not be necessarily an optimum one. This leads to choose an optimal layout of a certain structure or a structural component from the available domain of solutions which represent a physical model of the actual problem. Therefore optimized designs produce highly efficient and reliable results.

Thickness/Shape and topology optimization are the major objectives of structural optimization. The significant progress in these fields is a parallel progress in the fields of structural analysis, mathematical programming, geometric modeling and computer hardware. Specialized structural optimization software also emerged subsequently, utilizing more advanced approximation technology for enhancing overall efficiency.

Optimization theory focuses on the quantitative study of optima and methods for finding them. Real world problems are very complex and Classical methods of optimization are not sufficient to solve them. The
solution method developed in the past cannot tackle the complex problems in various fields of Engineering and science. Various review papers\textsuperscript{1-20}, technical papers\textsuperscript{21-15}, Master’s/Ph.D thesis\textsuperscript{216-225} and textbooks \textsuperscript{226-245} published in this area reflect the increasing interest in this field of structural optimization.

2.2 REVIEW OF LITERATURE

2.2.1 Thickness/Shape Optimization

Structural shape optimization combines mathematics and mechanics with engineering and has become a multidisciplinary field. It has wide applications in almost all the fields of engineering and science. In this form of optimization the topology of structure is known before but there can be some part and/or detail of the structure which can produce problems. Therefore the objective is usually to find the best shape that will result in the best solution. Parameters of shapes are dimensions of the optimized parts or a set of variables describing the shape such as coefficients of spline functions. In order to establish the functional requirements of structural shape optimization scheme it is desirable to examine the objectives

- To achieve a safe design which simultaneously satisfies constructional/manufacturing and functional requirements, and
- To reduce the cost of the construction/manufacture and maintenance of a structure or structural component.
Shape optimization is a mathematically inverse problem. Extensive literature survey on structural thickness/shape optimization can be found in the review papers by Barnes, Berke and others, Byugman, Jaroslav Mackerle, Pathak, Haftka and Grandhi, Ding and in books by many authors such as Allaire, Gajewski and Zyczkowski, Rao, S.S, Zienkiewicz and Campbell.

Various researches have proposed different methods for shape optimization. Bletzinger proposed inverse technique for form finding of shell structures. Authors such as Bennet and Botkin, Rasmussen, Kimmich and Ramm, Braibant and Fleury developed computer aided optimum design tools for plane stress/strain problems. A good introduction into shape optimization problems can be found in Gotsis, Botkin, Zolesio, Ramm, Bletzinger and Kimmich.

From the mathematical point of view, two representations of variables - continuous and discrete ones can be found within the shape optimization area. The first contributions to the analysis and theory of shape optimization of continuum structures was by Zienkiewicz and Campbell and the overview can be found in the works of Liang and Vanderplaat and the research on structural optimization of discrete structure is comprehensively summarized by Gutkowski and Bauer.

Authors such as Hinton and N.V.R. Rao, Hartman and Neumann studied the structural design and optimization of folded plates and prismatic shells using finite strip method with different objectives such
as strain energy minimization, maximizing the fundamental frequency etc. M. Ozacka and Tyasi\textsuperscript{86} conducted study on box-girder section straight and curved in plan for free vibrations.

Olhoff\textsuperscript{85} investigated optimization of circular plates for free vibrations for maximizing the fundamental frequency for a specific value of material volume. Shape optimization of structures for free vibrations was investigated by Bratus\textsuperscript{38}, Mota Soares\textsuperscript{77}, Bletzinger, Reitinger and Maute\textsuperscript{94}, Hemada\textsuperscript{53}, Thambiratnam and Thevendran\textsuperscript{43}, Roozen kroon\textsuperscript{222}, Kandasamy and Singh\textsuperscript{65} and other researchers.

Many of the researchers\textsuperscript{36,226} proposed general approaches for representation of shape using nodes, polynomials, spline control points and key points as design variables in obtaining the optimum shape.

- Zienkiewicz and Campbell\textsuperscript{118} used nodal coordinates
- Bhavikatti and Rama Krishnan\textsuperscript{30} used polynomials
- Braibant and Fleury\textsuperscript{37} used control points of quadratic bezier and cubic spline curves
- Antonio Tomas and Pascual Marti\textsuperscript{25}, Hinton\textsuperscript{55-59}, Rao and Seinz\textsuperscript{62}, Ozakca\textsuperscript{220}, Rao and Ozakca\textsuperscript{61} used position vectors of key points.
- Some authors such as Rupesh kumar and Rao\textsuperscript{107} defined direction movement of design variables to obtain the optimum shape.

Hyman\textsuperscript{64} carried out optimum design of cylindrical shells subjected to lateral pressure for stability requirements. Rotter\textsuperscript{103} stressed the need of new research trends on shell structures. Chapelle and others\textsuperscript{39} studied
the basic design aspects for the finite element analysis of shell structures. Studies on Buckling of cylindrical shells subjected to external pressure was carried by Araar\textsuperscript{26} to arrive at the new shapes. Adali\textsuperscript{21} carried investigations on optimization of laminated shells under axial load and external pressure considering sensitivity aspects. Sakamoto\textsuperscript{108} introduced a method of structural optimization by genetic algorithms. Optimal sizes of rotating disks were investigated by Bhavikatti and Ramakrishnan\textsuperscript{29}.

A few authors have developed computer aided optimum design tools. A great deal of research on structural dynamic optimization of beams, plates and shells, has been conducted during the past three decades. Leissa\textsuperscript{238} provided an extensive review of vibration analysis of shells that includes a various mathematical techniques and listed the merits and demerits of various shell theories.

Most of the work treated the dynamic frequency as a design constraint. Heller and others\textsuperscript{52} used standard non-gradient finite element methods in shape optimization. Some authors\textsuperscript{109} used gradient-less methods for shape optimization. Gradient-less methods avoid the singularities in calculating the eigenvalue derivatives with respect to the design variables due to the multiplicity of the objective function, which was measured by maximization of a weighted-sum of the system natural frequencies. Optimization of axisymmetric thin-shallow shells for maximum fundamental frequency was considered by Bratus\textsuperscript{38}. The
boundary conditions, material, surface area and uniform thickness of the shell were specified as constraints. Bimodal formulation was used and an iterative procedure based on the optimality condition was implemented. Akl\textsuperscript{24} and others presented a process for the design of stiffened shell structures subjected to hydrostatic pressure. Design variables included the number of stiffeners and their cross sectional dimensions. A multi-criteria optimization approach was utilized to select the optimal values of the design variables that minimize the maximum amplitude of vibration, sound intensity level and weight of rings, as well as the total cost of the structure.

Subsequently, some other review articles were published in the literature by others. Qatu\textsuperscript{10-11} published review articles on shape optimization and covered the period of 1989 – 2000. N. Camprubi, K. U. Bletzinger\textsuperscript{78} have adopted inverse method to obtain deflection free optimum shapes.

Hernendez and Fontn\textsuperscript{54}, Lindby and Santos\textsuperscript{72} presented 3D-shell optimization procedures. Xie and Steven introduced evolutionary structural optimization (ESO). In the ESO method, inefficient finite element removal is dependent on sensitivity numbers to generate optimal designs.

In recent years many researchers worked out arriving at optimum shapes mimicking the nature by genetic algorithms and neural networks. Genetic algorithm in shape optimization was presented by
Sakamoto and Takada. A method for shape generation (morphogenesis) and structural optimization of a reinforced concrete roof shell, based on the application of a genetic algorithm and computer aided design was presented by Alberto Pugnale, Mario Sassone, Renner, G. and Ekart optimized the axisymmetric shells using genetic algorithm optimization.

The present work deals with the free vibration analysis of paraboloid shell structures obtained using inverse method, essentially the extension of the authors (Camprubi and Bletziengar). However, no attempt was made to optimize the inverse models further as they are inherently satisfying minimum strain energy and no bending criteria.

A few researchers proposed various convergence criteria for the optimum shape. Many of the researchers dealt with the structural shape optimization with different objectives and behaviour and geometric constraints and much literature is available on shape optimization. To the author’s knowledge, probably no attempted is made to optimize the mathematical inverse models. Recently author has carried out such investigations. The optimum shapes of deflection free inverse shell models for different objectives are investigated for various support conditions is investigated and the examples are presented. Besides the inverse models optimum shape were found for cylindrical shells with three support conditions and prismatic shells i.e box girder sections for bridges which are straight and curved in plan.
2.2.2 Topology Optimization

For the past three decades tremendous improvement is noticed in the application of topology optimization in generating efficient design concepts. Current structural optimization software applications have built-in topology optimization modules in addition to shape optimization capabilities. Altair Opti-Struct is one such efficient tool appeared in the last decade. However, special optimization modules equipped with fewer analysis capabilities than general FEM codes offer higher efficiency for optimization. The reasons for this are

- highly specialized codes are typically smaller and therefore more flexible for incorporating the latest developments than general codes, and
- for specialized codes, highest priority is devoted to its core technology of optimization.

Now-a-days topology optimization has been performed separately while thickness and shape optimization can be combined into a single process. Topologically optimized models as considered as base models in the initial stages of design for performing shape optimization. But industrial users stress on the possibilities to consider the interaction of some thickness and shape parameters along with topology optimization.

An extensive literature on topology optimization of shell structures was published by reviewers like O. Sigmund\textsuperscript{18}, M.P. BendsHe\textsuperscript{232,233},
Rozvany\textsuperscript{14-16}, Jaroslav Mackerle\textsuperscript{6} and in books by Sigmund, O.\textsuperscript{232}, Rozvany, G.I.N\textsuperscript{242}.

M.P. BendsHe and N. Kikuchi,\textsuperscript{124,125} optimized continuum structures topologically and applied to many structural design problems. Suzuki and Kikuchi\textsuperscript{174} considered topology optimization to generate the stiffest design of elastic structures using the homogenization method. Duysinx and BendsHe\textsuperscript{133} investigated topology optimization techniques for continuum structures subjected to local stress constraints.

Beckers\textsuperscript{123} studied linearly elastic continuous structures for topology optimization. His study considers material distribution in a given domain with an objective of structural compliance minimization. This problem was treated as discrete because the design variables take two values representing the material presence and absence (1 or 0).

Authors such as Anantha suresh, Kota, Kikuchi\textsuperscript{120}, Nishiwaki, M.I. Frecker and S. Min\textsuperscript{153} studied extensively compliant mechanism design using topology optimization techniques. The optimal stiffener design of shell/plate structures with the small deformation was studied by Luo and Gea\textsuperscript{147}, Pedersen\textsuperscript{161}, Chen and Wu\textsuperscript{129}, Gea and Fu\textsuperscript{136} carried studies on linearly elastic structures to maximize eigen frequencies for topology design. Hammer and Olhoff\textsuperscript{138} studied topology optimization of continuum structures subjected to pressure loading.
Consideration of Geometrical and material nonlinearities is a must to arrive at more practical designs in real world problems subjected to large deformations. Under large deformation, the structure exhibits nonlinear behavior which is because of two reasons i.e geometric non linearity and material non linearity. Geometric non linearity requires equilibrium updation and the material non linearity results in material model failure. Buhl\textsuperscript{127} and others studied topology optimization with comparison between linear analysis geometrically nonlinearity.

Combining Homogenization method and Optimality Criteria (OC) algorithm resulted optimum topology of continuum structures for given loading and boundary conditions. The results represented true behavior of optimal structure. This idea has opened up a new technique in structural optimization by many researchers like Bendsoe and Kikuchi\textsuperscript{124}, Bendsoe\textsuperscript{232}, Patnaik, Berke and others\textsuperscript{160} and it has been successfully applied to find the optimum topology of a linearly elastic structure for its global stiffness. This combination yielded good results for dynamic problems. In fact, the dynamic problem is more important than the static one in design practice, e.g., in a car body design, Bridge design, design of structures subjected to ground excitation.

Multiple load cases was discussed by Stolpe and Svanberg\textsuperscript{173} with the objective of minimum compliance to achieve optimum topology. The method of moving asymptotes (MMA) was introduced by Svanberg\textsuperscript{175}. To prove global convergence and to stabilize the algorithm, Zillober and
others added a line search procedure known as sequential convex programming (SCP) method. Paris, J., Mu, I., Navarrina, F., Colominas, I., Casteleiro M presented a weight minimization technique to obtain optimal topology with local stress constraints.

A heuristic approach for energy absorption applications was presented by Soto. Research works on topology optimization with material nonlinearity can also be found. However, reports on topology optimization of inverse models are not found in literature.

Z.-D. Ma, N. Kikuchi, I. Hagiwara considered a frequency response optimization problem for both the optimal layout and the reinforcement of an elastic structure. Many researchers studied drawbacks in Optimality criteria method. Generally SCP (Sequential convex programming) method is used for frequency problems. Zillober, Schittkowski and Moritzen very large scale optimization by Sequential convex programming.

Recently a new topology optimization scheme called the Element Connectivity Parameterization (ECP) Method was proposed for geometrically nonlinear structures. This method disconnects discretized elements, indirectly connects them by zero-length links and optimizes stiffness values of these links. Pomezanski studied changing the connections of structural elements during optimization process.

In the stiffness optimization problem, minimizing the structural compliance is the objective. But in optimization of vibrating structures, it
is necessary to consider several different cases: the eigenvalue problem, frequency response problem, transient response problem and others. These different problems may require different solution techniques, and solutions may be of very different natures. Bendshe, Diaz and Kikuchi applied the technique to obtain dynamically stiff structure. These problems were treated based on the OC algorithm.

O. Sigmund and P.M. Clausen adopted mixed Displacement-pressure (incompressible) formulation a new way to solve the structures subjected to pressure loads. Extensive work on optimum topology of discrete structures such as trusses and grid-type structures was done by Kirsch and Rozvany. For finding optimum topology Zhou, Pagaldipti, Thomas and Shyy investigated implementation of FEM codes with certain capabilities of topology optimization. Soto studied Optimal structural topology design for energy absorption of structures using heuristic approach.

Extensive studies on ESO have been published in the literature. The ESO technique was extended to a bi-directional evolutionary structural optimization (BESO) method, which allows for elements to be removed from the structure as well as to be added to the structure.

The performance-based optimization (PBO) method has recently been developed by Qing Quan Liang for topology design of continuum structures with stress, displacement and mean compliance constraints.
Topology optimization with genetic algorithms was studied with morphological geometric representation scheme by Tai and Akhtar\textsuperscript{176}, Tai and Chee\textsuperscript{177}, Wang and Tai\textsuperscript{178}.

In the present thesis Optimality Criterion (OC) Method is used for minimum compliance problems and Sequential Convex Programming (SCP) Method is used for frequency maximization problems for cantilever beam, Deep beam, Bridge pier, Inverse shell model and cylindrical shells for various support conditions.

2.2.3 Integrated Topology and shape optimization

In Recent years an integrated approach was developed to provide the user with the freedom of combining sizing, shape, and topology optimization in a single process. Design experience is required to set the initial topology of the structure as it is difficult to change it during the optimization process. For example, if a singly connected domain is assumed at the initial design, the final structure obtained by a usual shape optimization method will remain the same topology as the initial one, although the truly optimal structure may have holes in the domain.

Research in combined topology and shape optimization of beams was carried by many researchers\textsuperscript{193-215} for the past two decades. Computer-aided structural topology and shape optimization was developed by Olhoff\textsuperscript{211}. It is stated that topology optimization is a priori for detailed shape optimization. With the Objective of minimizing the maximum strain, Baud\textsuperscript{197} and Neuber\textsuperscript{209} developed a theory describing the
optimum contour for components. They have extended the work of Schnack\textsuperscript{212}, changing the given surface and area. Schnacks’\textsuperscript{212} gradient-less shape optimization was modified and the related optimization strategy according to the rules of nature by Mattheck\textsuperscript{75,76}. The same team studied the optimum criteria with respect to sizing and topology optimization purposes of 2D and 3D structures, loaded by single and multiple load cases.

Some authors like Ramm and Maute\textsuperscript{206,207} presented adaptivity in shape and topology optimization. Kikuchi,N, Hagiwara.L, Ma.Z.D\textsuperscript{205} studied shape and topology for a frequency response problem. Lin and Chao\textsuperscript{204} proposed automated optimization system for integrated shape and topology of shell structures.

A new approach for thickness, shape and topology optimization is presented by Nima Bakhtiary\textsuperscript{210} and others using CAOSS(Computer Aided Optimization System Sauter) and MSC/NASTRAN.

Geometric reconstruction technique was adopted by Tang and Chang\textsuperscript{213} supporting topology optimization using computer-aided approach. Geometry with holes is considered for integrating and arriving at optimal topology layout. The model generation method proposed requires significant user interaction and decision making. Yildiz\textsuperscript{214} and others presented an approach, combining topology and shape optimization based on neural network techniques.
Integrated procedure for 2D structures for optimum topology was developed by Bendsoe and Rodrigues\textsuperscript{198}. Utilizing the CAD Graphics facilities integration is done by drawing the shape of the initial form directly on the top of the screen view of the topology optimization result.

The density function approach was proposed by Kumar and Gossard\textsuperscript{203} combining both the shape and topology. The density function at the every nodes serve as the design variables of the problem.

Cappello, F., Mancuso, A.\textsuperscript{201} proposed combined topology and shape optimization using genetic algorithms. The structural evaluations are carried out with a FEM commercial code, linked to the algorithm. Zhou, Pagaldipti, Thomas, Shyy,\textsuperscript{215} studied integrated topology and shape optimization.

Recently, feature based neural network techniques overcome the drawbacks of conventional methods and to provide effective subsequent to design. Integrated optimal topology design and shape optimization using neural networks was presented by A.R. Yildiz, N. Ozturk, N. Kaya and F. Ozturk\textsuperscript{214}.

However, simultaneous treatment of thickness and shape optimization during the topology optimization phase is not permitted by many codes. This limits the design space and prevents arriving at possible better designs since the interaction of thickness and shape variables with topology modification was excluded. (Ph.D thesis\textsuperscript{223})