Chapter 1

Introduction
An option is a financial instrument which gives its owner the right to buy or sell certain amount of an underlying asset (such as common stock) at a predetermined price over a specified period of time. An American option can be exercised at any time up to the expiration date, whereas a European option can be exercised only on a specified future date. The development of a formula to price options remained elusive until 1973 when Fisher Black and Myron Scholes developed their famous formula to value equity options. It is considered to be major breakthrough in the field of finance as it has found immense application in almost all the spheres of the field.

The Black and Scholes formula is derived under the assumption of market efficiency i.e. options are correctly priced by the market and it is not possible to make profits by creating long and short positions in options and underlying stock.

Other assumptions are:

1. The underlying stock price follows a geometric Brownian motion with mean, $\mu$ and volatility, $\sigma$ constant.
2. The risk-free interest rate is constant.
3. The stock pays no dividend,
4. The options are European and can be exercised only on expiration date,
5. There are no transaction costs.
6. The short selling of stocks as well as options is permitted.
7. The security trading is continuous.

Under these assumptions it is possible to create a hedged position with a long position in the stock and a short position in the option and a Partial Differential Equation (PDE) is derived for the value of the option.

$$w_t = r w - r x w_t - \frac{1}{2} \sigma^2 x^2 w_{11}$$

The solution of this PDE is the Black-Scholes option pricing formulas.

$w(x,t)$ is the value of the option as a function of stock price and time.

The subscripts refer to the partial derivative of $w(x,t)$ w.r.t to its first argument.

$\sigma^2$ is the variance rate of return on the stock.
r is the risk-free rate.

Boundary conditions for the above PDE are:

Value of option at maturity is is \( w(x,t) = x - K \), for \( x > K \)

\[ = 0 \quad \text{for} \quad x < K \]

The solution of the above differential equation and the price of the call option under the assumed boundary condition is

\[ w(x,t) = x N(d_1) - Ke^{-n} N(d_2) \]

\[ d_1 = \frac{\ln \left( \frac{x}{K} \right) + \left( r + \frac{1}{2} \sigma^2 \right) t}{\sigma \sqrt{t}} \]

\[ d_2 = \frac{\ln \left( \frac{x}{K} \right) + \left( r - \frac{1}{2} \sigma^2 \right) t}{\sigma \sqrt{t}} \]

The function \( N(d) \) is the cumulative probability distribution function for a standardized normal distribution.

The first term in the price of the call option is \( x N(d_1) \). It is the present value of the stock the call holder expects to receive upon exercise where \( N(d_1) \) is the delta of the call option. Thus, \( x N(d_1) \) is the value of the stock currently embedded in the call (the current price of the stock times the call delta). The second term in the price is \( Ke^{-n} N(d_2) \). It represents the present value of what the call option holder expect to pay upon the exercise of the option. The exercise of the call results in the cash outflow of \( K \). according to the risk-neutral approach \( N(d_2) \) is the risk-neutral probability of the option finishing in-the-money. Thus, \( Ke^{-n} N(d_2) \) is the present value of an outflow of \( K \) at maturity \( T \) times the risk neutral probability of this outflow.

The Black-Scholes formula depends only on five variables that makes it easy to implement. Out of these five variables, two are contract variables (strike price and maturity), and two are market variables (stock price and interest rate), and only
one variable $\sigma$ (volatility of the underlying stock)- is not directly observable. Further, option prices do not depend on the expected return on the stock which is difficult to estimate. Lastly, the formula is developed under arbitrage-free condition; it can be used for delta-hedging of option positions.

1.1. Empirical performance of Black-Scholes model
The empirical performance of the Black-Scholes model is not very supportive of the model. The formula assumes that the logarithmic returns on the underlying asset follow normal distribution with a mean zero. But, a number of studies have observed significant deviation from normality for a number of financial markets. In order to account for non-normality of returns, skewness-kurtosis adjusted formula has been proposed. Another assumption underlying the Black-Scholes formula is that the underlying asset has a constant variance, however, it is observed that implied volatility is different for options with different exercise price and time to maturity. This has led to extension and modification in original Black and Sholes model such as stochastic volatility process, jump-diffusion process and GARCH variety model

1.2. Derivatives Market in India
In India, derivatives have been actively traded over the last decade. The use of derivatives in the commodity segment has been existent over several years, but these were mostly confined to futures and forward transactions. Options contract in the stock markets have become very popular in recent years. In the foreign exchange market, over the counter forwards have been prevalent for long, but formalized futures and options are yet to take shape. Trading of interest rate derivatives has been formally introduced in the stock exchanges but volumes are still low as compared to futures and options. Swap transactions are done on a customized one-to-one basis rather than as a formal standardized instrument. Credit derivatives have made an entry but are yet to become very popular.

1.2.1. Introduction of derivatives in India
In India trading in derivatives began in the year 2000 when both NSE and BSE commenced trading in equity derivatives. In June 2000, Index Futures became the first type of derivatives instrument to be launched in the Indian markets followed
by Index Options in June 2001, options in individual stocks in July 2001 and futures in single stock derivatives in November 2001. Since then equity derivatives have come a long way. New products; expanding list of eligible investors; rising volumes and best of risk management framework for exchange traded derivatives have been the hallmark of the journey of equity derivatives in India so far.

The underlying asset for the index options and index futures is S&P CNX .Nifty index. Nifty is a portfolio of fifty stocks and is calculated using market capitalized portfolio method. The index options have a maximum maturity of three months. At any point of time three contracts are available for trading; near month contract, next month and far month contract. The near month contract expires on the last Thursday of the month and the following day a new contract is introduced. On the day the contracts are introduced on the exchange: there are nine in-the-money contracts, one at-the-money contract and nine out-of-the-money contracts. The index options that are available for trading have European style exercise feature and both calls and puts are available for trading.

1.2.2. Growth of derivatives in India
India's experience with equity derivatives market has been extremely positive. The derivatives turnover on the NSE has surpassed the equity market turnover. The turnover of derivatives on the NSE increased from Rs 23, 654 million in 2000-01 to Rs 176, 636, 647 million in 2009-10 and Rs 124, 517, 441 million in the first half of 2010-11. The average daily turnover in this segment of the markets on the NSE was Rs 723, 921 million in 2009-10 compared to that of Rs 453, 106 million during 2008-09.

In Indian context, there is a dearth of studies on empirical performance on Black-Scholes model. The present study is an attempt to provide further evidence in this regard. The present thesis consists of five chapters.

Chapter 1 introduces the Black-Scholes option pricing formula. It outlines the assumptions and elements of the formula. Also discusses growth of derivatives market in India.
Chapter 2 reviews the studies conducted on the empirical testing on the Black-Scholes model. In particular, it examines studies on normality of returns, implied volatility and introduction of stochastic volatility/interest rate. It also discusses the studies done in Indian context.

Chapter 3 presents the motivation, objectives and hypotheses of the present study. It also describes the data used for the analysis and outlines the methodology for testing the hypotheses.

The Chapter 4 discusses the results of the study. It revolves around examination of the normality of Nifty returns, presence of implied volatility of index options, estimation and evaluation of the deterministic models of implied volatility, and comparison of the BS model with skewness-kurtosis adjusted BS model.

Chapter 5 presents the findings of the study, delineates the directions for future research and highlights the limitations of the present study.