Chapter 2
Review of literature
This chapter reviews the studies conducted on the empirical performance of the Black-Scholes model. Section 2.1 consists of studies related to implied volatility. In the Section 2.2 describes the studies on the extension and the modification of the model. The section 2.3 reviews performance of alternative option pricing models. Section 2.4 discusses studies on the informational content of implied volatility derived from option prices. Section 2.5 is based on the studies pertaining to return distribution. Section 2.6 provides studies on joint options and returns data. The final section presents studies undertaken in the Indian context.

2.1 *Implied volatility and its determinants*

One of the indirect tests of Black-Scholes model is whether implied volatility inferred from options is same across different strike prices for a given maturity. Displayed on a graph of implied volatility against strike price, it should be horizontal line.

Rubinstein (1985) used all reported trades and bid-ask quotes on the 30 most liquid CBOE option classes from August 1976 to August 1978, Rubinstein (1985) conduct non-parametric tests for the null-hypothesis that implied volatilities computed from Black-Scholes formula exhibit no systematic differences across strike prices or across time to maturity for otherwise identical options. The finding which was consistent across sub-periods- out-the-money call options’ implied volatility is systematically higher for options with short time to maturity. He divided the period of study in two sub-periods: period I from August 23, 1976 to October 21, 1977, and period II from October 24, 1977 to August 31, 1978. For at-the-money calls, Rubinstein (1985) finds that in period I, Implied Volatilities for options with short time to maturity are higher than for those with longer time to maturity, while opposite result to true for period II. Furthermore, in period I, Implied Volatilities are higher for options with lower strike prices, but again the result is reversed in period II.

Rubinstein (1994) report that the plot of implied volatility against strike price were downward sloping, a shape named option smirk. Rubenstein point out that before the 1987 crash, Plots of implied volatility exhibited flatter pattern. He points out that something is missing from the formula. He attributes the volatility
smile to “Crashophobics”. The traders take into account the possibility of stock market crash like 1987.

Culumovic & Welch (1994) use transactions data of Chicago Board of Options Exchange (CBOE) call options from 1987 to 1989 and show that the constant-variance model of Black and Scholes (and its dividend variant by Roll) has severe mispricing errors. Employing a matched-methodology (similar to Rubinstein’s (1985)) they find that the relative biases between two calls depend not only on the moneyness (S/X) ratio and maturity (T) of the options but also on the company and calendar time period, maturity (T), and the stock-to-strike price (S/X) ratio. However, the maturity bias is much more consistent than the moneyness bias. For example, the third quartile relative mispricing of 1450 matched calls (whose S/X ratio was between 0.90 and 0.95) was 22.1%. Specifically, the call with the shorter maturity (between 21 and 70 days) was underpriced by 22.1% relative to its mate (with a maturity between 71 and 120 days.) This is an example of the direct time bias that was pervasive in Rubinstein’s study (using 1976-1978 CBOE data) and indicates that the implied standard deviation (ISD) of the shorter-term call exceed the longer term’s ISD.

Aggarwal et al. (1999) studied twenty stock market indices across the world- six Asian indices, four Latin American indices, six developed market indices and four Morgan Indices over a 10-year period of May, 1985-April, 1995. The distributions of most markets were non-normal—most of the developed markets have significant negative skewness. Moreover, many of the emerging markets exhibit positive skewness.

Dumas et al. (1998) develop a deterministic volatility function, extending stock volatility to be a deterministic function of stock price and time. The deterministic volatility approach is popular among practitioners because of its simplicity. Second, it allows for the efficient non-parametric fitting of the volatility function. Dumas et al. (1998) computed the S&P 500 implied volatilities on April 1, 1992 for S&P 500 index call options with April, May and June option expiration. They found that the volatilities do not all lie on a horizontal line. This pattern is often called the volatility “smile” and constitutes evidence against the Black–Scholes model. Further, the “smile” actually appears to be more of a “sneer.” The
volatilities were symmetric around zero moneyness, with in-the-money and out-of-the-money options having higher implied volatilities than at-the-money options. The sneer pattern however, is more indicative of the pattern since the crash of 1987, with call (put) option implied volatilities decreasing monotonically as the call (put) goes deeper out of the money (in the money) resulting in. The sneer is influenced by the time to expiration of the underlying options with implied volatilities decreasing with time to maturity.

In a similar vein like Dumas (1998), Engstrom (2002) investigates the implied volatility pattern for Swedish equity options. Their results show a U-shaped smile pattern and dying smile. The shorter the time left, more pronounced the implied volatility and longer the time left, lower the implied volatility. He also employs different specifications such as time to expiration, and the moneyness to fit the volatility using six models. However, no model performs well for all moneyness levels.

The deterministic approach suffers from two main drawbacks. Firstly the mechanism by which the volatility smile is incorporated is not realistic—few market participants would attribute the existence of volatility smile solely to time and stock dependent volatility. Second, there is much empirical literature rejecting DVF models and their implications for hedging and market completeness.

Pena et al. (1999) report simple regressions and Granger causality tests in order to understand the pattern of implied volatilities across exercise prices. The data includes all calls and puts transacted between 16:00 and 16:45 on the Spanish IBEX-35 index from January 1994 to April 1996. They report that the Spanish index market tends to exhibit volatility smile throughout the sample period. They find that transaction costs (proxied by the bid-ask spread), time to expiration, the uncertainty associated with the market and the relative market momentum are the important determinants of implied volatility. The market conditions and transaction costs are relatively more important determinants for short-term options. Finally they find that there is a linear relationship between curvature of the smile and transaction cost.
Hafner & Vallmeier (2000) analyze the strike profile of implied volatilities of German DAX options for a time to expiration of 45 days, employing WLS spline regressions over the sample period from 1995 to 1999. They estimate a time series of smile characteristics in an attempt to attribute it to economic fundamentals. Their choice is motivated by common theoretical explanations of the smile. The strike pattern almost exclusively appears as a "skew" rather than a "smile". They find that the dynamics of the smile profile can be accurately modeled by a stationary AR (1) process. Market uncertainty, measured by volatility of volatility, and liquidity effects seem to play an important role in determining the pattern of DAX implied volatilities across exercise prices.

Dennis et al. (2006) study the asymmetric volatility pattern implicit in the option prices. Their study comprises of both the S&P 100 index as well as stocks of 50 firms with options traded on CBOE, for a sample period January 1998 to December 1995. The implied volatilities are computed using a 100-time steps binomial model, which account for early exercise. The index volatility represents systematic volatility while idiosyncratic volatility is represented by volatility of individual stocks. They find that there is a negative relationship between return on the underlying index and systematic volatility (index volatility). Secondly, there is almost no relationship between stock returns and idiosyncratic volatility (stock volatility). Thus, the asymmetric volatility pattern (skew) is primarily due to market-wide systematic factors rather than firm-specific idiosyncratic factors.

Stein (1989) examines the "term structure" of options' implied volatilities, using data on S&P 100 index options. The data consists two daily time series on implied volatilities for S&P 100 index options for the period December 1983 to September 1987, comprising of a total of 964 observations for each series. It consist of a "nearby" series- which represents options with the shortest time to expiration-between zero and one month, depending on the sampling date. Second, a "distant" series- which represents options with the next expiration data- between one and two months. In their empirical analysis they identify and estimate the stochastic process followed by implied volatility represented by these two series and find that an AR1 Process provides a good description of the data. Implied volatility is a strongly mean reverting process e.g. the implied volatility on a longer maturity.
option should move by less than one percent in response to one percent move in the implied volatility of a shorter maturity option. But their empirical study shows that this elasticity is larger than that predicted by rational expectations theory—long maturity options tend to “overreact” to changes in the implied volatility of short maturity options.

Shastri & Wethyavivorn (1987) examine the volatility pattern implied by the options written on Japanese yen, Swiss Franc, and West German Mark exchange rates on the floor of Philadelphia Stock Exchange (PHLX), covering the period from March 1, 1983 to August 31, 1984. 28,527 observations for the three currencies are classified into one of the 36 categories by time to expiration, and by spot exchange rate to exercise price ratio. Implied Volatility is then calculated for each option in the data set by using pure diffusion model, i.e. the Black-Scholes Merton model. They find that Implied Volatilities are U-shaped functions of the spot exchange rate to exercise price ratio (S/X), except for options with time to maturity greater than 220 days. They also find that there is no unique Implied Volatility pattern with respect to option maturity.

Bodurtha & Courtadon (1987) study the American option version of the Black and Scholes model to price foreign currency options traded on PHLX from February 28, 1983 to March 26, 1985. They find that the formula has the tendency to overprice in- and at-the-money calls and under-price out-of-the-money calls on foreign currencies during the period 1983 to 1985, indicating an implicit distribution more positively skewed than the lognormal. Second, the American model also exhibit a time to maturity bias. As time to maturity increases, the over-pricing of in-the-money and at-the-money options by the model decrease. Longer maturity options appear to trade at similar levels of implied volatility whether they are in, at or out-the-money.

Taylor & Xu (1994) provide both theoretical and empirical evidence of volatility smile for the foreign currency options. They use an approximation to the theoretical implied volatility is a quadratic function of $\ln (F/X)$ where $F$ is the forward price and $X$ is the strike price, and that this approximate function has a minimum at $X = F$. This theoretical result requires that the asset price and volatility differentials are uncorrelated and volatility risk is not priced. Second, the
determinants of this implied volatility represented by the quadratic function are the time to maturity of the option and several volatility parameters which includes the present level, any long run median level and the variance of future average volatility. The empirical tests conducted on the foreign currency option data on seven currencies and the ECU from the Philadelphia Stock Exchange (PHLX) over the period from November 1984 to November 1989 find little evidence of asymmetries in these implied volatilities across moneyness.

Bakshi & Chen (1997b) present closed form valuation formulas for currency options and currency futures option when there is a degree of correlation between domestic and foreign interest rates. Moreover, they computed the theoretical option price based on the proposed formula for certain assumed parameters (the interest rates and the currency volatility underlying the proposed option model are both stochastic over time), and then substituted the computed option price into the Garman-Kohlhagen formula (foreign currency equivalent of BS formula) to back out implied volatility. First, they develop implied volatility - strike price pattern for spot exchange rate assumed as 100 with three terms-to-expiration, \( \tau = 20 \) (short-term), 180 (medium-term) and 360 (long-term) days. Second, they develop implied volatility - term-to-expiration pattern for spot exchange rate assumed as 100 with strike prices \( K = 80, 100 \) and 120. The maturity - implied volatility shape is upward sloping, with longer-term calls having higher implied volatility. Holding term-to-expiration fixed, out-the-money calls have lower implied volatilities than both at-the-money and in-the-money calls.

Kim (2009) focuses on the usefulness of the traders' rules to predict future implied volatilities for pricing and hedging KOSPI 200 index options. The sample period extends from January 4, 2000 through June 30, 2007. The minute-by-minute transaction prices for the KOSPI 200 options were obtained from the Korea Stock Exchange. The 3-month CD rates were used as risk-free interest rates. There are two versions of ad hoc approach. The relative smile approach and absolute smile approach. In the “relative smile” approach, the implied volatility skew is a function of moneyness. While, in the “absolute smile” approach, the implied volatility skew is function of the strike price. Of the four specifications of implied volatility examined in this paper, two each belong to relative smile and absolute
smile approach. It is found that the pricing and hedging performance of "absolute smile" approach is better than both the Black & Scholes (1973) model and the stochastic volatility model.

Tompkins (2001) study options market on financial futures and examines the determinants of implied volatility. The sample consists of sixteen options markets over four countries (US, UK, Germany and Japan) and comprises four asset classes (stock index futures, bond futures, currency futures and deposit futures). The sample period varies with asset classes and options market. A number of contract specific factors (strike-price, time to maturity, at-the-money implied volatility etc.) and market specific factors (shocks, crashes etc.) were taken as independent variables in the proposed. The coefficients were estimated using weighted least square regression. The proposed model is able to explain around 85% of the variation in implied volatility. Moreover, for seven of the sixteen markets, the model is an unbiased estimator of the relative implied volatilities.

Rosenberg (2000) proposes and investigates dynamic implied volatility function (DIVF) by comparing its pricing performance with implied volatility functions (IVF) models. The DIVF separates time-invariant implied volatility function from the stochastic state variables that drive changes in implied volatilities. The sample consists of S&P 500 futures options traded on Chicago Mercantile Exchange (CME) over a period of 1983-1998. The implied volatilities were obtained using Black (1976) formula. The DIVF model outperforms time-invariant IVF models. The adjusted R-sq values for DIVF model is around 90% and that for IVF model the value is 30%. Similarly, the pricing errors for DIVF model are also lower.

The foregoing discussions highlight that the evidence of volatility smile is overwhelming in equity market as well as currency markets. The presence of smile/smirk implies that it misprices out-of-the money and in-the money options. According to Rubinstein (1994), apart from "Crash-o-phobia", smile may be result of jumps in the prices of underlying process, market imperfections such as liquidity, transaction costs and impediment to arbitrage and non-normality in the price process of underlying assets. Stein (1989) point out that it may be caused by market misreaction. Therefore, behavioral factors may be the potential source of smile.
2.2 Extensions of Black-Scholes model

The Black-Scholes model assumes constant volatility and no jumps in asset prices. The presence of volatility smile suggests that the model should be extended to capture the volatility smiles. The important classes of models that incorporate these modifications include stochastic volatility approach, jump-diffusion model and ARCH/GARCH models.

2.2.1 Stochastic volatility approach

In the stochastic volatility process, the volatility of the stock is assumed to be a mean reverting diffusion process, typically correlated with stock process itself. Depending on the correlation between the volatility process and the stock process, it is possible to generate variety of volatility smiles and skews using this model. Moreover, there is empirical evidence from the time series analysis of the presence of stochastic volatility in stock prices (Andersen et al., 1999). But, there are some limitations of this approach. First, in order to generate volatility smiles and skews which are consistent with traded options, unrealistically high degree of correlation between stock process and volatility process is required. Second, since stochastic volatility models are true multi-factor models, one would need a multi-dimensional lattice to evaluate American options. Finally, stochastic volatility models do not allow for perfect hedging using dynamic positions in the stock and money market account (which in the absence of other traded contracts form an incomplete market).

From the work of Merton (1976), Garman (1976), and Cox, Ingersoll & Ross (1985), the differential equation that the European call on a stock that has Stochastic Volatility must satisfy is known. The solution of this differential equation is independent of risk preferences if (a) the volatility is a traded asset or (b) the volatility is uncorrelated with aggregate consumption (market). If either of these conditions holds, the risk neutral valuation arguments Cox & Ross (1976) can be used to arrive at the solution.

Johnson & Shanno (1987) present a differential equation for a call option on a stock with stochastic volatility and provide its solution using Monte Carlo simulation. The result is then used to explain the switch in the exercise price bias observed by Rubinstein (1985). They studied the Black-Scholes formula when...
variance is changing randomly (stochastically) and is correlated with stock returns ($\rho$ is non-zero). First, he calculated call option prices via Monte Carlo simulation for $S = 40$ and $X = 35$, 40 and 45 for $\rho = \frac{1}{2}$ and $-\frac{1}{2}$. He then imputed implied volatility using Black-Scholes formula and found that implied volatility for out-of-the-money and at-the-money call options is larger for one-month maturity than four month maturity. Second, for longer maturity options, implied volatility is downward sloping with strike price when volatility is positively correlated with stock returns. When the correlation is negative the effect is reversed. Third, at-the-money options are insensitive to the value of $\rho$.

Scott (1987) compared the Black-Scholes model with his Random variance Model. No analytic formula is developed, but a model is derived that can produce accurate estimates of option prices when solved using Monte Carlo simulation. The sample period for daily stock returns data is from 1974 to June 1982. The Black-Scholes Model is used to compute prices for call options on Digital Equipment Corporation (DEC) for the period July 1982-June 1983. The parameters of asset price dynamics as well as volatility dynamics are calculated using method of moments and non-linear minimization technique. DEC is chosen because it does not pay cash dividends and thus allowed them to circumvent the dividend problem in the study. Using 728 options that are either in-the-money or out-of-the-money, sum of squared errors and mean squared errors are calculated. The Mean squared error for random variance model is 8.7% less than that for the Black-Scholes model. Moreover, it is found that the Black-Scholes model tends to overprice out-of-the-money options in relation to random variance model.

Hull & White (1987) use a general differential equation to derive a series solution (power series approximation) for the price of a call option on a security with a stochastic volatility that is uncorrelated with the security price (option price obtained from the series solution is considered as true price). It is shown for such a security that the Black-Scholes price overvalues at-the-money options and undervalues deep in- and out-of-the-money options. Moreover, if the Black-Scholes equation is used to determine the implied volatility of a near-the-money option, the longer the time to maturity the lower the implied volatility. The case in which the volatility is correlated with the stock price is examined using Monte-
Carlo simulation. When there is a positive correlation between the stock price and its volatility, out-of-the-money options are underpriced by the B-S formula, while in-the-money options are overpriced. When the correlation is negative, the effect is reversed. These general observations appear to be true for all maturities.

Stein & Stein (1991) study the stock price distributions that arise when prices follow a diffusion process with a stochastically varying volatility parameter and derive an explicit closed form solution (assuming zero risk premia) using Fourier inversion method for the case where volatility is driven by AR1 process. Option prices obtained from the proposed analytic solution are compared with prices from Black-Scholes model. For all entries, the price of underlying asset, P = 100, the risk-less rate r = 9.53%, and the volatility risk premium φ = 0. The black-Scholes price corresponds to the non-stochastic volatility setting where δ = k = 0. Various parameter values assumed are as follows: first, σ = 0 = 0.20, 0.25 and 0.35 (where, σ is volatility of the stock; 0 is long run average variance). Second, k ranges from 0.10 to 0.40 and δ ranges from 4 to 16. The black-Scholes model under-prices all call options, with far-maturity options most under-priced. The under-pricing also increases with increasing value of k. Moreover, the under-pricing increases with strike price, with in-the-money options most under-priced. is obtained when analytic prices are substituted in Black-Scholes model to impute implied volatility.

Heston (1993) derive a closed form solution for the price of a European call option on an asset with stochastic volatility (Fourier inversion method). The model allows arbitrary correlation between volatility and spot asset returns as well as stochastic interest rates. Thus, it can be applied to bond options and foreign currency options. SV model prices are compared with BS prices under two conditions (i) ρ = 0 - zero correlation has the effect of raising far-in-the-money and far-out-of-the-money option prices and lowering near-the-money prices (BS under-prices ITM and OTM options while it over-prices ATM options resulting in U-shaped smile. and, (ii) ρ = {-0.5, 0.5} - positive correlation increases the prices of out-of-the-money options and decreases the prices of in-the-money options relative to Black-Scholes model with comparable volatility (BS under-prices OTM options while it over-prices ITM options resulting in upward sloping smile.
While negative correlation has completely opposite effects resulting in downward sloping smile.

Ball & Roma (1994) compare the Black-Scholes prices with stochastic volatility option prices (obtained via Power series methods) when there is no correlation between volatility innovations and security prices. For one month maturity options, BS model under-prices both in-the-money and out-of-the-money option prices. While for six-month maturity options, BS model over-prices options across all strike prices with over-pricing of at-the-money options greater than that for in-the-money as well out-of-the-money options.

Analyzing liquid equity options on 9 stocks traded on the London International Financial Futures and Options Exchange (LIFFE) between August 1990 and December 1991., Duque & Lopes (1999) use the Hull & White (1998) stochastic volatility option pricing model to find the theoretical relations that should exist between shape of the smile, the level of volatility and the time to expiration. They test two different phenomena: (1) the increase of the "smile" as maturity approaches; (2) and the association between the smile and the volatility of the underlying stock. The empirical results lend support to the above two hypotheses. First, as the time to expiration decreases, the magnitude of the smile is found to increase. However, the changes in the smile pattern are asymmetric. The "wry grin" found for longer-term options is converted into a "reverse grin" for options near expiration. For medium term options, the smile is more symmetric. Second, a statistically significant positive relation between the smile intensity and the volatility of the underlying stock is also found.

2.2.2 Jump diffusion approach

This approach was originally suggested by Merton (1976), it can generate required volatility smiles and skews by adding discontinuous (Poisson) jumps to the Black-Scholes diffusion process. In particular, by setting the mean parameter of the jump process to be negative, steep short term skews for short maturity options can be easily captured in this framework, as in pointed out by Bates (1996) and Bakshi et. al (1997). The limitations of this approach are similar to stochastic volatility. First, there is absence of perfect hedging because of incomplete markets. Moreover, the degree of incompleteness is higher in a jump diffusion model than in a stochastic
volatility model, a stochastic volatility model can be made complete by the introduction of one or few traded options. But, a jump-diffusion model requires the existence of a continuum of options for the market to be complete. Second, they are a challenge to handle numerically.

Bates (1991) uses two approaches are used to examine the expectation of stock market crash of October 1987 using option prices. Calculating skewness premium and estimating jump-diffusion parameters implicit in option prices. Roughly two-thirds of the transactions during 1985-87 were in calls, one-third in puts. Skewness premium, defined as the percentage deviation of x% out-of-the-money call prices from x% out-of-the-money put prices, is used measure of deviation between OTM call and put prices. In the first week of August, 1987, 4% OTM puts were about 25% more expensive than correspondingly OTM calls, whereas standard distributional hypothesis imply the puts should have been 0-4% cheaper. Thus, it is shown that out-of-the-money puts became unusually expensive during the year preceding the crash. Second, a model is derived for pricing American options on jump-diffusion processes with systematic jump risk. The jump-diffusion parameters implicit in options prices are obtained. Prior to October 1986, parameters (with some exceptions) indicate an essentially log-symmetric, fat-tailed distribution, with jumps of zero mean and 2-8% standard deviations expected on monthly to annual frequencies. After October 1986, expectations of predominantly negative jumps (k < 0) are evident in option prices. Thus, it is found that a crash was expected and that implicit distributions were negatively skewed during October 1986 to August 1987. But, the above approaches indicate no strong crash fears during the 2 months immediately preceding the crash.

Bates (1996) extends the Fourier inversion option pricing methodology of Stein & Stein (1991) and Heston (1993). an efficient method is developed for pricing American options on stochastic volatility/jump-diffusion processes under systematic jump and volatility risk. First, the parameters implicit in deutsche mark (DM) options of the model and various sub models are estimated over the period 1984 to 1991 via nonlinear generalized least squares (NL-GLS). Second, they are tested for consistency with time series properties of log-differenced S/DM futures prices and the time series properties of implicit volatilities. Third, there was
substantial qualitative agreement between implicit and time series based distributions, most notably with regard to implicit volatility as forecast of future volatility. Finally, specific deficiencies of the postulated SV-JD model were also noted. The postulated one-factor SV model for expected variances is inadequate in capturing the evolution over time of implicit volatilities from multiple option maturities. Moreover, implicit skewness is both positive and negative and excess kurtosis is consistent feature over the sample period.

Andersen & Andreasen (2000) discusses the extension of implied diffusion approach of Dupire (1994) to asset prices with Poisson jumps. This extension yields important model improvements in capturing the dynamics of implied volatility surface. Second, a forward PIDE- which describes the evolution of European call prices as a function of strike and maturity-is derived and its applicability on European option prices on S&P 500 index is demonstrated. An unconditionally stable and computationally efficient ADI finite difference method is developed for numerical pricing of general contingent claims under jump diffusion process.

Andersen et al., (2002) extends the class of stochastic volatility diffusions for asset returns to encompass Poisson jumps of time-varying intensity using univariate time series S&P 500 index returns data over the sample period 1953-96. The parameters of the model are estimated using EMM approach proposed by Gallant & Tauchen (1996). First, the specification analysis suggests that an E-GARCH representation for the conditional variance process. Second, the unit root hypothesis is convincingly rejected in favor of stationarity of the return series. The objective of this paper is to identify a class of jump-diffusion (JD) models that are successful in approximating the S&P 500 return dynamics and should therefore constitute an adequate basis for continuous-time asset pricing applications. Our analysis indicates a general correspondence between the evidence extracted from daily equity-index returns and the stylized features of the corresponding options market prices. We find that any reasonably descriptive continuous-time model for equity-index returns must allow for discrete jumps as well as stochastic volatility with a pronounced negative relationship between return and volatility innovations to capture skewness in S&P 500 returns.
First, Bates (1991) found that implicit distributions were negatively skewed during October 1986-August 1987. Second, according to Bates (1996) - positive/negative implicit skewness and excess kurtosis were a consistent feature over the sample period. Third, Andersen et al. (2002) found that S&P 500 daily returns were negatively skewed with excess kurtosis over 1953-96.

2.2.3 Stochastic Volatility-Stochastic Interest rates models

In addition to constant volatility assumption, BS model also assumes constant risk-free rate. One class of model incorporates both stochastic volatility as well as stochastic interest rate.

Bailey & Stolz (1989) analyze the pricing of stock options in a simple general equilibrium model. The paper investigates the biases that arise when using the Black-Scholes model with assumed volatility and interest rate dynamics. First, a model is derived when volatility of the index is constant. Second, a model where volatility of the index is stochastic is investigated using CIR (1985b) model. Third, numerical solution is provided when not only the volatility of the index but also interest rate dynamics are stochastic. The findings in the paper are as follows: first, when there is positive correlation between index and volatility dynamics, the BS model under-prices stock index options. Second, when correlation is negative, the BS model over-prices index options. Finally, when is no correlation, the pricing errors are almost zero. Within positive correlation, deep-in-the-money options are most severely under-priced, with under-pricing decreasing with strike prices. Thus, deep-out-of-the-money options are least under-priced (downward-sloping smile). Thus, it is shown that the model can, in principle, explain the biases observed in empirical work on stock index options.

Amin & Ng (1993) investigate the valuation of individual stock options when the volatility of the underlying stock return is not only stochastic but also has a systematic component (non-diversifiable risk). It is evident from previous literature that empirical biases in the Black-Scholes option prices are different for options on high risk and low risk stocks. Low risk stocks are mainly large firm stocks with stochastic return volatilities that are highly correlated with the return volatility of the market, and high risk stocks are mainly small firm stocks with a less-important systematic volatility component. This paper demonstrates that the
direction of the bias inherent in prices obtained from the Black-Scholes model is different between stocks with a strong systematic volatility component and those with a strong idiosyncratic volatility component. This results from the fact that the systematic volatility component is related to the stochastic interest rate while the idiosyncratic volatility component is not. Since the effects of stochastic volatility and stochastic interest rates act in opposite directions, the actual bias depends on which effect dominates. Under-pricing of call option decreases as the option moves from ITM to ATM to OTM, with OTM options least under-priced.

Lo & Wang (1995) review the role of drift in the Black-Scholes option pricing. It is argued that although the Black-Scholes formula does not depend on the drift \( \mu \), but the drift may not be constant as in the case of Geometric Brownian Motion, but may be an arbitrary function of the underlying asset price, \( P \) and other economic variables. This implies that the Black-Scholes formula is applicable to a wide variety of processes, processes that exhibit complex patterns of predictability and dependence on other observed and unobserved economic factors. Second, they present an adjustment for the volatility parameter \( \sigma \) that accounts for most parsimonious form of predictability: autocorrelation in asset returns. To gauge the empirical relevance of the adjustment of correlation, a comparison of Black-Scholes prices under arithmetic Brownian motion and under the trending O-U process for various holding periods, strike prices, and auto-correlations for a hypothetical $40 stock. Third, to account for more general forms of predictability, two classes of linear diffusion processes are proposed: bi-variate and multi-variate trending Ornstein-Uhlenbeck (O-U) processes. Finally, a method is provided to estimate the parameters of these processes with discretely sampled data. The findings are as follows: first, even extreme auto-correlation in daily returns does not affect short maturity in-the-money call option prices very much. Second, even for short maturity, differences become more pronounced as the strike price increases. Third, as the time to maturity increases, the impact of daily auto-correlations also increases.

Bakshi & Chen (1997a) propose an option pricing model (SV-SI) with a closed form solution for bond, bond options, stock and stock option prices. The proposed model is an improvement over existing SV-SI models on the following
dimensions: first, the stock option price is jointly and simultaneously determined with bond and stock prices, which guarantees internal consistency. Second, the option pricing formula obtains whether the agent has a power or exponential utility function, as opposed to log utility commonly assumed in the existing literature. Finally, the underlying stock in the option pricing model pays a stochastic dividend yield and still yields a closed form solution. First they obtained (theoretical) option prices using their equity option pricing model, and then substituted those prices into the (dividend-adjusted) Black-Scholes formula to numerically back out the implied volatility. They present the following findings. First, the volatility smile is much stronger for call options with 30 days or less for expiration. For longer term options, however, the implied volatility is declining in strike price. Second, the implied volatility for in-the-money options is substantially higher, compared to those for both at-the-money and out-the-money options (downward-sloping smile). Finally, the term structure of implied volatility for both in-the-money and out-the-money options also display a volatility smile, the 30 days being the turning point of the U-curve, although for at-the-money calls, the implied volatility is virtually linear in the term to expiration.

2.3 Comparison of alternative option pricing models
The following studies compare the performance of various option pricing models: Bakshi et al. (1997) conducted an empirical study on the relative merits of competing option pricing models. They first developed in closed form an implementable option pricing model that admitted stochastic volatility, stochastic interest rates, and random jumps, which will be abbreviated as the SVSI-J model. The setup is rich enough to contain almost all the known closed form option formulas as special cases. S&P 500 call option prices were studied for the empirical work because the options written on the index are the most actively traded European-style contracts and the daily dividend distributions are available on the index. The sample period extends from June 1, 1988 through May 31, 1991. The BS implied volatility exhibits a strong U-shaped pattern (smile) as the call option goes from deep ITM to ATM and then to deep OTM, with the deepest ITM call-implied volatility taking the highest values (Downward-sloping smile).
ITM call-implied volatility taking the highest values (Downward-sloping smile). The out-of-sample pricing performance results are as follows: first, SV model reduces BS pricing error by half. Second, adding jumps does not improve SV pricing performance, except for short term options, and neither incorporating SI improves pricing performance. Finally, the SV, the SVJ and the SV-SI are still mis-specified (though to a lesser degree than BS).

Bakshi et al. (2000) list average daily BS implied volatilities across moneyness and maturity, using S&P 500 index data for a sample period September 1, 1993 to August 31, 1994 for a total of 12,094 puts. There are six moneyness categories and three maturity categories namely, less than 60 days, 180-365 days and greater than 365 days (called LEAPS). Two patterns are observed. First, for short term puts the implied volatility is U-shaped, whereas for medium and long-term puts the implied volatility is declining as the put goes from OTM to ITM (upward sloping smile). Among the three maturity groups, the LEAPS implied volatility exhibits the least variation with moneyness. Next, at different moneyness levels the term structure of implied volatilities can be U-shaped, or upward sloping, depending on whether it is deep OTM (or deep ITM), near the money, or ATM. The out-of-sample pricing results are as follows: first, SVJ model performs best among the four models in pricing short-term puts. Second, in pricing medium term put options, the SV-SI model does better in certain categories, while SV performs better in pricing other moneyness-maturity puts. Third, in pricing long-term puts, however, the SV model performs the best. Finally, all models are mis-specified statistically. For example, they have moneyness-maturity biases, though to varying degrees.

Heston & Nandi (2000) develop a closed-form option valuation formula for a spot asset whose variance follows a GARCH(p, q) process that can be correlated with the returns of the spot asset. The major empirical results are: first, the model provides readily computed option formula for a random volatility model that can be estimated and implemented solely on the basis of observables. Second, the single lag version of this model contains Heston (1993) stochastic volatility model as a continuous-time limit. Third, empirical analysis on S&P500 index options shows that the out-of-sample valuation errors from the single lag version of the
GARCH model are substantially lower than the ad hoc Black-Scholes model of Dumas, Fleming & Whaley (1998) that uses a separate implied volatility for each option to fit to the smirk/smile in implied volatilities. Finally, the GARCH model remains superior even though the parameters of the GARCH model are held constant and volatility is filtered from the history of asset prices while the ad hoc Black-Scholes model is updated every period. But, the GARCH model underperforms the simple BS model when it does not allow correlation between index returns and volatility. Thus, the improvement is largely due to the ability of the GARCH model to simultaneously capture the correlation of volatility with spot returns and the path dependence in volatility.

Jackwerth & Rubinstein (2001) compare the pricing performance of alternative option pricing models using $\text{RMSF}$ between market price and model price. The models include the benchmark BS model and AHBS model used by traders. The CEV model, the displaced diffusion and jump diffusion models, models incorporating stochastic volatility, stochastic interest rates and stochastic jumps and implied binomial trees to explain otherwise identical observed option prices that differ by strike prices, times-to-expiration and underlying asset. The data includes minute-by-minute trades and quotes covering S&P 500 European index options, S&P 500 index futures, and S&P 500 index levels from April 2, 1986, through December 29, 1995. The major empirical results are as follows: first, in the pre-crash period, performance of all models is similar to that of BS model. Second, in the post-crash period, the performance of predictive models used by traders is better than the more complicated models used by academicians. Third, the better performing models all incorporate the negative correlation between index level and volatility.

Eraker et al. (2003) presents the following findings: first, specification diagnostics tests indicate that in addition to SV model, a model with diffusive stochastic volatility and only jumps in return is also mis-specified. Second, implied volatility curves are developed for the SV, SV-J and SV-CJ models for three maturities using call options on a randomly selected average volatility day in the sample. The following patterns emerge: first, the SV-CJ and SV-IJ models deliver similar curves. Second, addition of jumps in returns and jumps in volatility significantly
increases the curvature of IV curves. For short maturity options, the difference between the SV, SVJ and SV-CJ IV curves for far in-the-money (ITM) or out-of-the-money (OTM) options is quite large. Third, the SV model generates flat IV curves, as it does not generate any substantial departure from normality. Finally, there is significant flattening out effect of IV curves as time to maturity increases for all models.

Kim & Kim (2004) examined the performance of alternative stochastic volatility option pricing models on the KOSPI 200 index option market. The sample period extends from January 3, 1999 through December 26, 2000. The minute-by-minute transaction prices for the KOSPI 200 options were obtained from the Korea Stock Exchange. The 3-month treasury yields were used as risk-free interest rates. The four models compared were AHBS, VG, SV and GARCH model. The performance was compared on the basis of three measures, i.e. in-sample, out-of-sample hedging performance. The major empirical findings are as follows: first, Heston (1993) SV model outperforms all other models on all three measures of in-sample pricing, out-of-sample pricing and hedging performance. Second, pricing and hedging errors are highest for out-of-the-money options, and decrease as the options move in-the-money in all four models. Third, the performance of SV is better than Dumas et. al (1998) AHBS in terms of mitigating the volatility smile effects found in the cross sectional data. Finally, Heston & Nandi (2000) GARCH model shows the worst performance.

Eraker (2004) performs the following analysis: first, parameter estimates obtained using joint options and return data are compared with those obtained from time-series returns data only. The estimation period for time-series data is January 1970 to December 1990. Second, option price fit or in-sample pricing performance studies how well various models fit historical option prices. Third, out-of-sample evaluation is done over January 1, 1991 to March 1, 1996. The empirical findings reported in this paper can be summarized as follows: First, parameters for the Heston (1993) stochastic volatility model as well as the Bates (1996) jump diffusion with jumps in prices are similar to those in Bakshi, Cao, and Chen (1997). Second, except for short and medium maturity contracts at high spot volatility and long maturity contracts with low spot volatility BS-IV for market
prices as well as model generated prices are similar. Thus, all the models do a reasonable job in fitting option prices in-sample. The BS-IV for market prices is steeper for short and medium maturity contracts but is flatter for long maturity contracts. But, models with more complicated jump components do not improve markedly upon simpler SV models. Third, out-of-sample absolute pricing errors are found to increase in magnitude as time to maturity increases. Moreover, pricing errors are larger out-of than in-sample. Finally, SV-J and SV-CJ models perform the worst.

Broadie et al. (2007) use extensive data set of S&P 500 futures option from January 1987 to December 2003 to compare empirical performance of SV, SV-J and SV-CJ models by addressing three questions. First, is there option implied time-series evidence for jumps in volatility. Second, are jumps in prices and volatility important factors in determining the cross-section of option prices? And third, what is the nature of factor risk premia embedded in the cross-section of option prices? To detect jumps in volatility, they first extract model based estimate of spot variance from option prices. Then calculate skewness and kurtosis statistics and finally simulate the statistics’ finite sample distribution. The tests reject the square root SV model and an extension with jumps in prices (the SV-J model). A model with contemporaneous jumps in volatility and prices (SV-CJ) easily passes these tests. Second, they report RMSE between the model fit and the interpolated implied volatility curves. The SV-J and SV-CJ model outperform the SV model in all categories, as they provide 15%-40% pricing improvement over SV model. But, SV-CJ is the only model capable of successfully explaining both cross-sectional and time-series properties. Third, estimates of risk-neutral mean price jumps are consistent across the three models, are of the order of 5% to 7%, are highly statistically significant, and imply a mean price jump risk premium of about 2% to 5%.

The empirical evidence of the jump diffusion model, stochastic volatility model, and other extension of the model such as stochastic volatility and interest rate models to account for biases in the original Black-Scholes models is mixed. The conclusion of Bakshi et al. (1997) is very relevant in this regard. He observed that Black-Scholes model remain misspecified whether SV, SVJ or SV-SI model is
applied. Although the pricing errors may be less in comparison to the original model. Black (1975) pointed out that one possible explanation for the mispricing pattern is that something has been left out of the formula. Despite varied extensions and modifications the misspecification patterns persists.

2.4 *Implicit volatility as forecast of future volatility*

Another approach to test BS model is to study the informational content of implied volatility. The studies of this category investigate the predictive power of implied volatility. A test on the forecasting power of option ISD is a joint test of option market efficiency and a correct option pricing model. Since, trading frictions differ across assets; cost of option replication and hedging also differs across assets. If the option market is informationally efficient and the B-S model is correct, implied volatility is expected to subsume all information contained in historical volatility and provides a more efficient forecast for future volatility. The results of any test of market efficiency and a correct option pricing model will vary across underlying assets. The following are the studies done on the forecasting power of option ISDs with the objective of studying market efficiency and model biasness using options written on individual stocks, stock market index and exchange rates.

Early studies find that implied volatility is a biased forecast of future volatility and contains little incremental information beyond historical volatility. First, Canina & Figlewski (1993) found that implied volatility from S&P 100 index options over the period 1983-87, is a poor forecast for the subsequent realized volatility of the underlying index. Second, in contrast, Day & Lewis (1992) - OEX options over the period 1983-89, Lamoureux & Lastmates (1993) report evidence supporting the hypothesis that implied volatility has predictive power for future volatility and is a biased forecast for future realized volatility. Third, more recent research attempts to correct various data and methodological problems in earlier studies. For example, using longer time series, high frequency asset returns, non-overlapping samples etc. thus, Christensen & Prabhala (1998), present evidence that implied volatility is a more efficient forecast for future volatility than historical volatility.
Macbeth & Merville (1979) analyzed daily closing prices of all call options written in 1976 on 6 stocks (American options) assuming zero probability of early exercise namely: American Telephone and Telegraph, Avon products, Eastman Kodak, Exxon, International Business Machines, and Xerox to compare the performance of Black-Scholes call option valuation model with Cox call option valuation model for CEV diffusion process. They conclude that the CEV model better explains market prices than the Black-Scholes model. Each day they computed the Black-Scholes implied volatility of an at-the-money option as true volatility, since in any constant elasticity of variance world the Black-Scholes model (approximately) prices at-the-money call options. They found that for their sample of long maturity option prices, the Black-Scholes model, with this true volatility, systematically under-priced in-the-money options and over-prices out-of-the-money options, for all values of $\theta$ (2, 0, -2, -4) and by construction, correctly priced at-the-money options.

Emmanuel & Macbeth (1982) expand the Macbeth-Merville (1980) to include (in addition to data of 1976) daily closing prices of call options written on the same stocks for each day in 1978. Assuming Black-Scholes model correctly prices long maturity at-the-money call options and, depending upon parameter values, the paper finds periods in which it under-prices in-the-money options and over-prices out-of-the-money options as well as period when it over-prices in-the-money options and under-prices out-of-the-money options. This changing nature of the mispricing is consistent across the stocks in the sample. Second, the elasticities of each stock are found to vary considerably over time but are generally negative. Third, when the prediction period is less than one month, the CEV model predicts market prices better than the Black-Scholes model.

Lamoureux & Lastrapes (1993) examine the behavior of measured variances from the options market with the time series volatility of underlying stock market. Under the joint hypotheses that markets are informationally efficient and that option prices are explained by a particular asset pricing model, forecasts from time-series models of the stock-return process should not have predictive content given the market forecast as embodied in option prices. This joint null hypothesis of orthogonality restriction for at-the-money call options on individual stocks is
tested by comparing the forecast performance of the implied variance from the HW model with time-series representations of the stock return volatility. Both in-sample and out-of-sample tests suggest that this hypothesis can be rejected. Three potential sources of bias between implied variance and subjective variance are identified: measurement error and non-synchronous option / stock prices; linearity of BS formula for ATM options and zero correlation between stock process and volatility process. Bias from the first source will be trivial as great care is taken in data selection. The magnitude of bias from the remaining two sources is measured by comparing the implied variance from the simulated data with the actual variance inherent in the empirical data. The variance extraction procedure under HW model appears to be insensitive to the non-linearity assumption and skewness in the context of empirical data.

Previous studies on the information content of implied volatilities from the prices of call options have used a cross-sectional regression approach. While Day & Lewis (1992) compares the information content of implied volatilities from call options on the S&P 100 index to GARCH and Exponential GARCH models of conditional volatility. By adding the implied volatility to GARCH and EGARCH models as an exogenous variable, the within-sample incremental information content of implied volatilities can be examined using a likelihood ratio test of several nested models for conditional volatility. The results imply that neither implied volatility nor the conditional volatilities from GARCH and EGARCH models completely characterize within-sample conditional stock market volatility. The out-of-sample predictive content of these models is also examined by regressing ex-post volatility on the implied volatilities and forecasts from GARCH and EGARCH models. The results are consistent with the hypothesis that implied volatility and the GARCH and EGARCH forecasts are unbiased.

Canina, Figlewski (1993) studied the implied volatilities for OEX index call options over a sample period between March, 1983 and March 1987. The implied volatilities for these American options were calculated using Binomial Model. They divided their observations into four maturity categories and eight moneyness categories (denoted by S-K, which is negative for out-the-money options and positive for in-the-money options). The mean implied volatility for the entire set
of 17,606 observations was 0.168, while the average implied volatility within four maturity groups declined monotonically from 0.195 for near-month options to 0.152 for options expiring in the fourth month. Similarly, when the options are classified according to moneyness, the implied volatility is lowest for at-the-money options while it becomes progressively higher as the options move further in-the-money or out-the-money. Moreover, in-the-money options have higher implied volatilities than out-the-money options.

Earlier studies observed that the volatility implied by S&P 100 index option prices to be a biased and inefficient forecast of future volatility and to contain little or no incremental information beyond that in past realized volatility. In contrast, Christensen & Prabhala (1998) find that implied volatility outperforms past volatility in forecasting future volatility and even subsumes the information content of past volatility in some of our specifications. The results differ from previous studies because we use longer time series and non-overlapping data. A regime shift around the October 1987 crash explains why implied volatility is more biased in previous work.

Britten-Jones & Neuberger (2000) derived a model-free implied volatility under the diffusion assumption. In this article, Jiang & Tiang (2005) extend the model-free implied volatility to asset price processes with jumps and develop a simple method for implementing it using observed option prices. Second, a direct test of the informational efficiency of the option market is performed using the model-free implied volatility. The results from the S&P 500 index (SPX) options suggest that: first, B-S implied volatility contains more information than the historical volatility of the underlying asset but is an inefficient forecast of future realized volatility. Second, the model-free implied volatility subsumes all information contained in the Black-Scholes (B-S) implied volatility and past realized volatility and is more efficient forecast for future realized volatility.

Jiang & Tiang (2005) derive model-free implied volatility under jump diffusion assumption and study its informational content using S&P 500 index options (SPX) traded on CBOE over a sample period June 1988-December 1994. Christensen & Prabhala (1998) study the informational content of implied volatility using S&P 100 index options over a sample period November 1983-

2.5 The role of higher moments in option pricing

The Black-Scholes model assumes that log-returns are normally distributed. However, the empirical findings virtually all financial markets provide contrary evidence. The deviations from the normality and impact of skewness and kurtosis offers potential explanations for phenomenon such as volatility smile and skew.

Corrado & Su (1996) study the extended version of Black-Scholes formula which incorporates non-normal skewness and kurtosis in index return distributions. This extended version uses Gram-Charlier series expansion of the normal density function to model the logarithmic returns distribution. The sample includes S&P 500 index options traded on CBOE over the period November 1990- December 1993. It is observed that the Black-Scholes model systematically over-values out-of-the-money options and under-prices in-the-money options. Moreover, the adjustment for skewness and kurtosis in the Black-Scholes formula removes these systematic strike price biases from the Black-Scholes model for S&P 500 index option prices.

Corrado & Su (1997) study the presence of non-normal skewness and kurtosis in return distribution implied from stock option prices and compare the performance of original BS model and skewness-kurtosis BS model is pricing stock options. The sample includes four stock options traded on CBOE. It is observed that there is significant negative skewness and excess kurtosis in implied distributions implicit in stock option prices. The authors report that adjustment for skewness and Kurtosis in the Black-Scholes formula removes systematic strike-price biases from Black-Scholes model for pricing deep out-of-the-money and deep in-the-money stock options. The implied volatilities obtained from skewness-kurtosis formula are unrelated to option moneyness.
Bakshi et al. (2003) document the differential pricing (volatility smiles) of individual equity options versus the market index and empirically relate it to the asymmetry and the heaviness of the risk-neutral distributions. The sample includes nearly 350,000 OTM call and put option quotes written on S&P 100 index and its 30 largest individual equity components, from January 1991 through December 1995. The options are American and are traded on CBOE. The results of OLS regression of implied volatility on moneyness are as follows: First, the volatility smile of individual equity options is less steep compared with that of the market index. Second, for index options implied volatility always decreases with increasing moneyness (measured as K/S: downward sloping smile) but not so for individual equities. Thus, OTM puts are consistently and substantially more expensive than OTM calls for the index. The results of multi-variate and univariate OLS regressions of implied volatility slope on skewness and kurtosis are as follows: first, irrespective of sample period and regardless of maturity of options, the coefficient of skewness, \( \beta \), is positive and statistically significant. Thus, the more negatively skewed the risk-neutral distribution of the stock, the steeper the smile. Second, average \( R^2 \) in short-term univariate regressions is 46.54% with skewness alone and 5.6% for kurtosis alone. Thus, the cross-sectional behavior of equity options represented by the slope of the smile is driven primarily by the degree of symmetry (skewness) of the risk-neutral distribution. Third, the sign of \( \beta \) remains unaltered between restricted (univariate) and unrestricted (multivariate) regressions, but the coefficient of kurtosis reverses sign and turns negative. Thus, in the presence of risk-neutral skewness, a higher risk-neutral kurtosis flattens the implied volatility.

Conrad et al. (2013) study the relationship between volatility, skewness and kurtosis of return distribution and the subsequent security returns. The data includes daily option price data for all out-of-the-money calls and puts written on all stocks from 1996 to 2005. The data on stock returns include daily and monthly returns from 1996 to 2005 for all individual securities. The moments of the risk-neutral density function are estimated using the methodology of Bakshi et al. (2003). The findings are as follows: individual securities’ risk-neutral volatility, skewness, and kurtosis are strongly related to future returns. First, high (low)
volatility firms are associated with lower (higher) returns over the next month. Second, skewness has a strong negative relation with subsequent returns; firms with less negative or positive skewness earn lower returns. Third, there is a positive relation between kurtosis and returns. Finally, an analysis is done to determine the extent to which these returns relationships are robust to controls for differences in firm characteristics, such as beta, size, and book to market ratios, adjustments for Fama & French (1993) risk factors and differences in co-moments.

2.6 Joint option prices and asset returns data

Chernov & Ghysels (2000) attempt to model the price process of the underlying asset and the derivative security using options written on S&P 500 index and the closing prices of the index over the period November 1985 to October 1984. They present the following findings: first, the SNP densities are estimated for two univariate types of data i.e. (i) log-returns on the S&P 500 and (ii) log of the BS implied volatilities of closest-to-maturity ATM call options. The estimated densities are peaked, leptokurtic and weakly skewed. Second, the SNP densities for the bivariate case, involving joint process of returns and BS implied volatilities, suggests the presence of slight negative correlation between returns and volatility (ρ < 0). Third, the estimation of SV models involving only ATM options outperforms BS and the bivariate approach in terms of pricing performance. Thus, the results based on the S&P 500 index contract, shows dominance of univariate approach, which relies solely on options data. In general, pricing errors are large with average dollar errors across three models range from 5.23 to 0.42. Moreover, large errors occur at longer maturities. Similarly, the relative errors range from 20.71 to 0.14, with the same pattern. Finally, all models perform relatively well at shorter maturities. But, the discrepancies emerge at longer maturities and OTM contracts.

Pan (2002) compares the performance of the models on the basis of absolute pricing errors between model-implied and the market observed option prices, both measured in terms of Black-Scholes implied volatility. The major empirical findings are: first, the ability of the SVJ0 model to capture volatility smirks is quite remarkable. Second, the SVJ0 model consistently under-prices medium and
long dated options on days of high volatility, and over-prices them on days of low
volatility. Thirdly, neither the SVO model nor the SV model is capable of
explaining options across moneyness. Finally, Jump-risk premia recovered from
the joint data responds quickly to market volatility, and becomes more prominent
during volatile markets. This form of jump-risk premia is important not only in
reconciling the dynamics implied by the joint data, but also in explaining the
volatility “smirks” of cross-sectional options data.

He concludes that fitting stochastic volatility model of Heston (1993) to the joint
time series data \((S_t, C_t)\), there is significant volatility risk premium as well as
improvement in goodness-of-fit. But, the model is rejected by joint data. Second,
fitting Bates (2000) model (which extends Heston (1993) model by incorporating
jump and jump risk premium) to the joint time series data \((S_t, C_t)\), there is
significant jump risk premium. Moreover, the model is not rejected by the joint
data \((S_t, C_t)\).

Rubinstein (1994) examines a possibility: that the Black-Scholes model is true but
the market for options is inefficient and provides a computationally effective way
to value options, even when the Black-Scholes model fails when some of its
assumptions are violated, i.e. local volatility of the underlying asset is non-
constant and presence of risk-less arbitrage opportunities. First, a new method for
inferring risk-neutral probabilities (or state-contingent prices) of the underlying
asset return is developed. These probabilities are inferred from the risk-less
interest rate and the concurrent market prices of the underlying asset and its
associated otherwise identical European options. Second, these probabilities are
then used to infer a unique fully specified recombining binomial tree or stochastic
process of the underlying asset that is consistent with these probabilities (and,
hence, consistent with all the observed option prices). Third, using the stochastic
process, a simple backwards recursive procedure is developed which solves for
the entire tree. Thus, value and hedge parameters of any derivative instrument
maturing with or before the European options can be calculated.

Grundy (1991) examines the relation between option prices and the true, as
opposed to risk-neutral, distribution of the underlying asset. Linking the risk-
neutral distribution implicit in option prices to the true distribution of the
underlying asset remains a comparative mystery. The study demonstrates that observed option prices, when used in conjunction with simple assumed restrictions on the true distribution, do contain information about the non-central moments of the true distribution not directly implied by those assumed restrictions alone. If the underlying asset follows a diffusion process with an instantaneous expected return at least as large as the instantaneous risk-free rate, observed option prices can be used to place bounds on the moments of the true distribution. The geometric Brownian motion assumption places such a strong restriction on the true distribution that once one calculates the volatility parameter implied from observed option prices, the coefficient of variation of the true return distribution is known. The study then examines what can be learned from option prices about the true distribution if it is known that the underlying asset follows a diffusion process with an instantaneous expected return that is always at least the instantaneous risk-free rate.

Duan (1995) develops an option pricing model for options on an asset whose continuously compounded returns follow a generalized autoregressive conditional heteroskedastic (GARCH) process. Moreover, it elucidates how this econometric model is different from existing contingent pricing literature. First, the GARCH option price is a function of risk-premium embedded in the underlying asset instead of preference-free option pricing results. Second, the GARCH model is non-Markovian instead of the diffusion process assumed in existing pricing literature. Third, numerical analysis using Monte Carlo simulation suggests that the GARCH model may be able to explain some well-documented systematic biases associated with the Black-Scholes model, like under-pricing of short-maturity options, under-pricing of low-volatility securities, under-pricing of out-of-the-money options and the U-shaped implied volatility curve in relation to exercise price. The development utilizes the locally risk neutral valuation relationship (LRNVR). The LRNVR is shown to hold under certain combinations of preference and distribution assumptions. The GARCH option pricing model is capable of reflecting the changes in the conditional volatility of the underlying asset in a parsimonious manner.

2.7 Studies on option pricing in India
Misra et al. (2006) investigate the pattern of volatility surfaces in case of NSE Nifty options and to find out the determinants of implied volatility. The sample includes index options traded on NSE over the period January 1, 2004 to December 31, 2004. A regression analysis is performed with implied volatility as dependent variable and moneyness, time to maturity, interaction of moneyness and time to maturity, number of contracts and two dummy variables. The major empirical findings are as follows: first, coefficient of moneyness is positive and significant. Thus, deeply in the money and deeply out of the money options are having higher volatility than at the money options. Second, the implied volatility of out of the money call options is more than in the money calls. Third, implied volatility is higher for far the month contracts than for near the month contracts (NOC < 500), the effect is reversed for more liquid options (NOC > 1000). Fourth, deeply in the money and out of the money options with shorter maturity have higher volatility than those of with longer maturity. Fifth, put options have higher volatility than call options. Finally, implied volatility of more liquid options is more than that of less liquid options.

Tiwari & Saurabh (2007) study the skewness-kurtosis adjusted Black-Scholes formula proposed by Corrado & Su (1997) using S&P CNX nifty options traded on NSE over a period of three months from 1st August 2007 to 24th October 2007. The skewness-kurtosis formula requires, in addition to implied volatility, implied skewness and implied kurtosis. These two additional parameters are calculated by minimizing the sum of square error between model prices and market prices for all available strike prices within the same maturity series (near-month maturity) on a given day. It is observed that skewness-kurtosis formula performs significantly better the original Black-Scholes formula when performance is compared on the basis of error sum of squares (ESS).

Deo et al. (2008) empirically examines the implied volatility function for selected individual equity call options written on randomly selected 24 companies traded on NSE. The sample period ranges from 1st January 2006 to 31st December 2006. The major empirical findings are as follows: first, the implied volatility exhibits a U-shaped smile pattern when volatilities are averaged within groups according to their moneyness. Second, the implied volatilities of in-the-money option are
higher than implied volatility of out-of-the-money option (downward sloping smile). Third, short term options exhibit symmetric U-shaped smile pattern. Finally, when different specifications (as selected by Engstrom (2001)) of volatility functions are fitted and estimation over the period, the linear and quadratic function of moneyness and time to expiration have greater explanatory power among other specifications of implied volatility in equity call options at all levels of moneyness.

Schgal & Vijaykumar (2008) examine two propositions for the Indian options market: the presence of volatility smile and its determinants. The sample data includes daily data for the S&P CNX Nifty index call and put near maturity options and the Nifty index for the period January 1, 2004 to December 31, 2005. Moreover, 91-day Treasury bill rates taken from RBI are used as risk-free rate. Two specification of implied volatility function. The major empirical findings are as follows: first, the volatility functions exhibit a positive slope in the Indian context using alternative measures of moneyness, thus implied volatility is higher for deep in-the-money and deep out-of-the-money options than at-the-money options. The evidence on smile asymmetry is in contrast with findings for mature markets, which exhibit negative asymmetry profiles in general. Second, the historical volatility and time to expiration are important determinants of volatility smile.

Kumar (2008) investigate the information content of the implied volatility estimators and the historical volatility in forecasting future realized volatility. The sample includes Nifty call and put options traded on NSE and extends over the period January 2002 to July 2006. Moreover, only non-overlapping near month option contracts are studied. Implied volatility is computed using BS formula and in the regression framework the relationship between different implied volatility estimators and the historical volatility estimator is examined. The estimators are compared using R-sq values and RMSE values. The findings are as follows: first, IV estimators not only possess information about the future volatility but also dominate the historical volatility. Second, IV estimated from call options fare better than that computed from put options. Finally, IV estimators are unbiased and efficient estimators of the ex-post realized volatility.
Tripathi & Gupta (2010) study the following: first, the performance of the Black-Scholes (BS) model in pricing the Nifty index option contracts. Second, they compare the performance of skewness and kurtosis adjusted BS model of Corrado & Su (1997) and the original BS model. Third, they examine the relationship between volatility smile in case of NSE Nifty options and the non normal skewness and kurtosis of stock returns. The sample includes S&P CNX NIFTY near-the-month call options for the period January 1, 2003 to December 24, 2008. The major empirical findings are as follows: first, the BS model is misspecified because of the presence of smile pattern in the implied volatilities obtained from the model and there is significant under-pricing by the original BS model and that the mispricing increases as the moneyness increases. Second, pricing errors are less in case of the modified BS model than in case of the original BS model when measured on the basis of Mean Absolute Error (MAE). Finally, volatility smile in case of NSE Nifty options for the study period cannot be attributed to the non normal skewness and kurtosis of stock returns.

Out of the six studies conducted in Indian context five of these focus on Nifty options whereas Deo et al. (2008) is based on Indian stock options. Kumar’s (2008) study is supportive of the BS model, whereas the findings of Tripathi & Gupta (2010), Tiwari & Saurabh (2007), Misra et al. (2006), and Sehgal & Vijaykumar (2008) provide evidence that is not in line with BS model. Saurabh & Tiwari (2007) and Tripathi & Gupta (2010) find that modified skewness-kurtosis adjusted Black-Scholes Model performs better than original Black-Scholes Model.

It is apparent from the review that there is a paucity of studies of empirical performance of the Black-Scholes model in Indian setting. As a result the evidence is insufficient and inconclusive in this regard. This is in contrast to western markets where the model has been studied extensively. This offers rationale for conduct of fresh study on Black and Scholes model in India.