Chapter 4

Fundamental Theory and Design of Micro Pressure Sensor

Pressure sensor fabricated in this work is based on the piezoresistors. These piezoresistors undergo a change in resistance due to the applied pressure. The resistance change is converted into voltage signal by means of a Wheatstone bridge and by applying known pressures on the diaphragm and recording the output voltages the sensor can be calibrated.

A piezoresistive pressure sensor consists of three major active components namely, a diaphragm, Wheatstone bridge and the piezoresistors. These three components, responsible for the sensitivity of the device, are described in the following.

4.1 Stress Analysis for Thin Diaphragm

Understanding the deflection behavior of micromachined diaphragms is necessary for the designing mechanical sensors such as pressure sensors and accelerometers. Several papers have recently been published regarding the deflection characteristics of the micromachined diaphragms [1, 2, 3, 4, 5, 6, 7]. For these sensors, the applied load is assumed to be constant over the diaphragm surface. Small deflection theory is inadequate for describing diaphragm behavior. Large deflection theory is better, but generally does not consider built in stress effects.

The large deflection problems for square diaphragm with clamped edges have been solved by different methods [8, 9, 10, 11]. Here we deal with energy method analysis [12] to solve small deflection problem for thin diaphragm.
In order to do stress analysis for uniformly loaded square thin diaphragm under the effect of pressure load, a two-dimensional trial function [12] can be considered:

\[ w = \frac{c}{4} \left[ 1 + \cos \left( \frac{2\pi x}{L} \right) \right] \left[ 1 + \cos \left( \frac{2\pi y}{L} \right) \right] \]  

(1)

where \( w \) is the trial variational displacement function, \( c \) is deflection of the diaphragm at the center \((x=0, y=0)\), \( L \) is the edge length of the square diaphragm, and \( x \) and \( y \) are the in-plane coordinates.

The energy-method analysis [12] with this trial function gives a pressure-deflection equation:

\[ P = \left( c_r \left[ \frac{\sigma_o H}{L^2} \right] + c_b \left[ \frac{EH^3}{(1-\nu^2)L^4} \right] \right) c + c_s f_s (\nu) \left[ \frac{EH}{(1-\nu)L^4} \right] c^3 \]  

(2)

where \( P \) is the pressure applied, \( c_r \) is the coefficient of the residual stress, \( \sigma_o \) is the residual stress, \( H \) is the thickness of the membrane, \( c_b \) is the coefficient of the diaphragm bending term, \( c_s \) is the coefficient of the large-amplitude in plane stretching term, \( \nu \) is the Poisson ratio, and \( E \) is Young’s modulus of the material.

But, for the present case, membrane is sufficiently thin that the bending term can be neglected (i.e. \( c_b = 0 \)). Further, we assume a small amplitude loading, in which case \( c_s = 0 \). Thus the equation (2) simplifies to;

\[ P = c_r \left( \frac{\sigma_o H}{L^2} \right) c \]  

(3)
Since for the case of membrane the stress term should be dominant at small deflections, hence in the equation (3) the right hand side term, linear in \( c \), becomes stress dominated when:

\[
\sigma_o \approx \frac{EH^2}{L^2} \tag{4}
\]

Hence, equation (3) becomes:

\[
P = c\left(\frac{EH^4}{L^4}\right) c \tag{5}
\]

In order to calculate the total shear stress at the diaphragm edge, we find the \( x \)-directed radius of curvature due to bending at the center of the edge \((y = 0)\) as follows:

\[
\frac{1}{\rho_x} = \left(\frac{\partial^2 w}{\partial x^2}\right)_{x = \frac{L}{2}, y = 0} = \left(\frac{2\pi}{L}\right)^2 \frac{c}{2} \tag{6}
\]

The magnitude of the \( x \)-directed surface stress is

\[
\sigma_x = \frac{EH}{2\rho_x} \tag{7}
\]

using equation (6) into equation (7), we find;

\[
\sigma_x = \frac{EH}{2} \left(\frac{2\pi}{L}\right)^2 \frac{c}{2} \tag{8}
\]

Substituting the value of \( E \) from equation (5) into equation (8), \( \sigma_x \) can be expressed as;

\[
\sigma_x = \frac{\pi^2}{c_r} \left(\frac{L}{H}\right)^2 P \tag{9}
\]
For the membrane limit (assuming \(c_b = 0\)) [12, 13], \(c_r = 13.64\), Thus equation (9) simplifies to:

\[
\sigma_x = 0.722 \left( \frac{L}{H} \right)^2 P
\]  

(10)

According to Clark and Wise [14], a full numerical simulation result for the bending of square diaphragm using finite difference methods is:

\[
\sigma_x = 0.294 \left( \frac{L}{H} \right)^2 P
\]  

(11)

Our variational solution gives numerical agreement with the exact stress solution to within a factor of 2. The \(y\)-directed stress at the center of the edge could be calculated as:

\[
\sigma_y = \nu \sigma_x
\]  

(12)

Figure 4.1. The \(x\)- and \(y\)-directed in plane axial stresses.
Referring to Figure 4.1, the two principal-axis stresses, $\sigma_x$ and $\sigma_y$ are equivalent to a single net shear stress in the rotated coordinate system given by

$$\sigma = \left( \frac{\sigma_x - \sigma_y}{2} \right)$$

(13)

Using (10) and (12) into (13), we find;

$$\sigma = 0.722 \left( \frac{L}{H} \right)^2 P \left( 1 - \frac{1}{2} \nu \right)$$

(14)

from equation (14), it clears that thin diaphragm will offer maximum stress and hence maximum sensitivity of the device.

Figure 4.2. Maximum pressures as a function of membrane area for a clamped membrane of composite layer. The membrane thickness is used as a parameter.
Figure 4.2 shows the relation between the maximum allowable pressure applied and the size of square composite membrane made of \((\text{Si}_3\text{N}_4 + \text{SiO}_2 + \text{Si}_3\text{N}_4)\) using the membrane thickness as a parameter. Such pattern has also been obtained for a circular diaphragm with diameter of 0.5mm and the thickness of 5 \(\mu\)m [15].

### 4.2 Wheatstone Bridge

The Wheatstone bridge configuration of the resistances over the diaphragm contributes maximum in influencing the sensitivity of the device. This also includes the shape and location of each piezoresistors of the configuration [14]. The ratio of the two resistances in each branch of the Wheatstone bridge is the parameter, which is crucial to optimize the sensitivity of the sensor as a whole [16]. The change in the resistances due to strain on the diaphragm develops potential difference, which translates the applied pressure. In order to increase the sensitivity of the bridge, the resistances are placed on the diaphragm so as to get maximum change in potential difference.

In order to understand the basics of the placement of the resistors on the diaphragm, consider the four resistances of the bridge \(R_1, R_2, R_3\) and \(R_4\), as shown in Figure 4.3.

![Wheatstone bridge configuration](image)

**Figure 4.3.** Wheatstone bridge configuration.
The bridge is biased between the points A and C and the potential difference developed across the points D and B is monitored. In case of ideal situation of point D and B at the same potential level, following relation holds:

\[
\frac{R_1}{R_2} = \frac{R_4}{R_3}
\]

However, for the purpose of sensing applications, a potential difference between points D and B is required, which is possible under following condition:

\[
\frac{R_1}{R_2} \neq \frac{R_4}{R_3}
\]

The potential developed between points D and B; \(V_{out}\) is a measure of pressure sensor sensitivity, and is given by

\[
V_{out} = V_{in} \left[ \frac{R_1}{R_1 + R_2} - \frac{R_4}{R_4 + R_3} \right]
\]

Obviously, for ideal condition (under no pressure), there is no output voltage.

The magnitude of \(V_{out}\) can be adjusted with the help of the two ratios of the piezo-resistances as shown above. In order to achieve highest possible sensitivity following condition is required to meet;

\[
\left| \frac{R_1}{R_2} - \frac{R_4}{R_3} \right| = \text{Maximum}
\]

It can be deduced from the above that the most desirable condition is to allow \(R_1, R_3\) and \(R_2, R_4\) to vary in the opposite way. This can be accomplished on a strained diaphragm where stress profile is composed of tensile and compressive fringes. In this
case the piezoresistors change their values in contrast while interacting with tensile and compressive regions of the stress. This is possible only when two of the resistors of the bridge are placed in the region of one type of stress while remaining two are placed in the region of another type of stress. But for very small diaphragm, as in our case, to accommodate all the four resistors on the diaphragm is difficult. In such case, one way to obtain the above condition of maximum sensitivity is to place two of the resistors, either $R_1$ and $R_3$ or $R_2$ and $R_4$ on the diaphragm where stress will occur and the other two outside the diaphragm where stress will not occur. In the design presented $R_2$ and $R_4$ are placed on the diaphragm and $R_1$ and $R_3$ outside the diaphragm. Figure 4.4 shows the top view (Layout) of the device.

If $R_2$ and $R_4$ are on the diaphragm (called as effective resistance $R_e$) and $R_1$ and $R_3$ outside the diaphragm (called as non-effective resistance $R_n$) then under the pressure applied on the diaphragm the output voltage signal is given by;

$$V_{out} = \frac{2 \cdot R_n \Delta R_e}{(R_e + R_n)^2} V_{in}$$

Where $\Delta R_e$ is the change in effective resistance.
Layout of the micro pressure sensor

Figure 4.4. Schematic diagram of the top view (Lay out) of the micro pressure sensor.
In Figure 4.4, the hatch-lined objects are the resistors, square box with dotted lines is the diaphragm, the two blacken rectangles are the etch holes through which etch material (wet etchant) enters in the bulk of the substrate, five square boxes towards the periphery are the contact pads and the lines from resistors to these pads are the metal contact lines. The corresponding design parameters are mentioned below;

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip size</td>
<td>1mm x 1mm</td>
</tr>
<tr>
<td>Diaphragm size</td>
<td>100 µm x 100 µm</td>
</tr>
<tr>
<td>Diaphragm thickness</td>
<td>0.7 µm</td>
</tr>
<tr>
<td>Resistor’s configuration</td>
<td>half- Wheatstone bridge</td>
</tr>
<tr>
<td>Resistor’s length</td>
<td>110 µm</td>
</tr>
<tr>
<td>Resistor’s width</td>
<td>10 µm</td>
</tr>
<tr>
<td>Resistor’s pad</td>
<td>20 µm x 20 µm</td>
</tr>
<tr>
<td>Resistor’s thickness</td>
<td>1.0 µm</td>
</tr>
<tr>
<td>Metal contact pad</td>
<td>100 µm x 100 µm</td>
</tr>
<tr>
<td>Metal contact line width</td>
<td>20 µm</td>
</tr>
<tr>
<td>Resistor’s value</td>
<td>0.22 KΩ</td>
</tr>
<tr>
<td>Sheet resistivity of resistors</td>
<td>20 Ω/□</td>
</tr>
</tbody>
</table>
Figure 4.5. Simulated stress profile of a strained composite membrane of thickness 0.8 µm on application of 344-psi pressure.

The simulated stress profile, shown in Figure 4.5, is the result of analysis of the device membrane using ANSYS software. From this, it is clear that stress is maximum at the edges and decreases as one moves away towards the center. Accordingly, the design layout with the resistors oriented in the transverse direction should give more sensitivity, as the resistors will experience maximum stress and hence maximum piezo-effect. The shapes of the resistors are chosen to ensure maximum length that can be accommodated.
in the diaphragm. The actual size of the diaphragm is kept 100\(\mu\)m x 100\(\mu\)m to realize a miniature sized pressure sensor - a micro sensor so that it will be compatible with other microelectronic devices. Keeping the width of the resistors 10\(\mu\)m and a square of 20\(\mu\)m x 20\(\mu\)m for contacts of contact lines, the total length comes out to be 150\(\mu\)m. Since the square of 20\(\mu\)m x 20\(\mu\)m is on either ends and is to be connected with the conducting metals, the net length of the resistors is only 110\(\mu\)m for 100\(\mu\)m x 100\(\mu\)m diaphragm.

### 4.3 Piezoresistors

Piezoresistors deposited over the diaphragm are made of boron-doped polysilicon because of good piezoeffect. Polysilicon has better stability and can be used in operating temperature up to 200 °C [17]. Certain materials like Si or poly Si are sensitive to change their resistance resulting from stress applied to the crystal lattice. Resistance, in particular, is dependent on the changes in length caused by stress. These Resistive changes are not isotropic, and can be divided into two independent functions, one component parallel to the direction of stress, and other component perpendicular to it, in the form of following expression:

\[
\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t
\]

Where \(\pi_l\) and \(\pi_t\) are the piezoresistive coefficients in longitudinal and transverse directions, respectively. Stresses in longitudinal and transverse directions are designated by \(\sigma_l\) and \(\sigma_t\). These coefficients depend on the orientation of resistors and as well as are the functions of temperature and doping concentration. The values of piezoresistive coefficients decrease with increase in impurity concentration and increase in temperature.

Low value of the resistors is preferred because when there is a slight change in the resistor’s value due to piezoresistive effect, the change in resistance compared to the original resistor’s value will be more noticeable rather than with higher valued resistors.
Since by relation, 

\[ \text{Resistance, } R = \frac{(\rho \cdot l)}{A}; \quad \text{where } l = \text{length and } A = t \cdot w; \]

\[ \text{where } t = \text{thickness, } w = \text{width} \]

Hence, the formula can be rewritten as;

\[ \text{Resistance, } R = \frac{(\rho / t) \cdot (l / w)}{} \]

Or

\[ R = \frac{\rho_s \cdot l}{w} \]

where \( \rho_s = \frac{\rho}{t} \) is called sheet resistivity of the surface of the resistor. The value of sheet resistivity can be controlled by:

1. Polysilicon layer thickness.
2. Type of dopant and the temperature at which dopant is diffused (as shown in Figure 2.2 in Chapter 2).

With increasing polysilicon layer thickness the sheet resistivity decreases. For 1.0µm thick poly Si layer the sheet resistivity can be brought down in the range of 15-20 \( \Omega/\square \). This way by controlling the polysilicon layer thickness, values of the polysilicon resistors can be designed for the given diaphragm.

The dimension of the sacrificial layer (polysilicon), to realize cavity later on, is kept 180µm x 100µm with 40µm each extending on either sides from the diaphragm dimensions. This is done so that etch holes of sufficient dimension can be accommodated beyond the actual diaphragm but within the dimension of sacrificial layer. This condition is necessary for sealing the cavity in the later stage. The dimension of the etch hole is kept 20µm x 100µm, maintaining 5µm from the diaphragm edge on either side. This
The width of the metal contact lines are kept 20µm and the minimum gap between two contact lines is tried to keep at least twice its width. The dimension for metal contact pads is kept 100µm x 100µm. Here five pads are used instead of four since one of the arms is opened. This provision is for the purpose of balancing the bridge externally if required and if not required then by shorting, it can be used normally. Leaving sufficient space on all the sides, the size of a single chip comes up to be 1mm x 1mm. This is a pretty small size. Leaving grid size of 0.1mm (the size that can be comfortably run by die-slicer blade), the number of devices that can be fabricated on a single wafer of 2 inches diameter after leaving sufficient space for alignment marks on either side are an array of 33 x 33 giving 1089 devices. Keeping all these in views, the layouts for the device are designed using the software L-Edit (layout editor) for layout design.
References:


