CHAPTER 4

EFFECTS OF STENOSIS AND POST STENOTIC DILATATION ON NON-NEWTONIAN FLUIDS
CHAPTER 4

4.1. Effects of stenosis and post-stenotic dilatation on Herschel-Bulkley fluid.

4.1.1 Introduction

In general, Herschel-Bulkley fluid is a non-Newtonian fluid which contains two parameters such as the yield stress and power law index. Maruthi Prasad and Radha Krisnamacharya [107] investigated the Herschel-Bulkley fluid flow in an inclined tube having non-uniform cross section with multiple constrictions. The study of Casson fluid flow in a radially non-symmetric constricted artery was done by A.K.Singh and D.P. Singh [46].

In all the above models, the studies considered the effect of single and multiple stenoses. Post stenotic dilatation is caused due to the shear stress and stenosis in coronary, which occurs at high flow rates (Tandon et al., [108], Kawaguti and Hamano [109]). By the motivation of the above studies, stenosis and post stenotic dilatation effects on Herschel-Bulkley fluid are studied.

4.1.2 Formulation of the problem

Consider the steady flow of Herschel-Bulkley fluid in a circular artery with multiple abnormal segments as shown in Fig. 4.1.1.

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Fig- 4.1.1 Geometry of an arterial segment under consideration.
The geometrical description of the Fig.4.1.1, is given by

$$h = \frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta_i}{2R_0} \left[ 1 + \cos \frac{2\pi}{l_i} (z - \alpha_i - \frac{l_i}{2}) \right], & \text{for } \alpha_i \leq z \leq \beta_i \\ 1, & \text{Otherwise} \end{cases}$$

(4.1.1)

Where $\delta_i$ represents the maximum height of the $i^{th}$ abnormal segment which is projected into the lumen and $\delta_i < 0$ for the aneurysms and $> 0$ for stenosis. $R$ represents radius of the artery at dilatation, $R_0$ is considered as radius of the artery under the normal condition, $l_i$ is the length of the $i^{th}$ abnormal segment, $\alpha_i$ represents distance between the origin and the start of the $i^{th}$ abnormal segment, which is defined as

$$\alpha_i = \left( \sum_{j=1}^{i} (d_j + l_j) \right) - l_i$$

(4.1.2)

$\beta_i$ is the distance between the origin and end of the $i^{th}$ abnormal segment

$$\beta_i = \left( \sum_{j=1}^{i} (d_j + l_j) \right)$$

(4.1.3)

Where $d_i$ is the distance between the end of $(i-1)^{th}$ abnormal segment till the start of the $i^{th}$, when $i = 1$ the distance is considered from the beginning of the segment.

The equation of momentum, is given as

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \tau_{rz} \right) = - \frac{\partial p}{\partial z}$$

(4.1.4)

Shear stress $\tau_{rz}$ for Herschel-Bulkley fluid, is given by

$$\tau_{rz} = \left( -\frac{\partial u}{\partial r} \right)^n + \tau_0$$

if $\tau_{rz} \geq \tau_0$ \hfill (4.1.5)

$$\frac{\partial u}{\partial r} = 0$$

if $\tau_{rz} < \tau_0$ \hfill (4.1.6)

Here $(r,z)$ are cylindrical polar co-ordinates, where $z$ is measured in the horizontal direction of the tube and $r$ is calculated in the direction perpendicular to the axis of the tube. ‘$p$’ denotes pressure, ‘$\tau_{rz}$’ is shear stress and ‘$\tau_0$’ is yield stress and ‘$u$’ represents velocity of fluid.
Boundary conditions:

(i) When \( r = 0; \quad \tau_{rz} \) is finite \hspace{1cm} \text{(4.1.7)}

(ii) When \( r = h(z); \quad u = 0 \) \hspace{1cm} \text{(4.1.8)}

4.1.3 Solution of the problem

Using boundary conditions (4.1.7) and (4.1.8) the solution of eq. (4.1.4) is obtained as

\[
\begin{align*}
    u &= \frac{h^{k+1} \mu}{2^{k(k+1)}} \left\{ \left( 1 - \frac{2r_0}{hP} \right)^{k+1} - \left( \frac{r}{h} - \frac{2r_0}{hP} \right)^{k+1} \right\} \text{ for } r \in [r_0, h] \quad \text{(4.1.9)}
\end{align*}
\]

Where \( P = -\frac{\partial p}{\partial z}, \quad k = \frac{1}{n} \)

Using the condition (4.1.6), \( r_0 \) is given as

\[
    r_0 = \frac{\tau_0}{P} \quad \text{(4.1.10)}
\]

and using the condition \( \tau_{rz} = \tau_h \) at \( r = h \)

\[
    \frac{r_0}{h} = \frac{\tau_0}{\tau_h} = \tau, \quad 0 < \tau < 1 \quad \text{(4.1.11)}
\]

When \( r = r_0 \) in Eq. (4.1.9) reduces to

\[
    u_p = \frac{h^{k+1} \mu}{2^{k(k+1)}} \left( 1 - \frac{2r_0}{hP} \right)^{k+1}, \quad r \in [0, r_0] \quad \text{(4.1.12)}
\]

Where \( u_p \) represents plug core velocity

The volumetric flow \( Q \) is defined as

\[
    Q = 2\left[ \int_0^{r_0} u_p r \, dr + \int_{r_0}^h u r \, dr \right] \quad \text{(4.1.13)}
\]

On integrating

\[
    Q = A[(k+2)(k+3) \left( 1 - \frac{\tau_0}{h} \right)^{k+1} - 2(k+3) \left( 1 - \frac{r_0}{h} \right)^{k+2} + 2 \left( 1 - \frac{r_0}{h} \right)^{k+3}] \quad \text{(4.1.14)}
\]
Where \( A = \frac{h^{(k+3)} p^k}{2^k (k+1)(k+2)(k+3)} \)

From Eq. (4.1.14),

\[
\frac{dp}{dz} = -p = \frac{\frac{1}{h^{\frac{3}{k+3}}}}{\frac{2Q^{\frac{1}{k+3}}}{(k+2)(k+3)(1-\tau)^{k+1-2(1-\tau)k+2}} \frac{1}{r^{\frac{k+2+\tau}{k}}}}
\]  

(4.1.15)

When \( k = 1, \ \tau_0 \to 0 \) Eq. (4.1.15) reduces to the results of Young [8].

By integrating Eq. (4.1.15), the pressure drop \( \Delta p \) between \( z = 0 \) to \( z = l \) is given by

\[
\Delta p = \int_0^l \frac{1}{h^{\frac{3}{k+3}}} \frac{2Q^{\frac{1}{k+3}}}{(k+2)(k+3)(1-\tau)^{k+1-2(1-\tau)k+2}} \frac{1}{r^{\frac{k+2+\tau}{k}}} \frac{dz}{r}
\]  

(4.1.16)

(4.1.17)

Substituting the non-dimensional variables

\[
\bar{z} = \frac{z}{L}, \ \bar{\delta} = \frac{\delta}{R_0}, \ \bar{R}(z) = \frac{R(z)}{R_0}, \ \bar{P} = \left( \frac{P}{\mu U/L} \right), \ \bar{\tau}_0 = \frac{\tau_0}{\mu/R_0}, \ \bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu/R_0}, \ \bar{Q} = \frac{Q}{\pi R_0^2 U}
\]

In Eq. (4.1.17), we finally get

\[
\Delta p = \int_0^1 \frac{1}{h^{\frac{3}{k+3}}} \frac{2Q^{\frac{1}{k+3}}}{(k+2)(k+3)(1-\tau)^{k+1-2(1-\tau)k+2}} \frac{1}{r^{\frac{k+2+\tau}{k}}} \frac{dz}{r}
\]  

(4.1.18)

The resistance to the flow \( \lambda \), in the obstructed tube is given as

\[
\lambda = \frac{\Delta p}{Q} = \frac{1}{Q} \int_0^1 \frac{1}{h^{\frac{3}{k+3}}} \frac{2Q^{\frac{1}{k+3}}}{(k+2)(k+3)(1-\tau)^{k+1-2(1-\tau)k+2}} \frac{1}{r^{\frac{k+2+\tau}{k}}} \frac{dz}{r}
\]  

(4.1.19)

\( \Delta p_N \) is the pressure drop without stenosis \( (h = 1) \), is obtained from Eq.(4.1.18) as

\[
\Delta p_N = \int_0^1 \frac{2Q^{\frac{1}{k+3}}}{(k+2)(k+3)(1-\tau)^{k+1-2(1-\tau)k+2}} \frac{1}{r^{\frac{k+2+\tau}{k}}} \frac{dz}{r}
\]  

(4.1.20)
\( \lambda_N \) is the resistance to the flow in the absence of stenosis and is obtained from Eq. (4.1.20) as

\[
\lambda_N = \frac{\Delta P_N}{Q}
\]  

(4.1.21)

the resistance to the flow \( \tilde{\lambda} \) is given by

\[
\tilde{\lambda} = \frac{\lambda}{\lambda_N}
\]  

(4.1.22)

The wall shear stress is given by

\[
\tau_h = -\frac{h \, dp}{2 \, dx}
\]  

(4.1.23)

### 4.1.4 Results and Discussions

In order to understand the complete nature of Herschel-Bulkley fluid flow through arterial segments, the expressions for velocity \( (u) \), plug core velocity \( (u_p) \), volumetric flow rate \( (Q) \), flow resistance \( \tilde{\lambda} \) and shear stress at the wall \( (\tau_h) \) are derived in eqs. (4.1.12), (4.1.14), (4.3.18), (4.1.22) and (4.1.23) respectively. For the purpose of numerical computation the following data is used. \( d_1 = 0.2, d_2 = 0.2, L_1 = L_2 = 0.2, L = 1 \) (Maruthi Prasad and Radha Krishnamacharya [107]). The numerical results are obtained for \( \tilde{\lambda} \), wall shear stress \( \tau_h \) by making use of the above data, and are exhibited in Figs. 4.1.2 - 4.1.17.

The resistance to the flow vs stenosis height is plotted for various values of dilatation heights in Fig. 4.1.2. It is noticed that, the flow resistance increases with the height of stenosis but decreases with dilatation height. In Figs. 4.1.3 & 4.1.4, the effect of dilatation height on flow resistance is shown. It is noticed that, the flow resistance decreases with respect to the dilatation height. It is observed that, from Figs. 4.1.4 & 4.1.5, the resistance to the flow increases with the length of the stenosis. The resistance to the flow decreases with the dilatation height and its length Fig.4.1.6.
The change in flow resistance with fluid index parameter \((n = \frac{1}{k})\), has been observed through the graphs Figs.4.1.7 -4.1.9. It is seen that, the flow resistance increases with fluid index parameter in the stenotic region but decreases in the dilatation region.

From Figs.4.1.10 - 4.1.12, it is interesting to note that, the flow resistance increases with yield stress when the height of stenosis increases, but decreases with the height of dilatation.

The variation of flow resistance with wall shear stress is shown in Figs.4.1.13 - 4.1.15. It is observed that, the flow resistance decreases with wall shear stress in the stenotic region and increases in dilatation region. The flow resistance of fluids such as Newtonian, Power law and Herschel-Bulkley fluid are compared in Figs.4.1.16 & 4.1.17. It is noticed that, the resistance of the Herschel-Bulkley fluid is more than the Newtonian fluid. These results have good agreement with the results obtained by Priyadarshini and Ponalagusamy [45].

4.1.5 Graphs

![Graph](image)

Fig- 4.1.2: Variation of flow resistance \(\bar{A}\) with \(\delta_1\) for different \(\delta_2\)

\((d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \tau = 0.02\) )
Fig- 4.1.3: Relation between $\bar{\lambda}$ and $\delta_2$ for various values of $\delta_1$

$(d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \tau = 0.02)$

Fig- 4.1.4: Comparison between $\bar{\lambda}$ and $\delta_1$ for various values of $L_1$

$(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = 0.0, \tau = 0.02)$
Fig- 4.1.5: Comparison between $\bar{A}$ and $\delta_1$ for various values of $L_1$

$(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = -0.02, \tau = 0.02)$

Fig- 4.1.6: Variation of flow resistance $\bar{A}$ with $\delta_2$ for different $L_2$

$(d_1 = d_2 = L_1 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_1 = 0.0, \tau = 0.02)$
Fig- 4.1.7: Variation of flow resistance $\bar{\lambda}$ with $\delta_1$ for different $k$

$$(d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, \tau = 0.02, \delta_2 = 0.0)$$

Fig- 4.1.8: Relationship between $\bar{\lambda}$ and $\delta_1$ for various values of $k$

$$(d_1 = d_2 = L_1 = L_2 = 0.2; L = 1; Q = 0.1; \tau = 0.02; \delta_2 = -0.02)$$
Fig. 4.1.9: Representation of $\tilde{\lambda}$ versus $\delta_2$ for distinct values of $k$

$$ (d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, \tau = 0.02, \delta_1 = 0.0) $$

Fig. 4.1.10: Variation of flow resistance $\tilde{\lambda}$ with $\delta_1$ for different $\tau_0$

$$ (d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = 0.0) $$
Fig- 4.1.11: Variation of flow resistance $\bar{\tau}$ with $\delta_1$ for different $\tau_0$

$$(d_1 = d_2 = L_1 = L_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = -0.02)$$

Fig- 4.1.12: Variation of flow resistance $\bar{\tau}$ with $\delta_2$ for different $\tau_0$

$$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_1 = 0.0, L_1 = L_2 = 0.2)$$
Fig- 4.1.13: Variation of flow resistance $\bar{\lambda}$ with $\delta_1$ for different $\tau_h$

$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = 0.0, \tau_0 = 1, L_1 = L_2 = 0.2)$

Fig- 4.1.14: Variation of flow resistance $\bar{\lambda}$ with $\delta_1$ for different $\tau_h$

$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_2 = -0.02, \tau_0 = 1, L_1 = L_2 = 0.2)$
Fig- 4.1.15: Variation of flow resistance $\bar{\lambda}$ with $\delta_2$ for different $\tau_h$

$\left( d_1 = d_2 = 0.2, L = 1, Q = 0.1, k = 2000, \delta_1 = 0.0, \tau_0 = 1, L_1 = L_2 = 0.2 \right)$

Fig- 4.1.16: Variation of flow resistance $\bar{\lambda}$ with $\delta_1$ for different fluids

$\left( d_1 = d_2 = 0.2, L = 1, Q = 0.1, \delta_2 = 0.0, L_1 = L_2 = 0.2 \right)$
Fig. 4.1.17: Variation of flow resistance $\bar{\alpha}$ with $\delta_2$ for different fluids

$d_1 = d_2 = 0.2, L = 1, Q = 0.1, \delta_1 = 0.0, L_1 = L_2 = 0.2$
4.2. Micropolar fluid flow in an artery with the effect of stenosis and post stenotic dilatation.

4.2.1 Introduction

Eringen[3] has introduced a model of micropolar fluid which is non-Newtonian in nature; consists of rigid, randomly oriented particles suspended in viscous medium for which the deformation of the particles is neglected. Ariman[110] observed the flow of blood in a rigid circular tube and concluded that the micropolar fluid model is the better model because it accounts for the microrotation of blood suspensions. Abdullah and Amin[111] studied a non linear micropolar fluid model for flow of fluid in a tapered artery with single constriction. Sanjeev kumar and Chandrashekhar Diwakar[44] investigated the effect of post stenotic dilatation and multiple stenosis through an artery by treating blood as Bingham plastic fluid.

Motivated by these studies, an effort has been made in the present work to analyze micropolar fluid flow through an artery with stenosis and dilatation.

4.2.2 Mathematical Formulation

The steady flow of an incompressible micropolar fluid of constant viscosity $\mu$, and density $\rho$, in a uniform tube of length $L$, is considered with multiple abnormal segments as shown in Fig 4.2.1.

![Fig.4.2.1. Geometry of arterial segment under consideration.](image-url)
The geometrical description of the wall, as shown in Fig. 4.2.1, is given in eq. 4.1.1.

The equations for steady flow of an incompressible micropolar fluid in the absence of body force and couple are given by

\[ \nabla \cdot U = 0 \] (4.2.1)

\[ \rho (U \cdot \nabla U) = -\nabla p + k \nabla \times U + (\mu + k) \nabla^2 U \] (4.2.2)

\[ \rho j(U \cdot \nabla V) = -2kV + k \nabla \times U - \gamma (\nabla \times \nabla \times V) + (\alpha + \beta + \gamma) \nabla (\nabla \cdot V) \] (4.2.3)

Where \( p \) is the pressure, \( U \) denotes velocity, \( V \) represents micro rotation vector, \( j \) denotes microgyration parameter. \( \mu, k, \alpha, \beta, \gamma \) are the material constants and satisfy the following inequalities [Eringen3].

\[ 2\mu + k \geq 0, k \geq 0, 3\alpha + \beta + \gamma \geq 0, \gamma \geq |\beta| \]

Here the flow is considered as axisymmetric, hence the flow variables are considered only in \( r, z \) directions. Hence, the velocity \( U = (u_r, 0, u_z) \) and the microrotation vector \( V = (0, \nu_\theta, 0) \), thus the equations (4.2.1)-(4.2.3) are expressed as (where \( u_r, u_z \) are the velocities in \( r \) and \( z \) directions)

\[ \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0 \] (4.2.4)

\[ \rho \left( u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + (\mu + k) \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} \right) + \frac{k}{r} \frac{\partial (ru_\theta)}{\partial r} \] (4.2.5)

\[ \rho \left( u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial p}{\partial r} + (\mu + k) \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right) - k \frac{\partial \nu_\theta}{\partial r} \] (4.2.6)

\[ \rho j \left( u_r \frac{\partial \nu_\theta}{\partial r} + u_z \frac{\partial \nu_\theta}{\partial z} \right) = -2k \nu_\theta - k \left( \frac{\partial u_z}{\partial r} - \frac{\partial u_r}{\partial z} \right) + \gamma \left( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (ru_\theta)}{\partial r} \right) \right) + \frac{\partial^2 \nu_\theta}{\partial z^2} \] (4.2.7)

By using non dimensional variables

\[ \bar{Z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{r} = \frac{r}{R_0}, \bar{\rho} = \frac{\rho}{\rho u_0^2}, \bar{u_z} = \frac{u_z}{u_0}, \bar{u_r} = \frac{Lu_r}{u_0}, \bar{\nu_\theta} = \frac{R_0 \nu_\theta}{u_0}, \bar{j} = \frac{j}{R_0^2} \] (4.2.8)
into Eqs. (4.2.4) - (4.2.7), the reduced equations in dimensionless form are (in the case of mild stenosis \( \frac{\delta}{R_0} \ll 1 \))

\[
\frac{\partial p}{\partial z} = \frac{1}{1-N} \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} + \frac{N}{r} \frac{\partial (r \nu_\theta)}{\partial r} \right) \quad (4.2.9)
\]

\[
\frac{\partial p}{\partial r} = 0 \quad (4.2.10)
\]

\[
2 \nu_\theta = -\frac{\partial u_z}{\partial r} + \frac{2-N}{m^2} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r \nu_\theta)}{\partial r} \right) \quad (4.2.11)
\]

Where \( N = \frac{k}{\mu + k} \) is the coupling number \( (N \in [0,1]) \) and \( m^2 = \frac{R_0^2 k (2 \mu + k)}{\gamma (\mu + k)} \) is the micropolar parameter.

The corresponding boundary conditions in dimensionless form are

\[
\frac{\partial u_z}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (4.2.12)
\]

\[
u_\theta = 0 \quad \text{at} \quad r = h \quad (4.2.13)
\]

\[
u_\theta = 0 \quad \text{at} \quad r = h \quad (4.2.14)
\]

\[
u_\theta \text{is finite at} \quad r = 0 \quad (4.2.15)
\]

\[
u_\theta \text{is finite at} \quad r = 0 \quad (4.2.16)
\]

4.2.3 Solution of the problem

Equation (4.2.9) can be expressed as

\[
\frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} + N r \nu_\theta - (1 - N) \frac{r^2 dp}{2 dz} \right) = 0 \quad (4.2.17)
\]

By integrating the above equation

\[
\frac{\partial u_z}{\partial r} = -N \nu_\theta + (1 - N) \frac{r dp}{2 dz} + \frac{c_1}{r} \quad (4.2.18)
\]

Using Eq.(4.2.18) in Eq. (4.2.11),
\[
\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} = \left( m^2 + \frac{1}{r^2} \right) v_\theta = \frac{m^2 (1-N)}{2^{-N}} \frac{r \ dp}{2 \ dz} + \frac{m^2}{(2-N) \ r} \ c_1 \quad (4.2.19)
\]

The general solution of Eq. (4.2.19) is
\[
v_\theta = c_2 I_1 (mr) + c_3 K_1 (mr) - \frac{1}{(2-N) \ 2} \int_0^h u_z r \ dr \quad (4.2.20)
\]

Where \( I_1 (mr) \) and \( K_1 (mr) \) are the modified first order Bessel functions of first and second kind.

Using Eq. (4.2.20) in Eq. (4.2.18), and solving for \( u_z \), using the boundary conditions (4.2.12) to (4.2.16), we get
\[
u_\theta = \frac{1-N}{2-N} \int_0^h u_z r \ dr \quad (4.2.21)
\]

The flow flux is defined by
\[
Q = 2\pi \int_0^h u_z r \ dr \quad (4.2.22)
\]

\[
Q = \pi \left( \frac{1-N}{2-N} \right) \int_0^h \left( -\frac{h^4}{4} + \frac{Nh^3 I_0 (mh)}{2m I_1 (mh)} - \frac{Nh^2}{m} \right) \ dr \quad (4.2.23)
\]

\[
\frac{dp}{dz} = \frac{Q}{\pi \left( \frac{1-N}{2-N} \right) \left( -\frac{h^4}{4} + \frac{Nh^3 I_0 (mh)}{2m I_1 (mh)} - \frac{Nh^2}{m} \right)} \quad (4.2.24)
\]

When the micropolar parameter \( N \to 0 \), the fluid becomes Newtonian fluid.

The pressure drop \( \Delta p \) in the presence of stenosis is calculated by integrating Eq. (4.2.24) and is given as
\[
\Delta p = \int_0^1 \frac{dp}{dz} \ dz = \int_0^1 \frac{Q}{\pi \left( \frac{1-N}{2-N} \right) \left( -\frac{h^4}{4} + \frac{Nh^3 I_0 (mh)}{2m I_1 (mh)} - \frac{Nh^2}{m} \right)} \ dz \quad (4.2.25)
\]

The flow resistance to the flow in the presence of stenosis \( \lambda \), is given as
\[
\lambda = \frac{\Delta p}{Q} = \int_0^1 \frac{1}{\pi \left( \frac{1-N}{2-N} \right) \left( -\frac{h^4}{4} + \frac{Nh^3 I_0 (mh)}{2m I_1 (mh)} - \frac{Nh^2}{m} \right)} \ dz \quad (4.2.26)
\]
The pressure drop in the normal artery $\Delta P_N$, is derived from Eq. (4.2.25) as

$$\Delta P_N = \int_0^1 \frac{Q}{\pi(2-N)\left(\frac{1}{4}\frac{N I_0(h)}{2m I_1(m)} - \frac{N}{m}\right)} \, dz$$  \hspace{1cm} (4.2.27)

From Eq. (4.2.27), the resistance to the flow in the absence of stenosis ($h = 1$) is obtained as

$$\lambda_N = \frac{\Delta P_N}{Q} = \int_0^1 \frac{1}{\pi(2-N)\left(\frac{1}{4}\frac{1}{2m I_1(m)} - \frac{N}{m}\right)} \, dz$$  \hspace{1cm} (4.2.28)

The resistance to the flow

$$\bar{\lambda} = \frac{\lambda}{\lambda_N}$$  \hspace{1cm} (4.2.29)

The wall shear stress is given by

$$\tau_h = \frac{-1}{(1-N)} \left(\frac{\partial u_z}{\partial r} + N v_{\theta}\right) \bigg|_{r=h}$$  \hspace{1cm} (4.2.30)

From Eq. 4.2.14 $v_{\theta} = 0$ at $r = h$

$$\tau_h = \frac{-1}{(1-N)} \left(\frac{\partial u_z}{\partial r}\right) \bigg|_{r=h}$$  \hspace{1cm} (4.2.31)

From Eq. 4.2.12 & 4.2.18

$$\frac{\partial u_z}{\partial r} = \left((1 - N) \frac{r}{2 \, dx}\right) \bigg|_{r=h}$$  \hspace{1cm} (4.2.32)

From Eqs. 4.2.31 & 4.2.32

$$\tau_h = -\frac{h \, dp}{2 \, dx}$$  \hspace{1cm} (4.2.33)
4.2.4 Results and Discussions

The change in flow pattern and the micropolar fluid nature of blood in an artery with stenosis and post stenotic dilatation are studied. The analytic solutions are obtained for flow resistance ($\bar{\lambda}$) and wall shear stress ($\tau_w$) and are given in the equations (4.2.29) and (4.2.33) respectively. The results are analyzed for different values of stenosis and dilatation heights, coupling number and micropolar fluid parameter.

The influence of various parameters on flow resistance($\bar{\lambda}$) and wall shear stress($\tau_w$) has been studied through the graphs (Figs.4.2.2-4.2.18), by considering the parameter values as $d_1 = d_2 = 0.2; L = 1; L_1 = L_2 = 0.2$; (Maruthi Prasad and Radha Krishnamacharya [107]).

The variation of flow resistance with height of stenosis for different values of dilatation height is shown in Fig. 4.2.2. It is observed that, the flow resistance increases with the height of stenosis but it decreases with dilatation height. The reverse effect is noticed in Fig. 4.2.3.

The variation in stenosis length increases the resistance to the flow with the increase in height of stenosis (Fig. 4.2.4 & 4.2.5). It is noticed from the Fig. 4.2.6, that the flow resistance decreases with the length of dilatation.

Figs. 4.2.7 - 4.2.9 explains the effect of coupling number on flow resistance for various values of stenosis and dilatation height. It is observed that, the flow resistance increases with stenosis height and the coupling number, but decreases with stenotic dilatation. It can be noticed that the influence of coupling number on flow resistance is insignificant when $\delta \leq 0.05$ (Figs. 4.2.7 & 4.2.8).

The change in flow resistance with respect to stenosis height for different values of micropolar fluid parameter is depicted in Figs.4.2.10 & 4.2.11. It can be seen that, the flow resistance decreases with micropolar fluid parameter and increases with stenosis.
height. From Fig.4.2.12, it is observed that, the flow resistance decreases in dilatation region with micropolar fluid parameter.

Figs.4.2.13-4.2.15 reveals the influence of coupling number on the wall shear stress. It is noticed that, the wall shear stress increases with coupling number and height of stenosis, but it decreases in case of stenotic dilatation.

The micropolar fluid parameter effect on wall shear stress is shown in Figs. 4.2.16-4.2.18. It can be seen that, the wall shear stress decreases with the micropolar fluid parameter when the height of stenosis increases and decreases with stenotic dilatation.

4.2.5 Graphs

Fig-4.2.2: Graphical representation of $\bar{\lambda}$ with $\delta_1$ for distinct values of $\delta_2$

$(d_1 = d_2 = 0.2; L = 1; Q = 0.1; N = 0.2; L_1 = L_2 = 0.2; m = 1)$
Fig-4.2.3: Graphical representation of $\bar{\lambda}$ with $\delta_2$ for distinct values of $\delta_1$

$(d_1 = d_2 = 0.2; L = 1; Q = 0.1; L_1 = L_2 = 0.2; N = 0.2; m = 1 )$

Fig-4.2.4: Graphical representation of $\bar{\lambda}$ with $\delta_1$ for distinct values of $L_1$

$(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, N = 0.2, m = 1, \delta_2 = 0.0 )$
Fig-4.2.5: Graphical representation of $\bar{\lambda}$ with $\delta_1$ for distinct values of $L_1$

\begin{align*}
(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, N = 0.2, m = 1, \delta_2 = -0.02)
\end{align*}

Fig-4.2.6: Variation of flow resistance $\bar{\lambda}$ with $\delta_2$ for different $L_2$

\begin{align*}
(d_1 = d_2 = 0.2, L = 1, Q = 0.1, N = 0.2, m = 1, \delta_1 = 0.0, L_1 = 0.2)
\end{align*}
Fig-4.2.7: Effect of $\delta_1$ on $\bar{\alpha}$ for various values of $N$

$(d_1 = 0.2, L = 1, L_1 = L_2 = 0.2, Q = 0.1, d_2 = 0.2, m = 1, \delta_2 = 0.0)$

Fig-4.2.8: Graphical representation of Variation of $\bar{\alpha}$ and $\delta_1$ for various values of $N$

$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, m = 1, \delta_2 = -0.02, L_1 = L_2 = 0.2)$
Fig-4.2.9: Graphical representation of Variation of $\tilde{\lambda}$ and $\delta_2$ for various values of $N$

$\left( d_1 = 0.2; L = 1, Q = 0.1; d_2 = 0.2; m = 1; \delta_1 = 0.0; L_1 = L_2 = 0.2 \right)$

Fig-4.2.10: Variation of flow resistance $\tilde{\lambda}$ with $\delta_1$ for different $m$

$\left( d_1 = d_2 = 0.2; L = 1, Q = 0.1; N = 0.2; \delta_2 = 0.0; L_1 = L_2 = 0.2 \right)$
Fig-4.2.11: Variation of flow resistance $\bar{\lambda}$ with $\delta_1$ for different $m$

\[(d_1 = d_2 = 0.2; L = 1, Q = 0.1; N = 0.2; \delta_2 = -0.02; L_1 = L_2 = 0.2)\]

Fig-4.2.12: Variation of flow resistance $\bar{\lambda}$ with $\delta_2$ for different $m$

\[(d_1 = d_2 = 0.2; L = 1, Q = 0.1; N = 0.2; \delta_1 = 0.0; L_1 = L_2 = 0.2)\]
Fig-4.2.13: Variation in $\tau_h$ with respect to $\delta_1$ for varying values of $N$

$(d_1 = 0.2; L = 1, Q = 0.1; d_2 = 0.2; m = 1; \delta_2 = 0.0; L_1 = L_2 = 0.2)$

Fig-4.2.14: Comparison of wall shear stress $\tau_h$ with $\delta_1$ for distinct values of $N$

$(d_1 = 0.2; L = 1, Q = 0.1; d_2 = 0.2; m = 1; \delta_2 = -0.02; L_1 = L_2 = 0.2)$
Fig-4.2.15: Comparison of wall shear stress $\tau_h$ with $\delta_2$ for distinct values of $N$

\[(d_1 = 0.2; L = 1, Q = 0.1; d_2 = 0.2; m = 1; \delta_1 = 0; L_1 = L_2 = 0.2)\]

Fig-4.2.16: Graphical representation of $\tau_h$ and $\delta_1$ for various values of $m$

\[(d_1 = d_2 = 0.2, L = 1, Q = 0.1, N = 0.2, \delta_2 = 0.0, L_1 = L_2 = 0.2)\]
Fig-4.2.17: Graphical representation of $\tau_h$ and $\delta_1$ for various values of $m$

$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, N = 0.2, \delta_2 = -0.02L_1 = L_2 = 0.2)$

Fig-4.2.18: Relationship between $\tau_h$ and $\delta_2$ for various values of $m$

$(d_1 = d_2 = 0.2, L = 1, Q = 0.1, N = 0.2, \delta_1 = 0.0, L_1 = L_2 = 0.2)$
4.3 Effects of stenosis and post stenotic dilatation on flow of Jeffrey fluid through an artery

4.3.1 Introduction

The study of blood flow in stenosed arteries has been investigated by many researchers. Mishra and Verma [112] studied the blood flow in an artery with stenosis having a uniform cross section. Gupta et al., [113] studied the multiple stenoses effects with viscosity variation for power law fluid model.

Abd-Alla et al., [114] observed the peristaltic flow in an asymmetric channel by considering the magnetic field and gravity field. Sudhakar Reddy et al., [115] studied viscosity effect on peristaltic flow of Jeffrey fluid in a uniform tube.

The above literature is concerned with different shapes stenosis with uniform or non-uniform cross sectional tubes by considering a non-Newtonian fluid.

A study conducted on post-stenotic dilatation by considering blood as Casson fluid (Tandon,et al.,[108]). A.K. Singh and D.P.Singh [46] studied the effects of post-stenotic dilatation by treating the fluid as Bingham fluid.

With this motivation, the influence of stenosis and post-stenotic dilatation on Jeffrey fluid has been analyzed.

4.3.2. Mathematical Formation of the problem

Consider the flow of Jeffrey fluid in an axi-symmetric artery containing multiple abnormal segments as shown in figure 4.2.1. The equations expressing the geometrical characteristics of the wall, as shown in Fig.4.2.1, are given by eq.4.1.1.  

The constitutive equations for the fluid are (M.Sudhakar Reddy et al., [115])

\[ T = -pl + S \]  

\[ S = \frac{\mu}{1+\lambda_1} \left( \frac{\partial^2 \gamma}{\partial t^2} + \lambda_2 \frac{\partial^2 \gamma}{\partial t^2} \right) \]
Here $T$ represents Cauchy stress tensor, $S$ represents the extra stress tensor, $p$ represents the pressure, $I$ represents the identity tensor, $\lambda_1$ represents the ratio of the relaxation to retardation times, $\lambda_2$ represents the retardation time, $\mu$ represents the dynamic viscosity and $\gamma$ represents the shear rate.

The equations for Jeffrey fluid are

$$
\left( \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) = 0
$$

(4.3.3)

$$
\rho \left( u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) u = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \frac{\partial}{\partial z} \left( S_{rz} \right) - \frac{S_{\theta \theta}}{r}
$$

(4.3.4)

$$
\rho \left( u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) w = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rz} \right) + \frac{\partial}{\partial z} \left( S_{zz} \right)
$$

(4.3.5)

Where

$$
S_{rr} = \frac{2\mu}{1+\lambda_1} \left( 1 + \lambda_2 \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \frac{\partial u}{\partial r} \right)
$$

$$
S_{rz} = \frac{\mu}{1+\lambda_1} \left( 1 + \lambda_2 \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \frac{\partial u}{\partial r} + \frac{\partial u}{\partial z} \right)
$$

$$
S_{zz} = \frac{2\mu}{1+\lambda_1} \left( 1 + \lambda_2 \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \frac{\partial u}{\partial z} \right)
$$

$$
S_{\theta \theta} = \frac{2\mu}{1+\lambda_1} \left( 1 + \lambda_2 \left( \frac{u}{\partial r} + \frac{w}{\partial z} \right) \frac{u}{r} \right)
$$

Considering the non dimensional variables given below

$$
Z = \frac{z}{L}, \delta = \frac{\delta}{R_0}, \bar{R} = \frac{R}{R_0}, \bar{P} = \frac{P}{\mu \bar{L}} , \bar{u} = \frac{u}{\bar{u}}, \bar{w} = \frac{L}{\bar{u} \bar{L}} w, Re = \frac{\rho R_0 u}{\mu}, \bar{\mu} = \frac{\mu}{\mu_0}
$$

(4.3.6)

Where $U$ represents the average velocity.

in Eqs. (4.3.3)- (4.3.5), considering mild stenosis $\frac{\delta}{R_0} \ll 1$ takes the form (after dropping the bars)

$$
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0
$$

(4.3.7)

$$
\frac{\partial p}{\partial r} = 0
$$

(4.3.8)
\[ -\frac{\partial p}{\partial z} = \frac{1}{1 + \lambda_1} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \]  \hspace{1cm} (4.3.9)

The corresponding non dimensional boundary conditions are

\[ \frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \]  \hspace{1cm} (4.3.10)

\[ u = 0 \text{ at } r = h \]  \hspace{1cm} (4.3.11)

4.3.3. Method of Solution

Solving the equation (4.3.9) using the boundary conditions Eqs. (4.3.10) and (4.3.11), yields

\[ u = \left( \frac{h^2 - r^2}{4} \right) (1 + \lambda_1) \frac{dp}{dz} \]  \hspace{1cm} (4.3.12)

The volumetric flow rate is defined by

\[ Q = 2\pi \int_0^h ur \, dr \]  \hspace{1cm} (4.3.13)

After integrating,

\[ Q = \frac{\pi h^4}{8} (1 + \lambda_1) \frac{dp}{dz} \]  \hspace{1cm} (4.3.14)

\[ \frac{dp}{dz} = \frac{8Q}{\pi h^4 (1 + \lambda_1)} \]  \hspace{1cm} (4.3.15)

When \( \lambda_1 = 0 \), the equations indicate the Newtonian fluid.

The pressure drop \( \Delta p \) in the presence of stenosis is obtained by integrating Eq. (4.3.15), as

\[ \Delta p = \int_0^1 \frac{dp}{dz} \, dz \]  \hspace{1cm} (4.3.16)

\[ \Delta p = \int_0^1 \frac{8Q}{\pi h^4 (1 + \lambda_1)} \, dz \]  \hspace{1cm} (4.3.17)
the flow resistance $\lambda$, is described as

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q} \int_0^1 \left( \frac{8Q}{\pi h \left(1+\lambda_1\right)} \right) \, dz$$  \hspace{1cm} (4.3.18)

The pressure drop without stenosis is represented as $\Delta P_N$, is derived from Eq. (4.3.15).

$$\Delta P_N = \int_0^1 \frac{8Q}{\pi (1+\lambda_1)} \, dz$$  \hspace{1cm} (4.3.19)

The resistance to the flow in the normal artery is given by $\lambda_N$ and is derived from Eq. (4.3.19)

$$\lambda_N = \frac{\Delta P_N}{Q} = \frac{1}{Q} \int_0^1 \frac{8Q}{\pi (1+\lambda_1)} \, dz$$  \hspace{1cm} (4.3.20)

The resistance to the flow is

$$\bar{\lambda} = \frac{\lambda}{\lambda_N}$$  \hspace{1cm} (4.3.21)

The expression for wall shear stress is given by

$$S_{rz} = \left\{ \frac{1}{1+\lambda_1} \left[ \frac{\partial u}{\partial r} + \lambda_2 u \left( \frac{\partial^2 u}{\partial r^2} \right) \right] \right\} \bigg|_{r=h}$$  \hspace{1cm} (4.3.22)

4.3.4. Results and Discussions

The equations for velocity ($u$), flow resistance $\bar{\lambda}$ and shear stress at the wall ($\tau_h$) are given by (4.3.12), (4.3.21) and (4.3.22) respectively. The results are computed numerically by taking $d_1 = d_2 = 0.2, L_1 = L_2 = 0.2, L = 1$ (Maruthi Prasad and Radha Krishnamacharya [107]). The influence of different variables on the flow resistance ($\bar{\lambda}$), pressure drop ($\Delta p$), wall shear stress ($\tau_h$) are obtained and depicted in Figs. 4.3.1-4.3.16.

The height of stenosis and dilatation on flow resistance are shown in Figs. 4.3.1 & 4.3.2. It is noticed that, the flow resistance increases with the stenosis height but decreases with dilatation height. From Fig. 4.3.3-4.3.5, it is observed that the resistance to the flow increases with height of stenosis, but it decreases with height of dilatation.
Figs. 4.3.6-4.3.8, illustrates the pressure gradient variation with the height of stenosis and dilatation for different values of volumetric flow rate. It is noticed that, pressure gradient increases with the stenosis height and volumetric flow rate, but decreases in case of dilatation. The effect of Jeffrey fluid parameter on pressure gradient is shown in Figs. 4.3.9 - 4.3.11. The pressure gradient decreases with Jeffrey fluid parameter, but the reverse effect occurs in the case of dilatation.

It is observed that, the velocity of the fluid decreases with Jeffrey fluid parameter (Fig.4.3.12), but increases in the case of dilatation as shown in Fig.4.3.13. The variation of wall shear stress with Jeffrey fluid parameter for various values of stenosis and dilatation heights is presented in Figs.4.3.14 -4.3.16. The wall shear stress decreases in the dilatation region with the increasing values of Jeffrey fluid parameter \( \lambda_1 \), but it increases in the stenotic region with the decreasing values of Jeffrey fluid parameter \( \lambda_1 \).

4.3.5 Graphs

![Graphical representation of \( \tilde{\lambda} \) with \( \delta_1 \) for various values of \( \delta_2 \)](image)

Fig.4.3.1: Graphical representation of \( \tilde{\lambda} \) with \( \delta_1 \) for various values of \( \delta_2 \)

\[
( d_1 = d_2 = 0.2; L = 1; Q = 0.1; \lambda_1 = 0.2; L_1 = L_2 = 0.2 )
\]
Fig-4.3.2: Graph depicting the changes in $\bar{\lambda}$ with $\delta_2$ for different values of $\delta_1$

$$(d_1 = d_2 = 0.2; L = 1; Q = 0.1; \lambda_1 = 0.2; L_1 = L_2 = 0.2)$$

Fig-4.3.3: Effect of $\delta_1$ on $\bar{\lambda}$ for various $L_1$

$$(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, \delta_2 = 0.0, \lambda_1 = 0.2)$$
Fig 4.3.4: Effect of $\delta_1$ on $\bar{\lambda}$ for various $L_1$

$\left(d_1 = 0.2, d_2 = 0.2, L_2 = 0.2, L = 1, Q = 0.1, \delta_2 = -0.02, \lambda_1 = 0.2 \right)$

Fig 4.3.5: Effect of $\delta_2$ on $\bar{\lambda}$ for various $L_2$

$\left(d_1 = d_2 = 0.2, L = 1, Q = 0.1, \delta_1 = 0, L_1 = 0.2, \lambda_3 = 0.2 \right)$
Fig 4.3.6: Comparison between $\Delta p$ and $\delta_1$ for distinct values of $Q$

$\left(d_1 = d_2 = 0.2, L = 1, \lambda_1 = 0.2, \delta_2 = 0, L_1 = L_2 = 0.2 \right)$

Fig 4.3.7: Comparison between $\Delta p$ and $\delta_1$ for distinct values of $Q$

$\left(d_1 = d_2 = 0.2, L = 1, \lambda_1 = 0.2, \delta_2 = -0.01, L_1 = L_2 = 0.2 \right)$
Fig-4.3.8: Pressure drop $\Delta p$ verses $\delta_2$ for various values of $Q$

\[(d_1 = d_2 = 0.2, L = 1, \lambda_1 = 0.2, \delta_1 = 0.0, \text{ etc.})\]

Fig-4.3.9: Variation of pressure drop $\Delta p$ with $\delta_1$ for different $\lambda_1$

\[(d_1 = d_2 = 0.2, L = 1, Q = 0.1, L_1 = L_2 = 0.2, \delta_2 = 0.0)\]
Fig-4.3.10: Variation of pressure drop $\Delta p$ with $\delta_1$ for different $\lambda_1$

$d_1 = d_2 = 0.2, L = 1, Q = 0.1, \delta_2 = -0.01, L_1 = L_2 = 0.2$)

Fig-4.3.11: Variation of pressure drop $\Delta p$ with $\delta_2$ for different $\lambda_1$

$d_1 = d_2 = 0.2, L = 1, Q = 0.1, \delta_1 = 0.01, L_1 = L_2 = 0.2$)
Fig-4.3.12: Variation of velocity $u$ with $\delta_1$ for different $\lambda_1$

$(Q = 0.1, \delta_2 = 0, r = 0.02, P = 0.1)$

Fig-4.3.13: Variation of velocity $u$ with $\delta_2$ for different $\lambda_1$

$(Q = 0.1, \delta_1 = 0.01, r = 0.02, P = 0.1)$
Fig-4.3.14: Relationship between \( \tau_h \) and \( \delta_2 \) for various values of \( \delta_1 \)

\[
(Q = 0.1, \lambda_1 = 0.2)
\]

Fig-4.3.15: Relationship between \( \tau_h \) with \( \lambda_1 \) for various values of \( \delta_2 \)

\[
(Q = 0.1, \delta_1 = 0.0)
\]
Fig 4.3.16: Comparison between $\tau_\lambda$ & $\lambda_1$ for various values of $\delta_1$

$Q = 0.1, \delta_2 = 0.01$
A comparative study between the Herschel-Bulkley, micropolar and Jeffrey fluid has been studied by taking flow resistance, stenosis height and dilatation height into consideration. This is shown in Table 4.1. This comparison has been done for the fixed values of $d_1 = d_2 = 0.2, L_1 = L_2 = 0.2, L = 1$.

**Table 4.1** Comparative study between Herschel-Bulkley, micropolar and Jeffrey fluid.

<table>
<thead>
<tr>
<th>$\delta_2 =$</th>
<th>Herschel-Bulkley fluid</th>
<th>Micropolar fluid</th>
<th>Jeffrey fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$\dot{\lambda}$</td>
<td>$\lambda$</td>
</tr>
<tr>
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<td>1.0061</td>
<td>1.0042</td>
<td>1.0023</td>
</tr>
<tr>
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<td>1.0082</td>
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<td>1.0043</td>
</tr>
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<td>1.0084</td>
<td>1.0065</td>
</tr>
<tr>
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</tr>
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<td>1.0111</td>
</tr>
<tr>
<td>0.1</td>
<td>1.0172</td>
<td>1.0152</td>
<td>1.0133</td>
</tr>
</tbody>
</table>

From the above table, it is observed that the flow resistance is more in Herschel-Bulkley fluid compared to micropolar and Jeffrey fluid. Hence Herschel-Bulkley fluid is more appropriate fluid for blood flow through stenosis and post stenotic dilatation.