MATERIALS AND METHODS

The design of compact and inexpensive ultra-wideband antennas with notch characteristics and the designing of UWB MIMO antenna was our target in this thesis. This chapter focuses on the theoretical background and design methodology used in designing those antennas and also the design of MIMO UWB system. Starting from the initial design for the microstrip-line-fed printed circular disc monopole antenna followed by Elliptical monopole antenna with CPW fed we developed different novel antenna structures for future UWB and UWB MIMO applications.

This chapter is organized as follows: Section 3.1 presents the numerical and analytical techniques used in designing all UWB antennas and UWB MIMO. All the antennas designed in this thesis are based on Finite Element (FE) method as used in Ansoft High Frequency Structure Simulator (HFSS). The fundamental antenna parameters are explained in section 3.2. The operation principles and design methodologies of UWB disc monopole antennas and the UWB MIMO antennas with dual band notched characteristics of the antenna performance and characteristics are addressed in Section 3.3.

3.1 Numerical/Analytical Techniques

Ansoft HFSS uses the FE numerical technique in order to generate an EM field solution for different 3D problems. First, the finite element technique is based on dividing the whole big problem space into small regions or sub-regions called elements. Then the first simulation software program is Ansoft High Frequency Structure Simulator (HFSS). Ansoft HFSS advanced solver and high-performance compute technology have made it an essential powerful EM field simulation tool for engineers, researchers and
scientists in academia doing accurate and rapid design of high-frequency and high-speed electronic components. It is based on a three-dimensional (3D) full-wave finite element (FE) method which is a frequency-domain numerical technique for solving Maxwell’s equations. Ansoft HFSS uses the FE numerical technique in order to generate an EM field solution for different 3D problems. First, the finite element technique is based on dividing the whole big problem space into small regions or sub-regions called elements. Then the fields in each finite element are formulated by local functions. Ansoft HFSS automatically converts the whole problem structure into a finite element mesh which consists of a large number of very small 3D tetrahedral shapes as shown in Figure 3.1. Each single tetrahedron is a four-sided pyramid as presented in Figure 3.2. It can be seen that the meshing or discretization operation done by Ansoft HFSS is very coarse in almost the whole structure while it is very fine at some regions which need more accuracy such as near wave port, metallic edges or discontinuities. After finalizing the mesh operation of the whole structure, the solution process starts with two-dimensional (2D) port solutions as the structure excitation then followed by the field solution of the full 3D problem including fields at all vertices, midpoints and interior points as in Figure 3.2. The program exploits the computed 2D fields on ports to be used as boundary conditions to solve the 3D fields of the whole structure, the fields in each finite element are formulated by local functions.

![Figure 3.1 Meshing based on tetrahedral shapes of an antenna structure in Ansoft HFSS](image-url)
Fig 3.2 A single tetrahedral mesh shape used in Ansoft HFSS.

**FR4 substrates**

FR4 substrates are used in the designing of all antennas in the thesis.

FR4 epoxy glass substrates are the material of choice for most PCB applications. The material is very low cost and has excellent mechanical properties, making it ideal for a wide range of electronic component applications. As more and more microwave systems aimed at consumer markets are developed, there is a considerable interest in minimising the cost of these systems. Substantial cost savings could be realised by using FR4 in place of costly PTFE based substrates for microwave circuits and antennas. The use of FR4 is unlikely to be viable for antenna feeding structures due to its high losses. However, for high density microwave circuits where path lengths are short and for broadband antenna elements, where losses and absolute dielectric constant values are less critical, the material could be used in place of more conventional microwave substrate materials, offering significant cost savings.

### 3.2 Fundamental Antenna Parameters:

To describe the performance of an antenna, definitions of various parameters are necessary. In practice, there are several commonly
used antenna parameters, including bandwidth, radiation pattern, directivity, gain, input impedance, and so on.

**Bandwidth:**

Bandwidth ($BW$) is the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard. The bandwidth can be considered to be the range of frequencies, on either side of the center frequency, where the antenna characteristics are within an acceptable value of those at the center frequency. Generally, in wireless communications, the antenna is required to provide a return loss less than -10dB over its frequency bandwidth.

The frequency bandwidth of an antenna can be expressed as either absolute bandwidth ($ABW$) or fractional bandwidth ($FBW$). If $f_H$ and $f_L$ denote the upper edge and the lower edge of the antenna bandwidth respectively. The $ABW$ is defined as the difference of the two edges and the $FBW$ is designated as the percentage of the frequency difference over the center frequency, as given in Equation (3-1) and (3-2) respectively.

$$ABW = f_H - f_L \quad (3-1)$$

$$FBW = 2(f_H-f_L) / (f_H+f_L) \quad (3-2)$$

For broadband antennas, the bandwidth can also be expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable, as shown in Equation (3-3).

$$BW = f_H/f_L \quad (3-3)$$

**Radiation Pattern:** The radiation pattern (or antenna pattern) is the representation of the radiation properties of the antenna as a function of space coordinates. In most cases,
it is determined in the far-field region where the spatial (angular) distribution of the radiated power does not depend on the distance. Usually, the pattern describes the normalized field (power) values with respect to the maximum values.

The radiation property of most concern is the two or three-dimensional (2D or 3D) spatial distribution of radiated energy as a function of the observer’s position along a path or surface of constant radius. In practice, the three-dimensional pattern is sometimes required and can be constructed in a series of two-dimensional patterns. For most practical applications, a few plots of the pattern as a function of $\phi$ for some particular values of frequency, plus a few plots as a function of frequency for some particular values of $\theta$ will provide most of the useful information needed, where $\phi$ and $\theta$ are the two axes in a spherical coordinate system.

For a linearly polarized antenna, its performance is often described in terms of its principle $E$ plane and $H$-plane patterns. The $E$-plane is defined as the plane containing the electric-field vector and the direction of maximum radiation whilst the $H$-plane is defined as the plane containing the magnetic-field vector and the direction of maximum radiation.

There are three common radiation patterns that are used to describe an antenna's radiation property:

- **Isotropic**: A hypothetical lossless antenna having equal radiation in all directions. It is only applicable for an ideal antenna and is often taken as a reference for expressing the directive properties of actual antennas.

- **Directional**: An antenna having the property of radiating or receiving electromagnetic waves more effectively in some directions than in others. This is
usually applicable to an antenna where its maximum directivity is significantly
greater than that of a half-wave dipole.

- **Omni-Directional:** An antenna having an essentially non-directional pattern in
  a given plane and a directional pattern in any orthogonal plane.

**Directivity:**
To describe the directional properties of antenna radiation pattern, directivity $D$ is
introduced and it is defined as the ratio of the radiation intensity $U$ in a given direction
from the antenna over that of an isotropic source. For an isotropic source, the radiation
intensity $U_0$ is equal to the total radiated power $P_{rad}$ divided by $4\pi$. So the directivity can
be calculated by:

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \quad (3-4)$$

If not specified, antenna directivity implies its maximum value, i.e. $D_0$

$$D_0 = \frac{U_{max}}{U_0} = 4\pi \frac{U_{max}}{P_{rad}} \quad (3-5)$$

**Gain:**
The antenna absolute gain according to is defines as “the ratio of the intensity in a given
direction to the radiation intensity that would be obtained if the power accepted by the
antenna were radiated isotropically.”

Antenna gain $G$ is closely related to the directivity, but it takes into account the
radiation efficiency $e_{rad}$ of the antenna as well as its directional properties, as given by:

$$G = e_{rad} \cdot D \quad (3-6)$$

Similarly, the maximum gain $G_0$ is related the maximum directivity $D_0$ by:

$$G_0 = e_{rad} \cdot D_0 \quad (3-7)$$
Figure 3.3 shows the equivalent circuit of the antenna, where $R_r$, $R_L$, $L$ and $C$ represent the radiation resistance, loss resistance, inductor and capacitor, respectively. The radiation efficiency $e_{rad}$ is defined as the ratio of the power delivered to the radiation resistance to the power delivered to $R_r$ and $R_L$. So the radiation efficiency can be written as:

$$e_{rad} = \frac{\frac{1}{2}|I|^2 R_r}{\frac{1}{2}|I|^2 R_r + \frac{1}{2}|I|^2 R_L} = \frac{R_r}{R_r + R_L}$$  \hspace{1cm} (3-8)

**VSWR:**

VSWR stands for Voltage Standing Wave Ratio, and is also referred to as Standing Wave Ratio (SWR). VSWR is a function of the reflection coefficient, which describes the power reflected from the antenna. If the reflection coefficient is given by $\Gamma$, then VSWR is defined as:
The VSWR is always a real and positive number for antennas. The smaller the VSWR is, the better the antenna is matched to the transmission line and the more power is delivered to the antenna. The minimum VSWR is 1.0. In this case, no power is reflected from the antenna, which is ideal.

Often antennas must satisfy a bandwidth requirement that is given in terms of VSWR. For instance, an antenna might claim to operate from 100-200 MHz with VSWR<3. This implies that the VSWR is less than 3.0 over the specified frequency range. This VSWR specification also implies that the reflection coefficient is less than 0.5 over the quoted frequency range.

**Impedance Bandwidth:**
Impedance bandwidth indicates the bandwidth for which the antenna is sufficiently matched to its input transmission line such that 10% or less of the incident signal is lost due to reflections. Impedance bandwidth measurements include the characterization of the VSWR and return loss throughout the band of interest.

**Polarization:**
Antenna polarization indicates the polarization of the radiated wave of the antenna in the far-field region. The polarization of a radiated wave is the property of an electromagnetic wave describing the time varying direction and relative magnitude of the electric-field vector at a fixed location in space, and the sense in which it is traced, as observed along the direction of propagation. Typically, this is measured in the
direction of maximum radiation. There are three classifications of antenna polarization: linear, circular and elliptical. Circular and linear polarizations are special cases of elliptical polarization. Typically, antennas will exhibit elliptical polarization to some extent. Polarization is indicated by the electric field vector of an antenna oriented in space as a function of time. Should the vector follow a line, the wave is linearly polarized. If it follows a circle, it is circularly polarized (either with a left hand sense or right hand sense). Any other orientation is said to represent an elliptically polarized wave.

3.3 Model Analysis of Microstrip Antennas:
The most widely used microstrip patch configuration is the rectangular patch. Analysis of this patch is easy using transmission-line and cavity models. The transmission-line model is the easiest of all and yields accurate results.

Transmission-Line Model:
This model represents the microstrip antenna by two slots separated by a transmission line of length $L$. Fringing effects occur at the edges of the patch and is a function of $L$, width $W$ of the patch and height $h$ of the substrate.

Due to fringing effects, the length of the patch is extended on the ends by a distance $\Delta L$, which is a function of the effective dielectric constant $\epsilon_{r(ef)}$ and width-to-height ratio. The approximate relation for extended length is given by:

$$\frac{\Delta L}{h} = 0.412 \frac{\left(\epsilon_{r(ef)}+0.3\right)\left(\frac{W}{h}+0.264\right)}{\left(\epsilon_{r(ef)}-0.258\right)\left(\frac{W}{h}+0.8\right)}$$  \hspace{1cm} (3-10)

Where $\epsilon_{r(ef)}$ is given by:

$$\epsilon_{r(ef)} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + 12 \frac{h}{W} \right]^{-1/2}$$  \hspace{1cm} (3-11)
Hence, the effective length of the patch is

\[ L_{\text{eff}} = L + 2\Delta L \]

The resonant frequency of the microstrip antenna is related to its length as:

\[ f_r = \frac{1}{2L\sqrt{\varepsilon_r/\mu_0\varepsilon_0}} = \frac{v_o}{2L\sqrt{\varepsilon_r}} \]

**Practical Design Procedure:**

For a specified dielectric constant of the substrate \( \varepsilon_r \), resonant frequency \( f_r \), and height of the substrate \( h \), the following design equations are used to calculate the length and width of the patch:

For efficient radiation the width (\( W \)) is given by:

\[ W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r + 1}} = \frac{v_o}{2f_r} \sqrt{\frac{2}{\varepsilon_r + 1}} \]  

(3-12)

Where \( v_o \) is the free-space velocity of light.

\( f_r = \) Resonant frequency.

Now, the extended length \( \Delta L \) can be calculated.

The actual length of the patch can be determined using

\[ L = \frac{1}{2f_r\sqrt{\varepsilon_r(\varepsilon_{eff})/\mu_0\varepsilon_0}} - 2\Delta L \]  

(3-13)
3.4 UWB Antenna Design Methodology

Design and analysis of UWB antenna is more challenging compared to narrowband antennas because it possess a wide operating band as specified by FCC and required compactness in antenna with the desired antenna characteristics. Such design and development of UWB antenna possesses several advantages over conventional data transmission in case of indoor communication. Hence, a compact UWB antenna has to be designed very carefully to enhance the system performance. The size of the antenna can be reduced significantly with the use of high dielectric (\(\varepsilon_r\)) substrates [Balanis, 2005]. High \(\varepsilon_r\) substrates help to reduce size but at the same time it also deteriorates the radiation efficiency of the antenna.

Earlier the author Bokhari et al. in 1996 uses the meandering technique in antenna design by cutting slots in the non-radiating part of the design to achieve compactness. This technique helps to increase the effective electrical path length but it reduces the operating bandwidth due to capacitive loading. It is demonstrated that by etching rectangular and L-shaped slot in the ground plane; fractional bandwidth up to 125% is achieved. However, most of these antennas are larger in dimensions, which make them difficult to integrate with portable commercial devices. Thus, these techniques are not very efficient for compact wideband antennas.

The applications of UWB technology for indoor communication are very promising. However, it offers a great challenge among the antenna designer community because of its detrimental interference issues with other existing narrowband systems and services such as worldwide interoperability for microwave access (WiMAX) in 3.3-3.8 GHz, wireless local area network (WLAN) in 5.15-5.85 GHz and X-band in 7.9-8.4 GHz, which operates in the frequency range of 3.1-10.6 GHz. It is desirable in wireless communications to avoid any interference between different users present in the UWB
spectrum. Although, FCC has allowed UWB devices operation in this wide range with a restricted power level compliant with the emission mask, to avoid the concern related to potential interference. UWB antenna should have band rejection characteristics in the possible interfering bands. Hence, the design of an UWB antenna with multiple band notch characteristics is very challenging task.

The rapid growth in wireless communication forces the regulatory authorities to allow the transmission in higher and wider frequency spectrum in order to achieve high wireless channel capacity. It is achieved by using the diversity/MIMO technology in rich scattering environment without additional power or spectrum. MIMO technology uses multiple antennas at the transmitter and receiver terminal of transmission system. The UWB system is also susceptible to multipath fading problems similar to other wireless communication systems. The use of MIMO technology helps to improve diversity gain and multipath fading. UWB MIMO technology requires high isolation among antenna elements to combat multipath fading. However, the compact UWB MIMO antenna for portable applications in a given smaller area causes the degradation in diversity performance due to presence of various mutual coupling among the antenna elements. In addition, UWB system faces the severe interference challenges with existing narrowband system such as WLAN from 5.15-5.825 GHz, which lies in the UWB spectrum. Hence, the design of a compact UWB MIMO antenna with band notch characteristics is a very challenging task.

**MIMO Channel Model:**
The MIMO channel communication takes advantage of multipath propagation. The MIMO channel can be described by the following matrix:

\[ y = Hx + n \]
Where $y$ is the received signal vector, $x$ is the transmitted signal vector, $H$ and $n$ are the channel matrices.

![MIMO Channel Model](image)

Fig. 3.4 MIMO Channel Model.

In order to understand MIMO better, it is necessary to look into its channel model as shown in Figure. For a system with $M_T$ transmitters and $M_R$ receivers, the MIMO channel at a given time may be represented by $M_R \times M_T$ matrix as demonstrated below,

$$
H = \begin{bmatrix}
H_{1,1} & H_{1,2} & \cdots & H_{1,M_T} \\
H_{2,1} & H_{2,2} & \cdots & H_{2,M_T} \\
\vdots & \vdots & \ddots & \vdots \\
H_{M_R,1} & H_{M_R,2} & \cdots & H_{M_R,M_T}
\end{bmatrix},
$$

Where $H_{m,n}$ is the channel gain between the $m$-th receive and $n$-th transmit antenna. The $n$-th column of $H$ is called as the spatial signature of the $n$-th transmit antenna. The geometry of $M_T$ differentiates the signals launched from the transmitter.

**Forms of MIMO:**

The Multiple Input multiple Output (MIMO) method can be divided into various forms depending on uses [12]. MIMO is basically the combination of all the multiple antenna techniques such as SISO, SIMO and MISO. It can use the beam forming or the spatial
Multiplexing methods. MIMO can be categorized into two types, multi-antenna types and multi-user types. Multi-antenna types are listed below:

- **SISO (Single-input Single-output)** - is a conventional radio system where neither the transmitter nor receiver have multiple antenna.

- **SIMO (Single-input Multiple-output)** - is a special case when the transmitter has a single antenna.

- **MISO (Multiple-input Single-output)** - is a special case when the receiver has a single antenna.

Spatial multiplexing techniques make the receivers very complex, and therefore they are typically combined with Orthogonal Frequency Division Multiple Access (OFDMA) modulation, where the problems created by a multi-path channel are handled efficiently. The IEEE 802.16e standard incorporates MIMO-OFDMA. MIMO is also planned to be used in Mobile radio telephone standards such as recent 3GPP and 3GPP2. MIMO technology can be used in non-wireless communications systems. Home networking standard ITU-T G.9963, which defines a power-line communications system uses MIMO techniques to transmit multiple signals over multiple AC wires (phase, neutral and ground).

Envelope correlation coefficient (ECC) and channel capacity loss are the major parameters which characterize the diversity performance of a UWB MIMO antenna. The ECC can be defined in terms of S-parameters as follows:

\[
\rho_e = \frac{|S_{11}^*S_{12} + S_{21}^*S_{22}|^2}{(1-|S_{11}|^2)(1-|S_{21}|^2)(1-|S_{22}|^2)(1-|S_{12}|^2)}
\]  

(3-14)
Along with ECC, the channel capacity loss (b/s/Hz) is also a vital parameter that influence the quality of MIMO antenna. It is the upper limit of information rate at which reliable communications happen over the channel. For good MIMO performance, it must be within the acceptable limit of 0.4 b/s/Hz Shin H et al. (2003). It is calculated with the use of correlation matrix defined in Valderas D et al (2010) and is expressed as follows:

\[ CL_{\text{loss}} = -\log_2 \det(\Psi_R) \]  \hspace{1cm} (3-14)

where \( \Psi_R \) is the correlation matrix of receiving antenna and the elements \( \rho_{ij} \) are the correlation coefficients and defined as follows:

\[ \rho_{ii} = 1 - |S_{ii}|^2 - |S_{ij}|^2 \quad \text{and} \]

\[ \rho_{ij} = -(S_{ii}^*S_{ij} + S_{ji}^*S_{jj}) \text{ for } i, j = 1 \text{ or } 2. \]

Two types of UWB antenna designs are presented the first antennas design is based on the UWB printed disc monopole antenna, and the other designs are UWB antennas with CPW Feed achieving UWB impedance bandwidth with an almost stable dipole-like radiation pattern and constant gain across the whole desired frequency range. Taking these antennas as the basic design methodologies the other UWB antennas and UWB MIMO antennas are designed which are discussed in chapter 4

### 3.4.1 A Hot Air Balloon shaped Microstrip fed UWB Antenna

A hot air balloon shaped, microstrip fed UWB Antenna is designed and analyzed. In the ground plane the steps are introduced, under the microstrip feed line to enhance the bandwidth of the proposed antenna. Therefore, With the introduction of steps in the ground plane additional resonances will be excited and hence much wider impedance bandwidth can be achieved. The overall size of the suggested antenna is 35x20mm\(^2\).
The Ansoft HFSS is used as the simulation tool for designing and optimization of the antenna.

In this design, two steps of dimension $L_{n1} \times W_{n1}$ and $L_{n2} \times W_{n2}$ are etched in the ground plane under microstrip line to enhance the bandwidth. The optimal dimensions of the desired antenna are as follows: $L = 35$ mm, $W = 20$ mm, $L_g = 9$ mm, $W_f = 3.2$ mm, $L_f = 8$ mm, $R = 9$ mm, $L_1 = 10$ mm, $L_{n1} = 5$ mm, $L_{n2} = 4$ mm, $W_{n1} = 1.2$ mm, $W_{n2} = 1$ mm.

![Figure 3.5 Hot air balloon shaped UWB antenna](image)

The triangular radiating patch whose upper edge is modified by adding a semicircular geometry of diameter $R$ giving the shape of hot air balloon. The monopole antenna is fed by 50 Ω microstrip line printed on FR4 substrate of thickness 1.6mm, permittivity 4.4. The basic antenna structure entails of modified radiating patch, feed line, on the other side of the substrate conducting ground plane is placed.
Fig 3.6 VSWR of Hot air balloon shaped UWB antenna

It is clear from the Fig. 3.6 that with the introduction of slots enhances the bandwidth of UWB antenna.

Fig 3.7 Variation for different values of Wn1, keeping other geometrical parameters constant

It is clear from figure 3.7 as \( W_{n1} \) increases the additional resonances are created at the higher frequencies.

3.4.2 An Elliptical CPW Fed UWB Slot Antenna

An elliptical ultra-wide band CPW fed antenna is designed and analyzed. The antenna has elliptical slot structure which is fed by the CPW. This elliptical slot UWB antenna is
surrounded by the extended ground plane in order to reduce the size of the antenna. This antenna is designed with the optimal design parameters for UWB spectrum. The designed antenna is fabricated on FR4 substrate and the return loss is measured using vector network analyzer. The measured results hold good with simulated one. The overall size of the antenna is 18.3 mm × 23 mm × 1.6 mm

The geometry of the CPW-fed elliptical antenna is shown in Figure 3.8

Fig.3.8 Schematic configuration of the compact elliptical UWB antenna

Conventionally, circular monopoles are used for the design of UWB antenna. Elliptical monopoles are preferred over the conventional circular monopoles due to its better antenna characteristics such as gain, impedance bandwidth, radiation pattern etc.. The FR4 substrate having dielectric constant 4.4 and thickness of 1.6 mm is used for the design of elliptical slot antenna. The feedline of CPW is having width of W = 4.34 mm and a gap g =0.4 mm for 50 Ω transmission line. An UWB elliptical slot of dimensions
a₁ and a₂ are connected to CPW ground plane. As shown in Figure 3.8, a₁ and a₂ are semi-major axes and b₁ and b₂ are semi-minor axes of outer and inner elliptical slot structure. Both the elliptical structures are concentric in nature. The two ground planes are etched on the same side of the elliptical slot antenna. The CPW ground plane is extended vertically toward the both side of elliptical slot antenna. The overall antenna size is 18.2 mm × 23.14 mm × 1.6 mm.

Fig3.9 Simulated return loss of the UWB elliptical antenna with nominal parameters

The simulated return loss shown ultra wide bandwidth from 3.1 GHz to 10.9 GHz with the three resonant frequencies at 3.8 GHz, 7.8 GHz and 9.6 GHz. Particularly, the simulated bandwidth of the UWB elliptical antenna covers the entire UWB spectrum which ranges from 3.1 GHZ to 10.6 GHz. The elliptical slot UWB antenna is surrounded by the extended metallic plane which helps in the reduction of the overall size of the antenna. The gap ‘p’ between the UWB antenna and the ground plane is a
major factor which gives rise to over strong capacitive coupling. The return loss of the
antenna without extended ground plane is shown in Figure 3.9 by dashed lines. It has
resonant frequencies at 5.8 GHz, 9.7 GHz, and 13.0 GHz. So, the introduction of
extended ground plane in the proposed UWB antenna helps in the size reduction.

The nominal parameters of UWB elliptical antenna are presented in Table 3.1

Table 3.1: Optimal Design Parameters of Elliptical Ring UWB antenna

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units (mm)</th>
</tr>
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<tbody>
<tr>
<td>W</td>
<td>4.34</td>
</tr>
<tr>
<td>S</td>
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</tr>
<tr>
<td>W₂</td>
<td>1.0</td>
</tr>
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