Introduction

The introduction of mathematical literature of $\Gamma$-algebraic system dates back to 1964. The notion of $\Gamma$ in algebraic structure was first introduced by N. Nobusawa [44]. In 1964, N. Nobusawa published a paper [44] entitled “On a generalisation of ring theory” which opened a new horizon in the research of $\Gamma$-algebraic structure. In this paper [44] Nobusawa introduced a new type of algebraic system which is known as $\Gamma$-ring. The class of $\Gamma$-rings contains not only all rings but also Hestenes ternary rings [29]. Many fundamental results of ring theory were extended to $\Gamma$-rings. There is a large literature dealing with $\Gamma$-rings, some of them are in [3], [4] [5] and [6].

Now we consider the following example:

Let $A$ be a non-empty set and $S$ be the set of all mappings from $A$ to $A$. Then $S$ is a semigroup with respect to the usual composition of mappings, which is known as full transformation semigroup on $A$. But this result does not happen if we consider $S$ to be the set of all mappings from a non-empty set $A$ to another non-empty set $B$. Now, if $\Gamma$ is the set of all mappings from $B$ to $A$ and $aab$, $aa\beta$ denote the usual product of mappings, where $a, b \in S$ and $\alpha, \beta \in \Gamma$ then $aab \in S$ and $aa\beta \in \Gamma$. Moreover, $(aab)\beta c = a(ab\beta)c = a\alpha(b\beta)c$ for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$.

Considering this as a natural example, in 1981, M. K. Sen first introduced the notion of $\Gamma$-semigroup as follows:

Let $S$ and $\Gamma$ be two nonempty sets. $S$ is called $\Gamma$-semigroup if there exist mappings $S \times \Gamma \times S \rightarrow S$, written as $(a, \alpha, b) \rightarrow aab$, and $\Gamma \times S \times \Gamma \rightarrow \Gamma$, written as $(\alpha, a, \beta) \rightarrow \alpha a\beta$, satisfying the identities $a\alpha(b\beta c) = a(ab\beta)c = (aab)\beta c$ for all $a, b, c \in S$ and $\alpha, \beta \in \Gamma$.

In 1986, M. K. Sen and N. K. Saha [58] weakened the defining conditions of $\Gamma$-semigroup and redefined $\Gamma$-semigroup as follows:
Let $S = \{a, b, c, \ldots\}$ and $T = \{a, \beta, \gamma, \ldots\}$ be two nonempty sets. $S$ is called a $\Gamma$-semigroup if

(i) $aab \in S$, for all $\alpha \in \Gamma$ and $a, b \in S$ and

(ii) $(aab)\beta c = a\alpha(b\beta c)$, for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$. The notion of this $\Gamma$-semigroup is usually known as one sided $\Gamma$-semigroup.

Let $S$ be an arbitrary semigroup. Let 1 be a symbol not representing any element of $S$. Let us extend the binary operation defined on $S$ to $S \cup \{1\}$ by defining $11 = 1$ and $1a = a1 = a$ for all $a \in S$. It can be shown that $S \cup \{1\}$ is a semigroup with identity element 1. Let $\Gamma = \{1\}$. If we take $ab = a1b$, it can be shown that the semigroup $S$ is a $\Gamma$-semigroup where $\Gamma = \{1\}$. Thus a semigroup can be considered to be a $\Gamma$-semigroup.

N. C. Adhikari [14] studied both sided $\Gamma$-semigroup defined by conditions (i) $a\alpha(b\beta c) = a(\alpha b)c = (a\alpha b)c$ and (ii) $a\alpha(b\gamma) = \alpha(a\beta b)\gamma = (a\alpha b)\beta c$ for all $a, b, c \in S$ and $\alpha, \beta, \gamma \in \Gamma$.

Let $S$ be the set of all integers of the form $6n+1$ and $\Gamma$ be the set of all integers of the form $6n + 5$ where $n$ is an integer. If $aab$ is $a + \alpha + b$ and $\alpha a\beta = a + a + \beta$ for all $a, b \in S$ and $\alpha \in \Gamma$ then $S$ is a both sided $\Gamma$-semigroup.

The following example shows that there exists a one sided $\Gamma$-semigroup which is not a both sided $\Gamma$-semigroup.

Let $S$ be a set of all negative rational numbers. Obviously $S$ is not a semigroup under usual product of rational numbers. Let $\Gamma = \{-\frac{1}{p} : p \text{ is prime} \}$. Let $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. Now if $aac$ is equal to the usual product of rational numbers $a, \alpha, b$, then $aac \in S$ and $(aac)\beta c = a\alpha(b\beta c)$. Hence $S$ is a one sided $\Gamma$-semigroup. It is also clear that it is not a both sided $\Gamma$-semigroup.

The theory of semigroup was enriched by many mathematicians. More information about semigroup can be found in [31] and [12]. We have noticed that every semi-
group can be considered as a $\Gamma$-semigroup and also there was a remarkable growth of $\Gamma$-semigroup theory for example [33], [38], [67]. However, there remains a lot more to be explored. In the present thesis, we study some more interesting properties of $\Gamma$-semigroups.

This thesis consists of six chapters. Chapter - 1 is essentially a preliminary survey of the basic definitions and known results concerning $\Gamma$-semigroup which are needed to develop this thesis.

In Chapter - 2, we introduce and study strongly prime and uniformly strongly prime $\Gamma$-semigroups. In the first section, we introduce Rees congruence on a $\Gamma$-semigroup $S$ and study some properties relating operator semigroup of $S$. In the second section, we study right strongly prime $\Gamma$-semigroups and right strongly prime ideals. We show that if $S$ is a prime $\Gamma$-semigroup with DCC on right annihilators, then $S$ is a right strongly prime $\Gamma$-semigroup. Moreover, we show that there is a one-one correspondence between the set of all right strongly prime ideals of $S$ and its operator semigroups. In the third section, we introduce uniformly strongly prime (usp) $\Gamma$-semigroups and uniformly strongly prime (usp) ideals of a $\Gamma$-semigroup. We also define essential extension of a $\Gamma$-semigroup and show that if $S$ is a uniformly strongly prime $\Gamma$-semigroup, then any essential extension of $S$ is a uniformly strongly prime $\Gamma$-semigroup. In the fourth section, we study usp radical of a $\Gamma$-semigroup and in the fifth section, we introduce the notion of the structure space of $\Gamma$-semigroups formed by the class of uniformly strongly prime ideals. We also study separation axioms and compactness property in this structure space.

In Chapter - 3, in the first section, we introduce right(resp.left) orthodox $\Gamma$-semigroup and try to generalize the properties of semigroup to the right(resp.left) orthodox $\Gamma$-semigroup. In the second section, we introduce right(resp.left) sandwich set of a $\Gamma$-semigroup which takes part an important role in $\Gamma$-semigroup. In the third section, we
introduce right inverse $\Gamma$-semigroups and study such type of $\Gamma$-semigroups. We also investi-
gate the maximum idempotent separating congruence on a right inverse $\Gamma$-semigroup.

In Chapter - 4, in the first section we study the notion of the semidirect product of
a semigroup and a $\Gamma$-semigroup. Necessary and sufficient conditions for this semidirect
product to be right(resp. left) orthodox $\Gamma$-semigroup and right(resp. left) inverse $\Gamma$-
semigroup are obtained. In section 2 we introduce the wreath product of a semigroup
and a $\Gamma$-semigroup with unities and study it.

In Chapter - 5, we generalize the property of a regular $\Gamma$-semigroup. A $\Gamma$-semigroup
$S$ is called an $E$- inversive $\Gamma$-semigroup if for each $a \in S$ there exist $x \in S$ and $\alpha \in \Gamma$
such that $aax$ is a $\beta$-idempotent for some $\beta \in \Gamma$. A $\Gamma$-semigroup is called a right $E$- $\Gamma$-
semigroup if for any $\alpha$- idempotent $e$ and for any $\beta$-idempotent $f$, $eaf$ is a $\beta$-idempotent.
In the first two sections we investigate some results relating $E$- inversive $\Gamma$-semigroups
and right $E$- $\Gamma$-semigroups respectively. In the third section we investigate $\Gamma$-group
congruences on an $E$-inversive $\Gamma$-semigroup and also give some equivalent expressions
for any $\Gamma$-group congruences on an $E$-inversive $\Gamma$-semigroup. We also give the least
$\Gamma$-group congruence on an $E$-inversive $\Gamma$-semigroup.

In Chapter - 6, for a given $\Gamma$-semigroup $S$, we define a hyperoperation $'o'$ on $S$ by
$a \circ b = \{aab : a \in \Gamma\}$ for $a, b \in S$. We have shown that $(S, \circ)$ is a semihypergroup. This
semihypergroup is called the semihypergroup associated with the $\Gamma$-semigroup $S$. In this
chapter we study different properties of semihypergroups associated with $\Gamma$-semigroups
and also we discuss some fundamental properties of semihypergroups. We also study
hyerideals, prime hyperideals, semiprime hyperideals in semihypergroups and show
that some results of $\Gamma$-semigroups published in some research papers can be generalized
to semihypergroups.