IMAGING CHARACTERISTICS OF A PERFECT LENS MASKED BY SECTOR-SHAPED AND SEMI-CIRCULAR POLARISATION MASKS

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It had been established earlier that all possible phase and amplitude masks may be simulated on the lens aperture by proper use of polarization devices. This paper attempts to study the effects of polarization masked sector apertures and traces the azimuthal variation of the PSF. The effect of polarization masking is to introduce polarization-induced phase and amplitude transmittances in the masked zones, with the additional advantage that they can be controlled independently. It is obvious that the asymmetric nature of this mask would serve only to degrade the image in a conventional diffraction-limited imaging system. This apparent disadvantage may be put to use in two ways: a) in image processing applications where information along certain azimuths need to be enhanced compared to others, and b) compensation of asymmetric aberrations. It is the former possibility that this paper attempts to study in terms of the azimuthal dependence of the PSF. Computational details and results are presented.

1. INTRODUCTION

The fact that circular symmetric phase and amplitude coating can modify the imaging qualities of a lens has often been used to advantage in the field of optical imagery [1–4]. These effects can be easily simulated by introducing appropriate polarization masks in the aperture plane of an optical imaging system. Application of this polarization induced phase (PIT) and polarization induced transmittance (PIP) to imagery has theoretically yielded a versatile imaging system where the imaging characteristics (PSF and OTF) can be controlled in situ [5]. It has been shown that all phase differences between $\pi$ and $-\pi$ may be introduced between two zones of the imaging aperture and that the PIP and the PIT may be decoupled and varied independently of each other.

Some authors have considered the effect of circularly symmetric polarization masks on the imaging characteristics of a lens. These systems naturally have circularly symmetric optical transfer functions. In this paper we attempt to study the imaging characteristic of linearly polarized sector apertures. The imaging aperture is broken down into sectors so that each sector transmits linearly polarized light, the polarization axis of which depends on the orientation of the masking polariser. Unlike circular symmetric polarization masked aperture sector aperture exhibit circularly asymmetric transfer functions. It is obvious that the asymmetric na-
ture of this mask would serve only to degrade the image in a conventional diffraction limited imaging system. However, the possible advantages derived from such systems are twofold. First, with suitable orientation of the masking polarisers, suppression of the asymmetric aberration terms is a feasible proposition. This is based on the fact that it is possible for the asymmetric PIP to compensate, at least partially, the off-axis aberrations. Secondly, the azimuthal dependence of the imaging characteristics allows spatial frequencies along certain azimuths of the image plane to be relatively enhanced or suppressed a possibility, which may be utilised in performing certain image processing applications. It is this second possibility that we attempt to study in this paper. If a circular aperture is formed by two semicircular apertures each having a different phase and/or transmittance, and an elliptically polarised light beam is used as the input beam this results in interesting natures of PSF along various azimuths.

A variation of the polarisation parameters involved results in wide variations of the PSF along a specific azimuth, thus showing the overall azimuthal dependence of the masked lens considered.

2.THEORY

Fig.1 and 2 show a typical sector-shaped aperture, the aperture configuration, and the co-ordinate systems. The aperture planes $A$ and the plane of observation $F$ in the far field of the aperture are mutually parallel. The origins of the co-ordinated systems are on the line $OO'$. The polar co-ordinates $r$, $\theta$, and $r'$, $\theta'$ are used on the planes $A$ and $F$, respectively.

![Fig.1 Aperture configuration](image-url)
If we assume that a plane wave is incident on the aperture, the amplitude distribution will be given on the plane F, by the integral.

\[ A(r, \theta) = C_0 \int_0^{2\pi} \int_0^R |t(r, \theta)\exp\left[i \frac{2\pi}{\lambda f} r' \cos(\theta - \theta')\right]| \, r \, dr \, d\theta \]  

where \( C_0 \) is a constant, \( t(r, \theta) \) is the amplitude transmittance of the aperture, \( R \) is the radius of the aperture, and \( f \) is the far-field distance. Let \( \phi \) be the angle covered by each clear sector and \( \varphi \) the angle between the positive direction of the x axis and the bisector of the angle of the sector that is nearest to the x axis in the clockwise direction (fig. 1). Since the aperture is made up of transparent and opaque sectors, the amplitude transmittance of the aperture may be written as,

\[ t(r, \theta) = \begin{cases} 1 & \text{for } \frac{2\pi}{N} (p-1) + \phi - \frac{\phi}{2} \leq \theta \leq \frac{2\pi}{N} (p-1) + \varphi + \frac{\phi}{2} \\ 0, & \text{elsewhere} \end{cases} \]

where \( N \) is the number of transparent sectors and \( p=1,2, \ldots, N \).

Hence, Eq. (1) takes the form

\[ A(u, \theta') = C_0 R^2 \sum_{p=1}^{N} \int_{\frac{\theta - \phi}{2}}^{\frac{\theta + \phi}{2}} \exp\left(iup \cos \omega\right) dp \, d\theta \]  

where \( \omega = \theta - \theta', \theta_p = \left(\frac{2\pi}{N}\right)(p-1) + \varphi, \rho = \left(\frac{r}{R}\right), u = \left(\frac{2\pi}{\lambda f}\right)Rr' \)

Integrations over \( \theta \) and \( \rho \) can be easily performed by the use of the method of Mahan. [6-10] We then have

\[ A(u, \theta') = C_0 R^2 \sum_{p=1}^{N} \left[C(u, \theta', \theta_p) - iS(u, \theta', \theta_p)\right] \]
where

\begin{align*}
C(u, \phi; \theta_p) &= \phi \frac{J_1(u)}{u} + \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)} \frac{J_{2n+1}(u)}{u} \times \sum_{m=1}^{n} \sin 2m \left( \frac{\theta_p + \theta - \phi}{2} \right) - \sin 2m \left( \frac{\theta_p - \theta - \phi}{2} \right) \\
S(u, \phi; \theta_p) &= 8 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{4n^2 - 1} \frac{J_{2n}(u)}{u} \times \sum_{m=1}^{n} \sin (2m - 1) \left( \theta_p + \theta' \right) - \sin (2m - 1) \left( \theta_p - \theta' \right) \\
\phi &= \frac{\pi}{2} \\
\phi &= \pi
\end{align*}

In case of a semicircular aperture,

\begin{align*}
C(u, \phi; \theta_p) &= \pi \frac{J_1(u)}{u} \\
S(u, \phi; \theta_p) &= 8 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{4n^2 - 1} \frac{J_{2n}(u)}{u} \times \sum_{m=1}^{n} 2 \sin (2m - 1) \theta'
\end{align*}

It may be observed that for a diametrically opposite sector,

\begin{align*}
A(u, \phi) &= C(u, \phi) + iS(u, \phi)
\end{align*}

If the upper half is masked by a phase mask having uniform phase and transmittance \( \Delta_1 \) and \( T_1 \) respectively, the complex amplitude distribution in the image of a point is now given by,

\begin{align*}
A_1(u, \phi) &= A_1(u, \phi) T_1 e^{i\Delta_1} \\
&= T_1 (C - iS) \cos (\gamma - \alpha_1) e^{i\Delta_1}
\end{align*}

where \( T_1 = T_1 \cos (\gamma - \alpha_1) \) is the modified PIT due to the introduction of an analyser \( P(\gamma) \) at the output side.

Similarly if the lower half is masked by a semi-transparent phase mask having uniform phase and transmittance \( \Delta_2 \) and \( T_2 \) respectively, then the complex amplitude distribution in the image of a point is now given by,

\begin{align*}
A_2(u, \phi) &= A_2(u, \phi) T_2 e^{i\Delta_2} \\
&= T_2 (C + iS) \cos (\gamma - \alpha_2) e^{i\Delta_2}
\end{align*}

where \( T_2 = T_2 \cos (\gamma - \alpha_2) \) is the modified PIT due to the introduction of an analyser \( P(\gamma) \) at the output side.

Now when the imaging beam is elliptically polarised and is represented by the Jones vector,

\begin{align*}
E_i = \begin{vmatrix} a \\ b \cos \delta \end{vmatrix}
\end{align*}
where \( a \) and \( b \) are amplitudes of the \( x \) and \( y \) components of the beam respectively and \( \delta \) is the phase difference between these two components. If the sector is masked by a polarizer \( P (\alpha) \) then the PIP and PIT introduced are respectively by,

\[
\Delta = \tan^{-1} \frac{b \sin \alpha \sin \delta}{a \cos \alpha + b \sin \alpha \cos \delta}
\]

\[
T = \left[ a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + a b \sin \alpha \cos \delta \right]^{1/2}
\]

If the polarisation induced phases in the upper and lower halves of the aperture are \( \Delta_1 \) and \( \Delta_2 \) respectively, then the PSF will be given by,

\[
\text{PSF} = \left| T_1 (C - iS) \cos (\gamma - \alpha_1) e^{i\Delta_1} + T_2 (C + iS) \cos (\gamma - \alpha_2) e^{i\Delta_2} \right|^2
\]

where,

\[
T_1 = \left[ a^2 \cos^2 \alpha_1 + b^2 \sin^2 \alpha_1 + a b \sin 2\alpha \cos \gamma \right]^{1/2}
\]

\[
T_2 = \left[ a^2 \cos^2 \alpha_2 + b^2 \sin^2 \alpha_2 + a b \sin 2\alpha_2 \cos \gamma \right]^{1/2}
\]

\[
\Delta_1 = \tan^{-1} \frac{b \sin \alpha_1 \sin \delta}{a \cos \alpha_1 + b \sin \alpha_1 \cos \delta}
\]

\[
\Delta_2 = \tan^{-1} \frac{b \sin \alpha_2 \sin \delta}{a \cos \alpha_2 + b \sin \alpha_2 \cos \delta}
\]

\[
\text{PSF} = C^2 \left( T_1^2 + T_2^2 + 2T_1 T_2 \cos (\Delta_1 - \Delta_2) \right) +
\]

\[
s^2 \left( T_1^2 + T_2^2 - 2T_1 T_2 \cos (\Delta_1 - \Delta_2) \right) +
\]

\[
4CST_1 T_2 \sin (\Delta_1 - \Delta_2)
\]

From the above expression it is clear that the introduced phases and transmittances are dependent

1) On the nature of the input beam
2) On the orientation of the masking polarises and
3) On the analyser orientation

3.COMPUTATIONAL RESULTS AND DISCUSSIONS

Since it is the relative phase or amplitude between two or more zones of an imaging aperture that matters, the imaging characteristics of a semicircular aperture would show no change with the introduction of uniform phase and / or transmittance over it. However, for the masking geometry of the aperture as shown in Fig.3. imagery with elliptically polarised light results in a phase differ-
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Fig. 3 Two halves of a semicircular aperture masked by two polarisers P(α₁) and P(α₂).

The transmittance \( T_1 \) and relative transmittance \( \frac{T_1}{T_2} \) between two halves of the circular aperture are introduced. Finally, an analyser at the output allows the contribution from the two halves to be coherently superposed. We shall limit the computational results to the relatively simple case of \( a/b=1, \lambda = 1, \alpha_1 = 0^\circ, \alpha_2 = 90^\circ \), since for these parameters the PIP difference simplifies to \( \Delta_1 - \Delta_2 = \delta \). The analyser \( P(\gamma) \) is maintained at \( \gamma = +45^\circ \) and \( -45^\circ \). The PSF is computed at \( 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ \) and \( 150^\circ \); it may be noted that intensity spread will be symmetrical about Y-axis.

Figure 4 and 5 depict the PSF curves and their azimuth dependence for \( \delta = 90^\circ \), when \( a=b=1 \) and \( \gamma \) is maintained at \( +45^\circ \) and \( -45^\circ \).

It needs to be mentioned that for \( \gamma = 0^\circ \), only the upper half performs imaging while at \( \gamma = 90^\circ \), it is the lower half that is effective. For \( \gamma = 45^\circ \), the contribution

Fig. 4 Azimuthal variation of PSF for the aperture shown in Fig. 3 for \( \alpha_1 = 0^\circ, \alpha_2 = 90^\circ, \gamma = 45^\circ, a/b=1 \) and \( \delta = 90^\circ \).
from the two halves are in phase resulting in the normal PSF of a circular aperture. For $\gamma = -45^\circ$ the contributions from both the halves are out of phase resulting in zero intensity along $\theta = 0^\circ$ when $a=b=1$. This has been shown in Fig.(6) and (7). This wide variation of the PSF along a particular azimuth just by rotating the analyser is noteworthy.

**Fig.5** Azimuthal variation of the PSF when $\gamma=-45^\circ$, other parameters remaining the same as in Fig.4.

**Fig.6** Azimuthal variation of PSF when $\alpha_1=0^\circ$, $\alpha_2=90^\circ$, $\delta = 0^\circ$, $\gamma=45^\circ$, $a/b=1$. This results in normal PSF of a circular aperture.
At the centre of the image plane the wave disturbance from the two halves arrive at increasing phase difference as $\theta$ is increased. This is reflected in the PSF curves which shows increasing dips at $z=0$. Since the intensity spread is symmetrical about Y-axis it results in identical PSFs for the values $\theta = 30^\circ$, $150^\circ$, and $\theta = 60^\circ$, $120^\circ$.

Fig. 8 and 9 shows the PSF curves and their azimuth dependence for $\delta = 90^\circ$ when $a=1$, $b=2$ and $\gamma$ is maintained at $+45^\circ$ and $-45^\circ$. 
From the different curves we have observed the strong azimuthal dependence of PSF. This phenomenon may prove to be useful in image processing applications where information only along certain azimuths of the object needs to be extracted.

REFERENCE

FOCAL SHIFT IN AN IMAGING SYSTEM WITH POLARIZATION-PHASE MODULATED APERTURE PLANE

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This study utilizes the concept of polarization phase to introduce a circularly symmetric phase step in the aperture plane of an imaging system. Since the polarization phase can be controlled by variation of the polarization parameters involved, the effective focal length of the imaging system can be varied within a small range. In the proposed technique, the modification of the focal length is obtained by rotating a retarder at the input of the imaging system. A study of the PSFs at the shifted focal planes shows that the quality of the image in these planes is comparable to that produced by a diffraction-limited system at Gaussian focus. The focal plane displacement achievable is about 300 μm of a f/10 system at 550 nm.

Keyword: Polarization-sensitive devices, polarization phase, polarization masking, focal shift, apertures.

1. INTRODUCTION

A diffraction-limited imaging system may be looked upon as an optical device that introduces a positive or a negative quadratic phase factor on the incident beam of light. A plane parallel wavefront incident on the entrance pupil of a positive lens is therefore rendered spherical and convergent and is focused at an axial point known as the focus. For fixed focus imaging system, the focal length is invariant for a particular wavelength and propagation media (normally air) and is determined by the magnitude of the spherical phase factor introduced by the imaging lens. In zoom lenses, large variation of focal length is brought about by imparting movements between the optical elements of the imaging system. A different and somewhat novel approach towards focal shift lies in introducing a spatially varying phase on the aperture plane which in effect modifies the sphericity of the emerging wavefront. The present study utilizes polarization phase modulated aperture plane to this end and the magnitude of the polarization phase can be varied substantially simply by varying the polarization state of the input beam, resulting in a small shift of the focal plane.

The use of semi-transparent phase masks on the aperture plane has been extensively explored and it has been shown that the imaging characteristic of the optical system can be modified by using these type of masks. The works done by Asakura and Castaneda need special mention in this regard. Lit showed that a suitable use of phase coating might markedly improve the resolution of the lens.Usu-
ally the imaging characteristic of the optical system due to the presence of these type of masks is pre-specified and any on-line modification is not possible.

The possibility of using polarization masks in the aperture plane has already been explored\(^1\). In such an imaging system, the aperture plane is partially masked by two different polarizers having different orientations of their principal transmission axes. Assuming that the input imaging beam is elliptically polarized, the beams emerging from the two polarizers differ in phase. Theoretical studies reveal that ingenious application of this polarization induced phase and polarization induced amplitude attenuation to imagery has the potential of production a versatile imaging system where the imaging characteristics (PSF and OTF) can be controlled in situ. It has been shown that all phase differences between \(\pi\) and \(-\pi\) may be introduced between two zones of the imaging aperture and that the polarization induced phase and polarization induced attenuation may be decoupled and varied independently of each other by suitable use of the polarization devices used in the set-up. It has also been shown that by suitable choice of the polarization parameters, the polarization phase introduced can compensate for spherical aberration of an imaging system\(^2\). The versatility of this technique for modifying the imaging characteristics of the system stems from the fact that the polarization-induced phase difference between the light emerging from the masking polarizers can be continuously varied either by changing the state of polarization of the incident beam or by reorienting the polarizers.

When the polarization mask is circularly symmetric, the polarization phase difference introduced between the polarization-masked zones of the aperture will also have circular symmetry. This polarization-induced phase in effect modifies the focal length of the imaging system, i.e. the best focus plane shifts from the Gaussian image plane. In the present paper, we will propose a polarization based imaging system where the polarization phase and hence the focal length can be controlled just by rotating a retarder placed at the input of the focusing system. It will be shown that, the quality of the image formed by the masked aperture at the shifted focus does not vary considerably from that formed at the Gaussian focus of an unmasked lens. This suggests a possibility of changing the focal length of an imaging system simply by rotating a retarder placed at the input. The computational results bear out this theoretical expectation.

2. PROPOSED SYSTEM

The input to the imaging system shown in Fig.1 is a polarizer-quarter waveplate combination, which renders the focusing beam elliptically polarized in general. The aperture of the focusing system consists of two polarization-masked zones: a central circular zone and outer annular zone masked with linear polarizers \(P(\alpha)\) and \(P(\beta)\) respectively, where \(\alpha\) and \(\beta\) are the orientation of the transmission axes of the polarizers with respect to the reference abscissa. The is followed by an analyzer \(P(\gamma)\) before the focal plane.
3. THEORY

With reference to the imaging system considered in Fig. 1, unpolarized light passes through a polarizer-compensator combination and is in general rendered elliptically polarized, the ellipticity and orientation of the vibration ellipse being determined by the relative orientation of the polarizer and compensator. So the polarization state of the input beam can be altered just by changing the orientation of either the polarizer or the compensator. Elliptically polarized light intercepts the two zones of the polarization mask and emerges, in general, with a phase and amplitude difference between them, as is shown in what follows. In presence of an analyzer before the image plane, light beam from these two zones are brought to the same polarization state and are made to interfere, yielding a redistribution of energy in the diffraction pattern. Since the polarization phase and amplitude are dependent on the state of polarization of the input beam, which in turn depend on the orientation of the retarder, the system PSF may be modified in-situ by reorienting the retarder placed at the input.

The general expression for the complex amplitude distribution at a point \( P(x,y,z) \) near the geometrical focus of a diffraction limited lens system of radius \( R \) and of focal length \( f \) is given by:

\[
U[P(x,y,z)]_0 = -\frac{2\pi R^2 A}{\lambda f^2} \exp \left( i \left( \frac{x}{R} \right)^2 \right) \int_0^1 \exp \left( iur/2 \right) J_0(\nu r) r dr
\]

\[
= KR^2 \int_0^1 \exp \left( iur^2/2 \right) J_0(\nu r) r dr
\]

(1)

where
\[ K = \frac{-2\pi A}{\lambda f} r \exp \left( i \left( \frac{f}{R} \right)^2 u \right) \]

\( r \) is the radial co-ordinate of the pupil plane, \( A \) is the amplitude of the incident light, \( v \) and \( u \) are the reduced radial and axial co-ordinates respectively at the plane of observation, and are given as follows,

\[ u = \frac{2\pi}{\lambda} \left( \frac{R}{f} \right)^2 z \quad (2) \]

\[ v = \frac{2\pi}{\lambda} \left( \frac{R}{f} \right) \left( x^2 + y^2 \right)^{\frac{1}{2}} \quad (3) \]

Defect of focus \( w_{20} \) is defined as,

\[ w_{20} = \frac{u}{2k} \quad (4) \]

where, \( k \) is the propagation constant. Eq.(2) is then modified to the following form:

\[ z = 8w_{20}(f_{\infty})^2 \quad (5) \]

and the amplitude point spread function is then written as,

\[ U[P(x,y,z)]_0 = KR^2 \int_0^r \exp(ikw_{20}r^2)J_0(\nu r)rdr \quad (6) \]

where, \( w_{20} \) is the effective defocus coefficient.

Similarly for an aperture of radius \( R' \) where \( R' = \in R \), the amplitude distribution is given by,

\[ G_4(P) = KR^2 \int_0^{R'} \exp(ikw_{20}r^2)J_0(\nu r)rdr \quad (7) \]

According to Babinet's principle, we may conclude that the amplitude distribution \( G_4(P) \) for the outer annular zone extending from \( r = \in \) to \( r = 1 \) is proportional to \( [U(P) - G_4(P)] \) or

\[ G_4(P) = \left[ \int_0^r \exp(ikw_{20}r^2)J_0(\nu r)rdr - \int_0^{R'} \exp(ikw_{20}r^2)J_0(\nu r)rdr \right] \quad (8) \]

It is to be noted that we have ignored that factor \( KR^2 \) which occurs in both \( U(P) \) and \( G_4(P) \).

Light from an incoherently illuminated or self-luminous object entering the masked lens is rendered elliptically polarized by the polarizer-retarder combination. To arrive
Fig. 2. Focal shift in μm is plotted against fast axis azimuth $\rho$ of the retarder. $\alpha=0^\circ$, $\beta=90^\circ$, $\gamma=45^\circ$ and $\theta=45^\circ$.

Fig. 3. Intensity PSF of a masked lens with reference to an ideal lens $\rho = 49.7^\circ$ and other parameters as in Fig. 2.
Fig. 4. Intensity PSF of a masked lens with reference to an ideal lens $\rho = 89.7^\circ$ and other parameters as in Fig. 2.

Fig. 5. Intensity PSF of a masked lens with reference to an ideal lens $\rho = 140^\circ$ and other parameters as in Fig. 2.
at an expression for the phase difference between the two light beams that finally emerge from the masked lens, we first determine the mature of the polarization ellipse along the reference coordinate system. It is then possible to reduce the elliptically polarized vibration to a Jones vector $J(\varepsilon)$ of the form given by,

$$
\varepsilon = \begin{bmatrix} a_x \\ a_y \end{bmatrix}
$$

(9)

where $a_x$ and $a_y$ are the amplitudes along the reference abscissa and ordinate, respectively, and $\Delta$ is the phase difference between these two components.

In the context of Jones calculus, the Jones vector $J(\varepsilon)$ is essentially the product

$$
J(\varepsilon) = W(\rho, \delta) J(\theta)
$$

(10)

where, $J(\theta)$ is the Jones vector for linearly polarized light with the direction of polarization along $\theta$ and is given by,

$$
J(\theta) = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}
$$

(11)

and $W(\rho, \delta)$ is the Jones matrix of a homogeneous linear retarder with fast-axis azimuth $\rho$ and retardance $\delta$ and is given by,

$$
w(\rho, \delta) = \begin{bmatrix}
\cos^2 \rho \exp(i\delta/2) + \sin^2 \rho \exp(-i\delta/2) & 2i \cos \rho \sin \delta \sin(\delta/2) \\
2i \cos \rho \sin \delta \sin(\delta/2) & \cos^2 \rho \exp(-i\delta/2) + \sin^2 \rho \exp(i\delta/2)
\end{bmatrix}
$$

(12)
Using these expressions for \( P(\theta) \) and \( W(\rho, \delta) \) Eq. (10) can be simplified into Eq. (9), where,

\[
a = \left[ (\cos \rho (1 - \cos \delta) \cos (\rho - \theta) + \cos \theta \cos \delta)^2 + (\sin \delta \sin \rho \sin (\theta - \rho))^2 \right]^{1/2}
\]

(13)

\[
b = \left[ (\sin \rho (1 - \cos \delta) \cos (\rho - \theta) + \sin \theta \cos \delta)^2 + (\sin \delta \cos \rho \sin (\rho - \theta))^2 \right]^{1/2}
\]

(14)

\[
\Delta = \tan^{-1} \frac{\sin \delta \cos \rho \sin (\rho - \theta)}{\sin \rho (1 - \cos \delta) \cos (\rho - \theta) + \sin \theta \cos \delta}
\]

(15)

The parameters \( a, b \) and \( \Delta \) determine the unique state of polarization produced by the polarizer-waveplate combination. Obviously, with a change of any of these parameters, which again are dependent on \( \theta, \delta, \) or \( \rho \) the state of polarization changes.

Referring to Eq. (9), the complex vector amplitude distribution due to the central circular region at the observation plane is given by,

\[
U_c = G_c P(\theta) |^{1/2} (16)
\]

and that due to the outer annular zone is

\[
U_o = G_o P(\theta) |^{1/2} (17)
\]

In the presence of an analyzer \( P(\gamma) \) before the image plane, the contribution from the two polarization masked zones are,

\[
U_c = P(\gamma) U_c (18)
\]

and

\[
U_o = P(\gamma) U_o (19)
\]

To find the polarization phase and amplitude it is convenient to express the above equation as,

\[
U_c = G_c R_1 \exp(i \Delta_1) |^{1/2} (21)
\]

where,

\[
R_1 = \cos(\gamma - \alpha) \left[ a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \delta \right]^{1/2}
\]

(22)

\[
R_2 = \cos(\gamma - \beta) \left[ a^2 \cos^2 \beta + b^2 \sin^2 \beta + ab \sin 2\beta \cos \delta \right]^{1/2}
\]

(23)

\[
\Delta_1 = \tan^{-1} \left( \frac{b \sin \alpha \sin \delta}{a \cos \alpha + b \sin \alpha \cos \delta} \right)
\]

(24)
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\[ \Delta_2 = \tan^{-1}\left(\frac{b \sin \beta \sin \delta}{a \cos \beta + b \sin \beta \cos \delta}\right) \]  

The phase difference \( \Delta' = \Delta_1 - \Delta_2 \) introduced between the two masked zones is thus given by,

\[ \Delta' = \tan^{-1}\left(\frac{ab \sin \delta \sin(\alpha - \beta)}{(a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta + ab \sin(\alpha + \beta) \cos \delta)}\right) \]  

From Eqa. (20) and (21), the IPSF may be written as,

\[ U = |U_{z1} + U_{z2}|^2 \]

or more explicitly,

\[ U = \left[ C_1 R_1^2 \cos^2(\alpha - \gamma) + R_2 C_2^2 \cos^2(\beta - \gamma) + 2C_1 C_2 R_1 R_2 \cos(\alpha - \gamma) \cos(\beta - \gamma) \cos(\theta_1 - \theta_2 + \Delta) \right] \]

where

\[ C_1 = \left( U_{z1}^2 + U_{z2}^2 \right)^{\frac{1}{2}} \]

\[ C_2 = \left( U_{z1}^2 + U_{z2}^2 \right)^{\frac{1}{2}} \]

\[ C_3 = \left[ (C_1^2 + C_2^2 - 2C_1 C_2 \cos(\theta_1 - \theta_2)) \right]^{\frac{1}{2}} \]

and in general,

\[ U_{z1} = \int_0^\infty \cos k(w_{z1} r^2) J_0(\nu r) r dr \]

Also,

\[ \theta_1 = \tan^{-1}\left( \frac{U_{z2}}{U_{z1}} \right) \]

\[ \theta_2 = \tan^{-1}\left( \frac{U_{z1}}{U_{z1}} \right) \]

\[ \theta_3 = \tan^{-1}\left( \frac{C_2 \sin \theta_1 - C_1 \sin \theta_2}{C_1 \cos \theta_1 - C_2 \cos \theta_2} \right) \]

Eq. (28) is a general expression for the intensity distribution at any point \( P(z, y, z) \). The relation between \( x, y, z \) and \( r, \nu, w_{z1} \) is given by Eqs. (2)-(5).

In the proposed technique, a quarter-waveplate placed at the input of the focusing system is rotated. As a result, the input beam parameters \( a, b \) and \( \Delta \) assume different values for different orientations of the fast axis of the quarter-waveplate.
with the reference abscissa. Essentially it means that the polarization phase may be continuously varied by rotating the retarder since the polarization phase depends upon the state of polarization of the input beam. The exact position (\(w_{20}\)) of the shifted focus with respect to the Gaussian focus can be located, i.e. the irradiance maxima from the Gaussian focus, by putting \(v=0\) in Eq. (28) and the amount of focal shift is calculated from Eq. (5). If the nature of the PSF at this shifted focal plane reasonably agree with that of an unmasked diffraction-limited imaging system, it is logical to conclude that shift of focus has indeed been achieved.

4. COMPUTATIONAL RESULTS

It is apparent from the earlier analysis that the focal shift is a function of the input beam parameters \(a, b\) and \(\Delta\) which in turn depend on the orientation of the retarder. Fig.2 shows the focal shift in micrometer with the rotation of the retarder. The plots in Fig.2 are for values of \(\rho\) from 0° to 360° at 5° intervals. Since for \(\theta = 45°\), there will be a periodic variation of the polarization-induced phase from 0 to 2\(\pi\) with the rotation of the retarder\(^9\), the shift of focus is also found to be periodic with \(\rho\). With increasing values of \(\rho\) from 0° to 360° the existence of the polarization induced circular symmetric phase is evident from the periodic nature of the focal shift. The PSFs at the shifted image points is shown in Fig.3, Fig.4, Fig.5 and Fig.6. The image points are chosen arbitrarily from different portions of the curve of Fig.2. The PSF of a diffraction limited lens aperture at the Gaussian focus plane is shown for comparison. In each of the cases the PSFs at the shifted axial point seem to agree well with the PSF of an ideal lens aperture. This suggests that full cycle of change of focus seem to correspond to a rotation of retarder from 0° to 180°. With a f/10 pencil for example (\(f/R=20\)) and with light of wavelength \(\lambda=550\) nm, the maximum focal shift obtained in the present configuration is about 150 \(\mu\)m on either side of the Gaussian focus, i.e., a focal shift of 300 \(\mu\)m is achievable.

5. DISCUSSION AND CONCLUSION

A technique for effecting small variations of focal length of a fixed focus imaging system, utilizing the concept of polarization phase, is presented. The aperture plane is modified by a circularly symmetric polarization mask. This introduces a variable phase step between the two zones of the mask resulting in a small focal shift. The variation in focal length can be achieved by rotating a retarder at the input of the imaging system, without any physical translation of the system elements. It appears that the outlined principles may be utilized in automatic focussing applications similar to optical read/write drives where it is necessary to dynamically track small displacements of the focal plane. Although an all-optical system is discussed, it is nevertheless possible to use electro-optic liquid crystal phase shifters to achieve the same effect with the added advantage that the control is fully electronic and incorporates no movable or rotatable elements.
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Performance of a polarization-masked lens aperture in the presence of spherical aberration

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Abstract

This paper suggests the possibility of partial compensation of spherical aberration with a polarization-masked lens aperture. The principle is based on the fact that any pre-specified phase step can be introduced between two zones of a lens aperture masked by suitably oriented linear polarizers and employing an elliptically polarized imaging beam. By comparison of the point-spread function of such a polarization-masked imaging system with that of an ideal lens, it has been shown that the effect of spherical aberration can be appreciably compensated by making an appropriate choice of the polarization parameters involved.

Keywords: Polarization phase, polarization masking, aberration compensation, spherical aberration

1. Introduction

Modification of the imaging characteristics of a diffraction-limited imaging system is often carried out by the use of amplitude and/or phase masks on the aperture plane. Although a substantial amount of scientific literature is dedicated to this technique, we refer the reader in particular to Tsujiuchi [1] and Jacquinet and Roizen-Dossier [2] for comprehensive reviews of this field. Practical imaging systems, however, are not ideal diffraction-limited systems, and we will consider the effect of aberration and a possible means for its compensation.

In a diffraction-limited imaging system, the point-spread function (PSF) is observed to be a scaled version of the Fraunhofer diffraction pattern of the exit pupil. The image produced by such a system is thus given by the convolution of the object intensity distribution with the PSF mentioned above. The presence of aberration, which further degrades the image, may therefore be regarded as due to a phase mask superposed on the aperture of the diffraction-limited imaging system. As a logical consequence, one of the approaches used to minimize the effect of aberration is to introduce a compensating phase mask in the aperture plane. For complete compensation of aberration, the phase distribution of the mask should be the complex conjugate of that introduced by the aberration terms under consideration. Such a mask is a phase mask whose phase characteristic is the inverse of that of the wave aberration, as seen in the correcting plate of a Schmidt camera. Since such a phase profile is difficult to achieve except for some terminal cases, partial compensation of aberration and modification of the imaging characteristics is often achieved by the use of appropriate semi-transparent phase steps. While the phase introduced by the mask attempts to modify the aberrant phase, the amplitude controls the relative contribution of the diffracted components arriving on the image plane. With a suitable combination of these two parameters, the resultant effect leads to an improvement of the system PSF and hence partial compensation of the wave aberrations.

The possibility of using polarization masks in the aperture plane has already been explored [3-9]. In such an imaging system the aperture plane is partially masked by two different polarizers having different orientations of their principal transmission axes. Assuming that the input imaging beam is elliptically polarized, the beams emerging from the two polarizers differ in phase. The intensity point-
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Figure 1. A schematic diagram of the proposed optical set-up. P: polarizer; L: imaging lens; C: compensator; F: fast axis of the compensator; S: slow axis of the compensator; θ: orientation of the transmission axis of P with respect to the reference abscissa; φ: orientation of the fast axis of the compensator C with respect to the abscissa; A: analyser; M: polarization mask; S: screen.

spread function (IPSF) of such a masked lens depends on the state of polarization of the input beam, the geometry and relative apportioning of the polarization-masked aperture, the relative orientations of the linear polarizers constituting the masks, and the position of the analyser placed at the output of the imaging system. Thus we have a number of degrees of freedom that can be utilized for modifying the imaging qualities of the polarization-masked imaging system with the added advantage that the imaging behaviour of the system can be modified in situ. It may be mentioned in this connection that the use of polarization properties of light for introducing a phase is not new. In 1969, Marathay [10] studied the possibility of producing complex spatial frequency filters using the properties of so-called vectograph films. The technique of subtraction of two images using a polarization-shifted periodic carrier suggested and successfully demonstrated by Dashiell and Lohmann [11] is essentially based on the possibility of introducing a phase difference by utilizing the polarization properties of light. Although the polarization phase and amplitude difference between the different zones of the polarization mask are in general coupled, it will be shown that the two terms can be varied independently for some specific orientations of the polarizers constituting the mask and the output analyser. The present paper explains the possibility of using polarization-masked lens apertures for partial compensation of spherical aberration. The technique described utilizes a polarized beam for aberration compensation and, in view of this, it is essential that the object does not modify the imaging beam’s polarization characteristics. The major advantage offered by this technique is its real-time compensation and optimization in degraded imaging systems. It may also prove to be useful in specific image processing applications, as is evident from the nature of the IPSFs achievable.

In what follows, an expression for the IPSF for the proposed system is derived and the quality of the image formed by the system is evaluated with the help of its IPSF. The Strehl ratio for the masked lens is plotted and the IPSF is evaluated at selected axial distances.

2. Proposed system

As shown in figure 1, the input to the imaging system under consideration is a polarizer-quarter-waveplate combination (PC), which renders the imaging beam elliptically polarized in general. F and S represent the fast and slow axes of the compensator. The orientation of the transmission axis of the polarizer P and that of the fast axis of the compensator C with respect to the reference abscissa are θ and φ. They determine the nature of the vibration ellipse of the elliptically polarized light incident on the mask M. The polarization-masked lens aperture consists of two polarization-masked zones: a central circular zone and an outer annular zone masked with linear polarizers with Jones matrices \( P(\alpha) \) and \( P(\beta) \), respectively, where \( \alpha \) and \( \beta \) are the orientations of the transmission axes of the polarizers with respect to the reference abscissa. This is followed by an analyser \( P(\gamma) \) before the image plane S.

3. Theory

In what follows we proceed to find the expression for the IPSF of the system described above. The following analysis is based on the simplifying assumption that the object beam can be described in terms of a two-element Jones vector given by

\[
\varepsilon_i = \begin{bmatrix} a \\ b e^{i\theta} \end{bmatrix}.
\]

The analysis using Jones calculus assumes that the polarization state description for plane waves is applicable to spherical waves as well. This vector approximation neglects the minor non-uniformities of the polarization states over a spherical wavefront—sometimes referred to as ‘polarization aberration’ [12–14]. Although this approximation will not radically influence the final observations, methods for incorporating these effects in the analysis, using specific system operators, may be utilized, if and when required [15,16].

With reference to figure 1, the two polarizers on the two zones of the mask M select components of the object beam along their respective transmission axes. These two components will in general differ in phase and amplitude. This effectively implies that the polarization mask introduces a phase step and an attenuation between the two portions of the wavefront. Without the analyser behind the lens, the image of a point source is a superposition of the PSFs due to the two linearly polarized masking zones with Jones matrices \( P(\alpha) \) and \( P(\beta) \). When \( \alpha \) and \( \beta \) are mutually orthogonal, the PSFs are fully incoherent. In the presence of the analyser, light beams from these two zones are brought to the same polarization state and interfere, yielding a redistribution of energy in the diffraction pattern. Hence modification of the
PSF is achievable. Furthermore, since the polarization phase and amplitude are dependent on the polarization parameters of the system, the system PSF may be modified in situ by reorienting the polarization components.

Following [17], for a point source, the general expression for the complex amplitude distribution at a point \( P(x, y, z) \) near the geometrical focus of a diffraction-limited lens system of radius \( R \) and of focal length \( f \) is given by

\[
U[P(x, y, z)]_0 = \frac{2\pi i R^2 \lambda}{f^2} \exp \left( \frac{2\pi i z}{\lambda} \right) \\
\times \int_0^1 \exp \left( i w_{390}^2 / 2 \right) J_0(\rho r) r \, dr \\
+ K R^2 \int_0^1 \exp \left( i w_{390}^2 / 2 \right) J_0(\rho r) r \, dr
\]

where

\[
K = \frac{2\pi i A}{R^2 f} e^{\frac{2\pi i}{\lambda}}.
\]

\( r \) is the radial coordinate of the pupil plane, \( A \) is the amplitude of the incident light, \( \rho \) and \( w_{390} \) (also known as the defocus coefficient) are the reduced radial and axial coordinate at the plane of observation and are given as follows:

\[
w_{390} = \frac{2\pi i}{\lambda} \left( \frac{R^2}{f} \right) z,
\]

\[
\rho = \frac{2\pi i}{\lambda} \left( \frac{R}{f} \right) r = \frac{2\pi i}{\lambda} \left( \frac{R}{f} \right) \sqrt{r^2 + y^2}.
\]

If defocusing and spherical aberration are introduced in an otherwise perfect optical system, equation (2) is modified to

\[
U[P(x, y, z)]_0 = K R^2 \int_0^1 \exp \left( i k(w_{390}^2 + w_{40}^4) \right) J_0(\rho r) r \, dr
\]

where \( w_{40} \) is the spherical aberration coefficient.

Similarly for an aperture of radius \( R' \) where \( R' = \epsilon R \), the amplitude distribution is given by

\[
G_\epsilon(P) = K R^2 \int_0^r \exp \left( i k(w_{390}^2 + w_{40}^4) \right) J_0(\rho r) r \, dr.
\]

As a consequence of Babinet's principle, the amplitude distribution \( G_\epsilon(P) \) for the outer annular zone is simply the difference between equations (4) and (5), i.e.,

\[
G_\epsilon(P) = K R^2 \left[ \int_0^1 \exp \left( i k(w_{390}^2 + w_{40}^4) \right) J_0(\rho r) r \, dr \right]
\]

\[\cdot - \int_0^r \exp \left( i k(w_{390}^2 + w_{40}^4) \right) J_0(\rho r) r \, dr \right].
\]

The term \( K R^2 \) being common to \( G_\epsilon \) and \( G_a \) may be left out of the subsequent analysis.

For the complex vector amplitude distribution due to the central circular region at the observation plane is given by

\[
U' = G_\epsilon P(\alpha) \begin{vmatrix} a \\ b \end{vmatrix}
\]

and that due to the outer annular zone is

\[
U'_a = G_a P(\beta) \begin{vmatrix} a \\ b \end{vmatrix}
\]

where \( a \) and \( b \) are the two orthogonal components of the elliptically polarized imaging beam subsequent to the polarizer-quarter-waveplate combination along the two axes of an arbitrary Cartesian coordinate system and \( \delta \) is the phase difference between them.

In the presence of an analyser with Jones matrix \( P(y) \) before the image plane, the contributions from the two polarization-masked zones are

\[
U_e = P(y)U' \tag{9}
\]

and

\[
U_a = P(y)U'_a. \tag{10}
\]

To find the polarization phase amplitude it is convenient to express the above equations as

\[
U_e = G_e R_1 \exp(i\Delta_1) \begin{vmatrix} \cos \gamma \\ \sin \gamma \end{vmatrix}
\]

and

\[
U_a = G_a R_2 \exp(i\Delta_2) \begin{vmatrix} \cos \gamma \\ \sin \gamma \end{vmatrix}
\]

where

\[
R_1 = \cos(\gamma - \alpha)[a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \delta]^{1/2}
\]

\[
\tan^{-1} \Delta_1 = (b \sin \alpha \sin \delta)/(a \cos \alpha + b \sin \alpha \cos \delta) \tag{13}
\]

\[
R_2 = \cos(\gamma - \beta)[a^2 \cos^2 \beta + b^2 \sin^2 \beta + ab \sin 2\beta \cos \delta]^{1/2}
\]

\[
\tan^{-1} \Delta_2 = (b \sin \beta \sin \delta)/(a \cos \beta + b \sin \beta \cos \delta). \tag{15}
\]

The phase difference \( \Delta_1 = \Delta_1 \sim \Delta_2 \) introduced between the two masked zones is thus given by

\[
\tan^{-1} \Delta_2 = b \sin \delta \sin(\alpha - \beta)/(a \cos \alpha \cos \beta + b \sin \alpha \sin \beta) \tag{16}
\]

For \( \alpha = 0^\circ \), \( \beta = 90^\circ \), and \( \gamma = 45^\circ \), equation (16) reduces to

\[
\Delta_2 = \delta. \tag{18}
\]

For this particular configuration of the polarization parameters, the ratio of the transmittance of the central zone to that of the annular zone is

\[
T_e = \frac{G_e R_1}{G_a R_2} = \frac{G_e}{G_a} \tan \gamma. \tag{19}
\]

From equations (18) and (19) it is obvious that the phase difference \( \Delta_2 \) between the masked zones and their relative transmittance \( T_e \) may be controlled independently since the former is simply equal to the input beam parameter \( \delta \) and the latter is a function of the analyser orientation \( \gamma \).

From equations (11) and (12), the IPSF may be written as

\[
U = |U_e + U_a|^2
\]

or, more explicitly,

\[
U = C_1^2 R_1^2 \cos^2(\alpha - \gamma) + R_2^2 C_2^2 \cos^2(\beta - \gamma)
\]

\[+ 2C_1 C_2 R_1 R_2 \cos(\alpha - \gamma) \cos(\beta - \gamma) \cos(\theta_1 - \theta_2 + \Delta) \tag{20}
\]
where

\[ C_1 = \left( U_1^2 + U_2^2 \right)^{1/2} \]
\[ C_2 = \left( \left( C_1^2 + C_2^2 - 2 C_1 C_2 \cos(\theta_1 - \theta_2) \right) \right)^{1/2} \]

and, in general,

\[ U_{1f} = \int_0^\infty \cos k\left(w_{20} r^2 + w_{40} r^4\right) J_0(\rho r) r \, dr \]

\[ U_{2f} = \int_0^\infty \sin k\left(w_{20} r^2 + w_{40} r^4\right) J_0(\rho r) r \, dr \]

Also,

\[ \theta_1 = \tan^{-1} \left( \frac{U_{2f}}{U_{1f}} \right) \]

\[ \theta_2 = \tan^{-1} \left( \frac{U_{2l}}{U_{1l}} \right) \]

\[ \theta_1 = \tan^{-1} \left( \frac{C_2 \sin \theta_1 - C_1 \sin \theta_2}{C_2 \cos \theta_1 - C_1 \cos \theta_2} \right) \]

Equation (20) is a general expression for the intensity distribution at any point \( P(x, y, z) \). The relation between \( x, y, z \) and \( \rho, \omega_{20} \) is given by equation (3).

4. Computation

In the presence of spherical aberration the best-focus plane is not necessarily the focal plane of the un aberrated lens system. It is therefore essential to locate the plane of best focus (image plane) for which the value of axial irradiance is maximum. This is estimated from the Strehl ratio curves for the masked lens. The Strehl ratio for the proposed system may be defined as

\[ SR(\alpha, \beta, \gamma, \alpha, \beta, \delta, \epsilon, w_{20}, w_{40}) \]

\[ = \frac{I(\rho = 0, \alpha, \beta, \gamma, \alpha, \beta, \delta, \epsilon, w_{20} = \text{best focus}, w_{40})}{I(\rho = 0, \alpha, \beta, \gamma, \alpha, \beta, \delta, \epsilon, w_{20} = \text{best focus}, w_{40} = 0)} \]

Each point on a Strehl ratio curve corresponds to a value of \( w_{20} \) for which the Strehl ratio assumes a maximum value; i.e. it corresponds to the axial point where the central intensity is maximum for the masked lens. Usually, the best performance of the imaging system occurs at this value of \( w_{20} \). It is at these axial points that the IPSF is plotted. Compensation of spherical aberration is indicated by a close match of the IPSF of the aberrated masked lens at these points to that of the IPSF of an ideal lens.
The phase $\Delta$ introduced by the polarization mask is 
a function of all the polarization parameters, i.e., $\Delta = \Delta(a, b, \delta, \alpha, \beta, \gamma)$. For most of the computation, the input beam is taken as $a = b = 1$, and the mask is taken as $\alpha = 0^\circ$, $\beta = 90^\circ$, since it was shown earlier that for these values of polarization parameters the polarization phase and transmittance can be varied independently.

In what follows, the Strehl ratio is plotted by (a) varying $\delta$, (b) varying $\varepsilon$, and (c) varying $\gamma$. The IPSFs are calculated at $w_{40}$- and $w_{20}$-values at the points where the Strehl ratio plots exhibit maxima.

(a) Variation of $\delta$. Figure 2 depicts the Strehl ratio versus the spherical aberration coefficient $w_{40}$ for $\delta = 0^\circ$, 30°, 60°, 90°, 120°, and 180°. The Strehl ratio of an unmasked or normal lens is also shown in the plot. Some representative IPSF plots for $\delta = 60^\circ$, 90°, 120° are shown in figures 2(b)–2(d). Each of these figures show the IPSFs of an ideal lens, an unmasked aberrated lens, and a polarization-masked aberrated lens at axial distances $w_{30}$ from the ideal focus where the axial intensity is maximum. The $w_{40}$-value corresponds to the magnitude of the spherical aberration at the peak of the Strehl ratio curve. In most of the cases, it can be observed that the IPSF for the polarization-masked lens almost matches that of the diffraction-limited lens for the given polarization parameters $\alpha$, $\beta$, $\gamma$ and $\delta$. This indicates compensation of spherical aberration.

(b) Variation of $\varepsilon$. Figure 3 shows the Strehl ratio versus the spherical aberration for $\varepsilon = 0.5, 0.707, 0.9$. As discussed earlier, it is not only the polarization phase difference, but also the relative areas of the aperture masked by $P(\alpha')$ and $P(\beta')$ that change the imaging performance and modify the IPSF. Figure 3(b) shows the IPSF curve for a masked lens; the value of $\varepsilon$ is 0.25. Not only is the resolution achieved better than that of an unmasked lens, it is almost the same as that of an ideal lens, indicating good compensation for the 3.522 spherical aberration caused by the masked aperture. Figure 3(c) shows the similarity of the intensity PSF curve of a masked lens for $\varepsilon = 0.5$ and that of an ideal lens. The sidelobes are appreciably reduced. Hence the system can be adapted both for enhanced resolution and apodization by changing the value of the design parameter $\varepsilon$.

(c) Variation of $\gamma$. Figure 4 depicts the Strehl ratio versus the spherical aberration for $\gamma = 0^\circ$, 30°, 45°, 90°, 135°, and 180°. It is found that for $\gamma = 0^\circ$ and for $\gamma = 180^\circ$, the Strehl ratio versus spherical aberration curves remain more
or less horizontal between spherical aberration values 0 and 2.5λ, implying that the imaging behaviour of such lenses does not change appreciably with the introduction of spherical aberration. This gives greater tolerance of spherical aberration. The imaging behaviour of the masked lens is again studied with the help of the intensity PSF curves. Figure 4(b) shows the near-identicality of the intensity PSF curve of a masked lens and that of an ideal lens. In this case, the analyser is kept at 30°. For γ = 90°, the intensity PSF curve shows increased resolution compared to that of an unmasked lens, closely resembling that of an ideal lens. This is shown in figure 4(c). The performance of the proposed system for γ = 135° was already shown in figure 3(c).

5. Conclusions

A scheme for partial compensation of aberration in optical systems using a suitable polarization mask has been suggested. In this paper we specifically analyse aberration compensation for spherically aberrated optical systems. The extent of the compensation is dependent on the polarization parameters involved and the magnitude of the spherical aberration. In some cases, apodization and enhanced resolution are observed. The results bear out our theoretical prediction that with appropriate selection of the polarization parameters, the polarization phase and amplitude introduced by the two zones of the mask can effectively compensate for the phase error due to the presence of spherical aberration. For computational ease we have restricted the number of polarization-masked zones of the aperture to two. However, as is obvious from the physical principle of aberration compensation using this technique, increasing the number of polarization-masked zones of the aperture will result in more effective compensation of aberration. For compensation of aberrations that yield circularly asymmetric PSFs, as in coma, the masking polarizers should also be circularly asymmetric. Some of these aspects will be discussed in future communications.

References

Possibility of an optical focal shift with polarization masks

Dola Roy Chowdhury, Kallol Bhattacharya, A. K. Chakroborty, and Raja Ghosh

The polarization phase shift (PPS) has emerged as an important analytical tool in optical metrology. The present study utilizes the concept of controlling the polarization phase in applications such as focal shift and automatic focusing. When elliptically polarized light, in general, is incident upon a circularly symmetric polarization mask consisting of circular and annular zones with each zone having a unique linear polarizability, the polarization-phase difference introduced between the polarization-masked zones is also circularly symmetric. With the mask at the lens aperture, the polarization phase introduced is multiplicative with the lens function and is shown to result in a shift of the Gaussian focus plane. Because the polarization phase can be controlled by variation of the polarization parameters, the effective focal length of the imaging system can be varied within a small range. A study of the point-spread function at the shifted focal planes has shown that the quality of the focal patch in these planes is comparable to that produced by a diffraction-limited imaging system at Gaussian focus. The shift of focus can be achieved by control of the polarization of the input beam. It is anticipated that this technique may find application in areas for which dynamic focusing within a small range is required.

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1. Introduction

Image formation at out-of-focus planes has been the subject of several papers because of interest in the focal shift effect,1,2 in automatic focusing,3 and in increasing depth of focus.4–7 The possibility of modifying the imaging properties of an optical system by using different kinds of pupil masks has also been extensively studied.8–12 The studies revealed that one can tailor the performance of an imaging system, characterized by the system’s point-spread function (PSF) and optical transfer function, by modifying its pupil function in terms of phase, amplitude, or both. It has also been recognized that one can achieve the same effect by masking the aperture plane with suitably oriented polarizers that introduce polarization-induced phase and amplitude in the pupil plane. The fact that one can vary the response of such a system continuously by changing the orientation of polarizing devices included in the system lends an enhanced flexibility to the system that is unobtainable by the use of conventional pupil masks.13–19 It has also been shown that by suitable choice of the polarization parameters the polarization phase that is introduced can compensate for spherical aberration of an imaging system.20 The versatility of this technique for modifying the imaging characteristics of the system stems from the fact that one can continuously vary the polarization-induced phase difference between light emerging from the masking polarizers either by changing the state of polarization of the incident beam or by reorienting the polarizers.

When the polarization mask is circularly symmetric, the polarization phase difference introduced between the polarization-masked zones of the aperture will also have circular symmetry. This polarization-induced phase in effect modifies the lens function and is shown to bring about a change in the best focal plane; i.e., the best focal plane shifts away from the Gaussian image plane. For the polarization-based focusing system considered, one can control the polarization phase and hence the best focal plane by modifying the input beam’s parameters or by changing the orientation of the polarizers included in the system. It will be shown that the quality of the image formed by the masked aperture at the shifted focus does not vary considerably from that formed at the Gaussian focus of an unmasked lens. This suggests a possibility of changing the focal length of an imaging system sim-
by adjusting the polarization parameters involved. The computational results substantiate this possibility.

2. Proposed System

The input to the focusing system shown in Fig. 1 is a polarizer–quarter-wave plate combination that renders the focusing beam elliptically polarized in general. The aperture of the focusing system consists of two polarization-masked zones: a central circular zone and outer annular zone masked with linear polarizers \( P(\alpha) \) and \( P(\beta) \), respectively, where \( \alpha \) and \( \beta \) are the orientation of the transmission axes of the polarizers with respect to the reference abscissa. They are followed by an analyzer \( P(y) \) before the focal plane.

3. Theory

The phase differences between pairs of transverse electric field vectors of elliptically polarized light are, in general, not the same. This implies that if two different linear components of elliptically polarized light are selected by two linear polarizers whose transmission axes do not coincide, the components will have a phase difference that depends on the orientation of the transmission axis of the polarizers and on the state of polarization of the incident beam. Thus there will be an interesting possibility of introducing any prespecified phase difference between two interfering light beams. The nature of the achievable phase and amplitude variation has been dealt with elsewhere. With reference to the imaging system considered in Fig. 1, elliptically polarized light intercepts the two zones of the polarization mask and emerges, in general, with a phase and amplitude difference between them, the expressions for which are given below. In the presence of an analyzer before the image plane, light from these two zones is brought to the same polarization state and interferes, yielding a redistribution of energy in the diffraction pattern. Because the polarization phase and amplitude depend on the polarization parameters of the system, one may modify the system’s PSF in situ by reorienting the polarization components.

The general expression for the complex amplitude distribution at a point \( P(x, y, z) \) near the geometrical focus of a diffraction-limited lens system of radius \( R \) and of focal length \( f \) is given by

\[
U[P(x, y, z)] = -\frac{2\pi i R^2 A}{\lambda f^2} \exp \left[ \frac{i}{R} \right] \int_0^1 \exp \left( i \frac{u r^2}{2} \right) J_0(pr) r dr \times \int_0^1 \exp \left( i \frac{u r^2}{2} \right) J_0(pr) r dr
\]

\[
= K R^2 \int_0^1 \exp \left( i \frac{u r^2}{2} \right) J_0(pr) r dr.
\]

where

\[
K = \int_0^1 \exp \left( i \frac{u r^2}{2} \right) J_0(pr) r dr.
\]

is the radial coordinate of the pupil plane. \( A \) is the amplitude of the incident light, and \( p \) and \( u \) are the reduced radial and axial coordinates, respectively, at the plane of observation and are given as follows:

\[
u = \frac{2\pi R^2}{\lambda f^2}
\]

\[
p = \frac{2\pi R^2}{\lambda f^2} \left[ \frac{x^2 + y^2}{2} \right]^{1/2}.
\]

Defect of focus \( w_{20} \) is defined as

\[
w_{20} = u/2k,
\]

where \( k \) is the propagation constant. Equation (2) is modified to the following form:

\[
z = 8w_{20} f_0^2,
\]

and the amplitude PSF is then written as

\[
U[P(x, y, z)] = K R^2 \int_0^1 \exp(ikw_{20}^2) J_0(pr) r dr,
\]

where \( w_{20} \) is the effective defocus coefficient. Similarly for an aperture of radius \( R' \), where \( R' = \frac{2\pi R^2}{\lambda f^2} \), the amplitude distribution is given by

\[
G_a(P) = K R^2 \int_0^1 \exp(ikw_{20}^2) J_0(pr) r dr.
\]

As a consequence of Babinet's principle, amplitude distribution \( G_a(p) \) for the outer annular zone is simply the difference between Eqs. (6) and (7), i.e.,

\[
G_a(P) = K R^2 \left[ \int_0^1 \exp(ikw_{20}^2) J_0(pr) r dr - \int_0^1 \exp(ikw_{20}^2) J_0(pr) r dr \right].
\]
assumption that the object beam can be described in terms of a two-element Jones vector given by

\[ \mathbf{e}_i = \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \]  

(9)

where \(a\) and \(b\) are the two orthogonal components of the elliptical focusing beam subsequent to the polarizer–quarter-wave plate combination along the two axes of an arbitrary Cartesian coordinate system and \(\delta\) is the phase difference between them.

Referring to Eq. (9), the complex vector amplitude distribution that is due to the central circular region at the observation plane is given by

\[ U = G_a P(\alpha) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

(10)

and that which is due to the outer annular zone is

\[ U' = G_b P(\beta) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

(11)

In the presence of analyzer \(P(\gamma)\) before the image plane, the contributions from the two polarization-masked zones are

\[ U_c = P(\gamma) U_c', \]  

(12)

\[ U_a = P(\gamma) U_a'. \]  

(13)

To find the polarization phase and amplitude it is convenient to express Eqs. (12) and (13) as

\[ U_c = G_a R_1 \exp(i\Delta_1) \frac{\cos \gamma}{\sin \gamma}, \]  

(14)

\[ U_a = G_a R_2 \exp(i\Delta_2) \frac{\cos \gamma}{\sin \gamma}, \]  

(15)

where

\[ R_1 = \cos \gamma - a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \delta, \]  

(16)

\[ R_2 = \cos \gamma - \beta^2 \sin^2 \alpha + b^2 \sin^2 \beta + ab \sin 2\beta \cos \delta, \]  

(17)

\[ \Delta_1 = \tan^{-1}(b \sin \alpha \sin \delta) / (a \cos \alpha + b \sin \alpha \cos \delta), \]  

(18)

\[ \Delta_2 = \tan^{-1}(b \sin \beta \sin \delta) / (a \cos \beta + b \sin \beta \cos \delta). \]  

(19)

The phase difference \(\Delta = \Delta_1 - \Delta_2\) introduced between the two masked zones is thus given by

\[ \Delta = \tan^{-1}(ab \sin \delta \sin(\alpha - \beta) / (a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta + ab \sin(\alpha + \beta) \cos \delta)). \]  

(20)

From Eqs. (14) and (15), the intensity PSF may be written as

\[ I = I_a + I_b, \]  

(21)

or, more explicitly,

\[ I = \left( C_1 + C_2 \cos(\theta_1 - \theta_2) + \Delta_1 \right) + \left( C_3 + C_4 \cos(\theta_1 - \theta_2) + \Delta_2 \right), \]  

(22)

where

\[ C_1 = \left( R_a + R_b \right)^2, \]

\[ C_2 = \left( R_{a1} + R_{b1} \right)^2, \]

\[ C_3 = \left( R_{a2} + R_{b2} \right)^2, \]

\[ C_4 = \left( R_{a3} + R_{b3} \right)^2. \]

In the presence of analyzer \(P(\gamma)\) before the image plane, the contributions from the two polarization-masked zones are

\[ U_{c1} = P(\gamma) U_{c1}', \]

\[ U_{c2} = P(\gamma) U_{c2}'. \]

Also,

\[ U_{a1} = P(\gamma) U_{a1}', \]

\[ U_{a2} = P(\gamma) U_{a2}'. \]

Referring to Eq. (9), the complex vector amplitude distribution that is due to the central circular region at the observation plane is given by

\[ U_c = G_a P(\alpha) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

(10)

and that which is due to the outer annular zone is

\[ U'_c = G_b P(\beta) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

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(14)

\[ U_a = G_a R_2 \exp(i\Delta_2) \frac{\cos \gamma}{\sin \gamma}, \]  

(15)

where

\[ R_1 = \cos \gamma - a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \delta, \]  

(16)

\[ R_2 = \cos \gamma - \beta^2 \sin^2 \alpha + b^2 \sin^2 \beta + ab \sin 2\beta \cos \delta, \]  

(17)

\[ \Delta_1 = \tan^{-1}(b \sin \alpha \sin \delta) / (a \cos \alpha + b \sin \alpha \cos \delta), \]  

(18)

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\[ \Delta = \tan^{-1}(ab \sin \delta \sin(\alpha - \beta) / (a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta + ab \sin(\alpha + \beta) \cos \delta)). \]  

(20)

From Eqs. (14) and (15), the intensity PSF may be written as

\[ U = |U_c + U_a|^2, \]  

(21)

or, more explicitly,

\[ U = \left( C_1 + C_2 \cos(\theta_1 - \theta_2) + \Delta_1 \right) + \left( C_3 + C_4 \cos(\theta_1 - \theta_2) + \Delta_2 \right), \]  

(22)

where

\[ C_1 = \left( R_a + R_b \right)^2, \]

\[ C_2 = \left( R_{a1} + R_{b1} \right)^2, \]

\[ C_3 = \left( R_{a2} + R_{b2} \right)^2, \]

\[ C_4 = \left( R_{a3} + R_{b3} \right)^2. \]

In the presence of analyzer \(P(\gamma)\) before the image plane, the contributions from the two polarization-masked zones are

\[ U_{c1} = P(\gamma) U_{c1}', \]

\[ U_{c2} = P(\gamma) U_{c2}'. \]

Also,

\[ U_{a1} = P(\gamma) U_{a1}', \]

\[ U_{a2} = P(\gamma) U_{a2}'. \]

Referring to Eq. (9), the complex vector amplitude distribution that is due to the central circular region at the observation plane is given by

\[ U_c = G_a P(\alpha) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

(10)

and that which is due to the outer annular zone is

\[ U'_c = G_b P(\beta) \left| \begin{bmatrix} a \\ b e^{i\delta} \end{bmatrix} \right| \]  

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\[ U_c = P(\gamma) U_c', \]  

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(14)

\[ U_a = G_a R_2 \exp(i\Delta_2) \frac{\cos \gamma}{\sin \gamma}, \]  

(15)

where

\[ R_1 = \cos \gamma - a^2 \cos^2 \alpha + b^2 \sin^2 \alpha + ab \sin 2\alpha \cos \delta, \]  

(16)

\[ R_2 = \cos \gamma - \beta^2 \sin^2 \alpha + b^2 \sin^2 \beta + ab \sin 2\beta \cos \delta, \]  

(17)

\[ \Delta_1 = \tan^{-1}(b \sin \alpha \sin \delta) / (a \cos \alpha + b \sin \alpha \cos \delta), \]  

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\[ \Delta_2 = \tan^{-1}(b \sin \beta \sin \delta) / (a \cos \beta + b \sin \beta \cos \delta). \]  

(19)

The phase difference \(\Delta = \Delta_1 - \Delta_2\) introduced between the two masked zones is thus given by

\[ \Delta = \tan^{-1}(ab \sin \delta \sin(\alpha - \beta) / (a^2 \cos \alpha \cos \beta + b^2 \sin \alpha \sin \beta + ab \sin(\alpha + \beta) \cos \delta)). \]  

(20)
Equation (22) is a general expression for the intensity distribution at any point \( P(x, y, z) \). The relation between \( x, y, z \) and \( r, \theta, w_{20} \) is given by Eqs. (2)–(5).

By virtue of the polarization mask's being circularly symmetric and centered on the axis of the imaging system, polarization-induced phase difference \( \Delta \) is of a similar nature. It is therefore reasonable to expect that the peak irradiance of the PSF will lie upon an axial point which in turn will depend on the polarization parameters involved. We find the exact location of the irradiance maxima from the Gaussian focus, \( w_{20} \), by substituting \( \theta = 0 \) into Eq. (22) and plotting the axial irradiance distribution. At points on the axis where the axial irradiance shows a maximum, the PSF is plotted for \( z = w_{20} \) with Eq. (22). If the nature of the PSF at this shifted focal plane agrees reasonably well with that of an unmasked diffraction-limited imaging system, it is logical to conclude that a shift of focus has indeed been achieved.

4. Computational Results
It is apparent from the analysis above that the axial irradiance distribution is a function of input light beam parameters \( \alpha, \beta, \) and \( \delta \) and system polarization parameters \( \alpha, \beta, \) and \( \gamma \). Of the large number of possible permutations of these parameters, only a few have been intuitively tried. Figure 2 shows the axial irradiance distribution for \( \alpha = \beta = 1, \alpha = 0^\circ, \beta = 90^\circ, \) and \( \delta = 45^\circ \), \( 90^\circ \), \( 135^\circ \), \( 180^\circ \), \( 225^\circ \), \( 270^\circ \), and \( 315^\circ \). Other parameters as in Fig. 2.

Fig. 3. In each case: top, comparison of the intensity PSF of a polarization-masked lens with an ideal lens for several values of \( \delta \); bottom, computed focal patch, as follows: (a) \( \delta = 45^\circ \), (b) \( \delta = 90^\circ \), (c) \( \delta = 135^\circ \), (d) \( \delta = 180^\circ \), (e) \( \delta = 225^\circ \), (f) \( \delta = 270^\circ \), (g) \( \delta = 315^\circ \).
Fig. 3. (Continued)

Fig. 4. Focal shift (in micrometers) plotted against input beam parameter $\delta$.

Fig. 5. Normalized axial irradiance for several values of $\gamma$ for $\alpha = 45^\circ, 60^\circ, 90^\circ$, $\gamma = 90^\circ$, $\epsilon = 0.707$.

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and $\gamma = 45^\circ$. It is interesting to note from Eq. (20) that, for these values of the polarization parameters, $\delta = \Delta$ and $R_1 = R_2$. The plots in Fig. 2 are for values of $\delta$ from 0° to 360° at 45° intervals. It is natural to expect that the axial irradiance for $\delta = 0^\circ$ (i.e., $\Delta = 0^\circ$) will be identical to that of an unmasked lens (curve O of Fig. 2). With increasing values of $\delta$ the existence of a polarization-induced circular symmetric phase is evident from the axial shift of the irradiance maxima. For $\delta = 180^\circ$ (curve D) the imaging system behaves as a double-focus system in which the center of the Gaussian image plane is dark and the axial irradiance maxima lie on either side.

The PSF at each of these shifted image points is shown in Fig. 3. Figure 3(a) shows the PSF for the masked system at an axial point $\rho = 5$ that corresponds to the axial intensity maximum of curve A of Fig. 2. The PSF of a diffraction-limited lens aperture at the Gaussian focal plane is shown for comparison. The lower half of Fig. 3(a) shows the computed focal patch, with its dynamic range compressed in intensity to reveal the secondary maxima. Identical pairs of plots at values of $\omega_m$ for which curves B, C, D, E, F, and G of Fig. 2 exhibit irradiance peaks, are shown in Figs. 3(b), 3(c), 3(d), 3(e), 3(f), and 3(g), respectively. It is interesting to note that for the double-focus configuration ($\delta = 180^\circ$) the PSFs at both foci are identical. In each case the PSF at the shifted axial point seems to agree well with the PSF of an ideal lens aperture. This suggests that one can achieve a continuous change of focus by varying input beam parameter $\delta$ from 0° to 360°. The shift of focus is found to be linear with $\delta$, as shown in Fig. 4. The extent of the shift can be calculated from Eq. (5).
With an f/10 pencil, for example (f/R ~ 20), and with light of wavelength \( \lambda = 550 \text{ nm} \), the maximum focal shift obtained in the present configuration is \(-30 \mu m\) on either side of the Gaussian focus; i.e., a focal shift of 600 \( \mu m \) is achievable.

It is expected that the axial intensity distribution of the proposed system will also depend on the analyzer’s orientation \( \gamma \), as this controls the relative contribution of light emerging from the two polarization masked zones of the aperture mask. Figure 5 shows the axial intensity distribution for \( \gamma = 0^\circ, 45^\circ, 90^\circ, 135^\circ \). For \( \gamma = 0^\circ, 90^\circ \) the aperture behaves simply as a reduced circular and a reduced annular aperture, respectively, and as such the effect of the polarization phase difference is not utilized. Figures 6(a), 6(b), 6(c), 6(d), and 6(e) show the nature of the PSF for \( \gamma = 45^\circ, 135^\circ, 75^\circ, 105^\circ, 150^\circ \) respectively. The quality of image at these shifted focal points is found by comparison with the PSF for an ideal imaging system at Gaussian focus. The shift of focus in this case is not linear with the angular orientation of the analyzer, as is shown in Fig. 7. The maximum shift achievable in this case is 150 \( \mu m \) on either side of the Gaussian focus.

5. Discussion and Conclusion

We have proposed and analyzed a technique for shifting the focus of an optical system within a small range without any physical translation of the system’s elements. The technique seems to be somewhat attractive and novel in the sense that the effective focal length of the optical system proposed is not determined wholly by the optical system’s configuration but can be controlled within a certain small limit by the polarization characteristics of the input light beam. It appears that the principles outlined may be utilized in automatic focusing applications that are similar to optical read–write drives for which it is necessary to track the focal plane dynamically. Input beam parameter \( \delta \) can easily be controlled in real time by a suitable compensator of the Babinet–Soleil type. Although an all-optical system has been discussed, it is nevertheless possible to use electro-optic liquid-crystal phase shifters to achieve the same effect, with the added advantage that the control is fully electronic and incorporates no movable or rotatable elements.

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References

Compensation of Seidel Aberrations and Defect of Focus of an Imaging System utilizing Vector Nature of Light

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Abstract
The imaging characteristics of a lens masked by two polarizers, one at the central circular zone, and the other at the outer annular zone is studied analytically. It is shown that the proposed system has larger depth of focus than conventional system. Expressions are obtained for the intensity point spread function (IPSF) and the axial irradiance distribution of the proposed system. Some specific cases are computed and illustrated graphically.

Keywords: Imagery, Polarization Masking, Depth of Focus

INTRODUCTION
Improvement in depth of field has been of interest in all areas of imaging, especially in high-resolution-microscopy. Various techniques have been proposed over the years[1-2]. In most of the conventional techniques for achieving high depth of focus we take advantage of the scalar wave properties of light. However it has been recognized that the polarization property of light beam offers a new dimension that may be fruitfully utilized for modification of the imaging characteristic of an optical system. Some recently proposed techniques discuss the use of polarization devices for increasing the depth of focus [3-4]. In the present article we report our studies on the imaging characteristics of a perfect lens masked by two linear polarizers of different orientations in two different zones of the lens aperture, one in the central circular zone, the other at the outer annular region. A general expression for the intensity distribution at any point in the far field diffraction pattern of the proposed system, followed by an analyzer is obtained by using Lommel function[5]. The expression is used to evaluate both the in-focus intensity distribution, i.e., the intensity point spread function (IPSF) and the axial intensity distribution for a number of cases.

THEORY
Let the central portion of radius R of a perfect lens be masked by a polariser P(α) whose transmission axis makes an angle α with the X-axis and let the annular zone having inner radius R' and unit outer radius be masked by a polariser P(β) whose transmission axis makes an angle β with X-axis. The beam incident on the mask is in general elliptically polarized. So the complex vector amplitude distribution due to the central circular region at the observation plane is given by,

\[ U_c = G_c P(\alpha) \begin{bmatrix} a \\ be^{i\delta} \end{bmatrix} \]

and that due to the outer annular zone is

\[ U_A = G_A P(\beta) \begin{bmatrix} a \\ be^{i\delta} \end{bmatrix} \]

where, Gc and Ga are the amplitude distribution due to the outer annular region and central region respectively. (inner and outer radii are R' and R respectively) a and b are the two orthogonal components of the input vibration along the two axes of an arbitrary Cartesian coordinate system and δ is the phase difference between them.

\[ G_c[P(x,y,z)] = C_T/\omega \text{exp}[iw/2](U_1-iU_2)-\text{exp}[iw/2](U_1'i-iU_2') \]

\[ G_A[P(x,y,z)] = C_T/\omega \text{exp}[iw/2] \begin{bmatrix} U_1 \\ U_1' \end{bmatrix} \]

where C = (i6R^2/\omega) exp[i(kr-\rho)]. k is the propagation constant, r is the radial coordinate of the pupil plane, ρ is the radial coordinate, f is the focal length of a lens system of radius R. The central region and the annular region of the system...
have different transmittance, denoted by T and T' respectively. U1(w,v) and U2(w,v) are the Lommel functions given by,

\[ \int_0^1 \exp \left( \frac{-iw^2}{2} \right) J_0 (vt) v dr = \frac{1}{w} \exp \left( \frac{iw}{2} \right) \left[ U_1(w,v) - iU_2(w,v) \right] \]

where in general Uc = Uc(w,v) and Uc' = Uc'(w',v'). v and w are the reduced radial and axial coordinates at the plane of observation

\[ w' = \frac{v^2}{w}, \quad v' = \frac{v}{w} \]

and

\[ \varepsilon = \frac{w}{R} \]

the normalized radius of the central zone.

So the resultant complex vector amplitude distribution due to the entire aperture will be the algebraic sum of Uc and UA. Hence

\[ U = U_c + U_A \]

If an analyser \( P(y) \) whose transmission axis makes an angle \( y \) with the X-axis is interposed between the lens and the observation plane then the vector amplitude distribution will be given by,

\[ U' = P(y) U \]

or

\[ U' = AG_c \cos(\alpha y) + BG_A \cos(\beta y) \]

where

\[ A = \cos \alpha + be^{i \sin \alpha} \]

\[ B = \cos \beta + be^{-i \sin \beta} \]

The intensity distribution at any point in the diffraction pattern is given by,

\[ I(w,v) = \left| AG_c \cos(\alpha y) + BG_A \cos(\beta y) \right|^2 \]

The computation will be confined to two regions of special interests, i.e., the geometrical focal plane \( w = 0 \) and along the optical axis \( v = 0 \).

**COMPUTATIONAL RESULTS**

We have taken the input beam parameters as \( a = b = 1 \). The proposed polarization mask is taken as \( \alpha = 0^\circ \) and \( \beta = 90^\circ \). Figure 1 shows the axial irradiance distribution of the proposed system for different values of \( \delta \). The analyzer is kept at 135\(^\circ\). For \( \varepsilon = 0.707 \) and \( \delta = 0^\circ \), a central zero is observed along with the two prominent irradiance peaks away from the geometrical focal plane. So the masked lens may therefore be considered as a double-focus system. With the increasing \( \delta \) the separation between the two foci widens, their intensities differ and the axial distribution becomes asymmetric about the geometrical plane. This clearly indicates that the focal depth of the proposed system can be controlled just by changing the input beam parameter \( \delta \). In figure 2 the variation of axial intensity with the effective defocus coefficient \( w \) is computed and plotted for different values of \( \varepsilon \). It is interesting to note that in the neighborhood of \( \varepsilon = 0.707 \) two prominent irradiance peaks occur on both sides of the Gaussian image plane. So the system behaves as a double focus system. In the process of transition from single focus to double focus system (with increase of \( \varepsilon \)) the depth of focus of the system gradually increases. In figure 3 the intensity point spread function is evaluated for different values of \( \delta \). The variation is most prominent when areas of the two masking regions are equal (\( \varepsilon = 0.707 \)). A central zero intensity is observed in this case for \( \delta = 0^\circ \). This implies that the proposed mask may be used to simulate the effect of a \( \pi \)-phase filter. It is to be noted that for \( \varepsilon = 0.5 \) the first zero occurs nearer to the center of the image plane when compared to that of the clear aperture, thereby indicating super-resolution. But with an increasing \( \varepsilon \) the shift of the first zero away from the origin clearly indicates the apodized behaviour of the imaging system. (Fig. 4).
CONCLUSION

A polarization masked imaging system is proposed for increasing the depth of focus and in-situ variation of resolution. In some cases the system behaves like a double focus lens. The present study is applicable to diffraction limited optical system. The results for aberrated systems will be reported in a forthcoming issue.

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