CHAPTER 6
THEORETICAL PREDICTION OF THE EXTRUSION LOAD AND THEORETICAL MODEL FOR THE FRICTION FACTOR

In this chapter, the theoretical prediction of the extrusion load determined by the Depierre modified approach is discussed. The relation between the instantaneous height and the movement of the billet in the container, is analyzed for the 12:8, 12:4, 12:2 and 8:4 extrusion dies. Three approaches, namely, Altan (1983), Avitzur (1965) and Bakhshi-Jooybari (2002) used to determine the experimental friction factor, are discussed.

6.1 THEORETICAL PREDICTION OF THE EXTRUSION LOAD- DEPIERRE MODIFIED APPROACH

The total extrusion load can be determined theoretically, using the Depierre approach (1970). The total extrusion load \( F_t \) is the sum of the friction load between the container and billet \( F_c \), and the die load \( F_d \). So the total extrusion load can be written as

\[
F_t = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^\beta - 1 \right) \left( \frac{\pi (D + d)}{2} \right) l
\]

(6.1)

In order to find the intermediate extrusion load for each and every movement of the billet in the container, the new approach has been developed. Equation 6.1 is modifying by replacing the extruded diameter \( d \)
and throat length \( (l) \) by the instantaneous diameter \( (D_i) \) and instantaneous height \( (y_i) \) respectively. The throat portion of the die is shown in Figure 6.1.

\[
F_t = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) (R^{\beta}-1) \left( \frac{\pi (D + D_i)}{2} y_i \right)
\]

(6.2)

![Figure 6.1 Throat portion of the die](image)

where

- \( F_t \) - Total extrusion load (N),
- \( D \) - Initial billet diameter (mm),
- \( d \) - Extruded billet diameter (mm),
- \( \alpha \) - Semidie angle (degree),
- \( \sigma \) - Material flow stress (N/mm\(^2\)),
- \( L \) - Billet length in the container (mm),
- \( l \) - Throat length or length of the die (mm),
- \( \tau \) - Billet shear stress (N/mm\(^2\)),
- \( R \) - Extrusion ratio,
- \( y_i \) - Instantaneous height (mm),
- \( x_i \) - Instantaneous diameter at \( y_i \) (mm),
- \( \beta \) - Semidie angle constant,
- \( D_i \) - Instantaneous diameter (mm).
The determination of the diameter at any point in the throat portion of the die is shown in Figure 6.1.

\[
\tan \alpha = \frac{x_i}{y_i}
\]

\[
x_i = y_i \tan \alpha
\]

\[
D_i = D - 2x_i
\]

For the total load required for extrusion, substitute \(D_i = D - 2y_i \tan \alpha\) in Equation 6.2

\[
F_i = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) (R^\beta - 1) \left( \frac{\pi (D + D - 2y_i \tan \alpha)}{2} y_i \right)
\]

(6.3)

Determination of the instantaneous height \(y_i\):

As the process gets initiated, a small amount of billet volume is filled in the throat part. Equating this volume of the billet and the volume of the throat filled,

\[
\frac{\pi}{4} D^2 h = \frac{\pi}{12} \left( D^2 + D_i^2 + D D_i \right) y_i
\]

where

\(h\) - Decrease in the billet height in the container (mm)

So, the distance moved in die part \(y_i = \frac{\frac{\pi}{4} D^2 h}{\frac{\pi}{12} \left( D^2 + D_i^2 + D D_i \right)}\)
\[ y_i = \frac{3 D^2 \; h}{\left(D^2+D_i^2+D \; D_i\right)} \]

\( D_i \) is replaced by \( D - 2 \; y_i \tan \alpha \)

\[ y_i = \frac{3 D^2 \; h}{\left(D^2+(D - 2 \; y_i \tan \alpha)^2 + D \; (D - 2 \; y_i \tan \alpha)\right)} \quad (6.4) \]

For a 12:8 die, substituting \( D = 12 \) and the semidie angle \( \alpha = 26.5 \) in equation 6.4,

\[ y_i^3 - 36 \; y_i^2 + 432 \; y_i - 432 \; h = 0 \quad \text{for} \quad 0 \leq h \leq 2.8 \quad (6.5) \]

\( y_i \) can be determined from Equation 6.5, by substituting from \( h = 0.2 \) to 2.8, and the corresponding \( y_i \) values are shown in Table 6.1. Equation 6.5 is used to find out the instantaneous height to be filled in the throat part, with respect to the decrease billet length in the container.

**Table 6.1 Instantaneous value with the height in the container part**

<table>
<thead>
<tr>
<th>Decrease in billet height in the container ( h ) (mm)</th>
<th>Instantaneous height ( y_i ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.20</td>
</tr>
<tr>
<td>0.4</td>
<td>0.41</td>
</tr>
<tr>
<td>0.6</td>
<td>0.63</td>
</tr>
<tr>
<td>0.8</td>
<td>0.86</td>
</tr>
<tr>
<td>1.0</td>
<td>1.09</td>
</tr>
<tr>
<td>1.2</td>
<td>1.34</td>
</tr>
<tr>
<td>1.4</td>
<td>1.60</td>
</tr>
<tr>
<td>1.6</td>
<td>1.87</td>
</tr>
<tr>
<td>1.8</td>
<td>2.16</td>
</tr>
<tr>
<td>2.0</td>
<td>2.47</td>
</tr>
<tr>
<td>2.2</td>
<td>2.80</td>
</tr>
<tr>
<td>2.4</td>
<td>3.15</td>
</tr>
<tr>
<td>2.6</td>
<td>3.55</td>
</tr>
<tr>
<td>2.8</td>
<td>4.0</td>
</tr>
</tbody>
</table>
In Equation 6.3, L decreases in steps of 0.2 mm, which is filled in the die part; in other words, \( y_i \) increases in the die part.

Equation 6.3 is valid until the material is filled in the die part up to 2.8 mm movement in the container. For any further movement in the ‘h’ value, and for every 0.2 mm movement ‘h’ in the container, the same will be filled in the throat part. It can be determined by

Small volume of movement in the container part = Volume of the extruded part

\[
\frac{\Pi}{4} D^2 h = \frac{\Pi}{4} \left( d^2 \right) h_e
\]

where

\[ h_e \] - Extruded billet height (mm)

\[
\frac{\Pi}{4} 12^2 0.2 = \frac{\Pi}{4} \left( 8^2 \right) h_e
\]

Therefore, \( h_e = 0.45 \).

**Table 6.2 Extruded billet movement with the container height**

<table>
<thead>
<tr>
<th>Decrease in billet height in the container ( h ) (mm)</th>
<th>Extruded billet height ( h_e ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.45</td>
</tr>
<tr>
<td>0.4</td>
<td>0.90</td>
</tr>
<tr>
<td>0.6</td>
<td>1.35</td>
</tr>
<tr>
<td>0.8</td>
<td>1.80</td>
</tr>
<tr>
<td>1.0</td>
<td>2.25</td>
</tr>
<tr>
<td>1.2</td>
<td>2.70</td>
</tr>
<tr>
<td>1.4</td>
<td>3.15</td>
</tr>
<tr>
<td>1.6</td>
<td>3.60</td>
</tr>
<tr>
<td>1.8</td>
<td>4.05</td>
</tr>
<tr>
<td>2.0</td>
<td>4.50</td>
</tr>
<tr>
<td>2.2</td>
<td>4.95</td>
</tr>
<tr>
<td>2.4</td>
<td>5.40</td>
</tr>
</tbody>
</table>
The extruded billet movement with the container height is shown in Table 6.2. 2.8 mm of billet is required to fill the throat volume and 2.2 mm is required for the extruded part; so, totally 5 mm is required for extrusion.

Therefore, for the total extrusion load, Equation 6.3 can be written as

\[ F_i = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^\theta - 1 \right) \left( \frac{\pi (D + D - 2 y \tan \alpha)}{2} y_i \right) + \pi d h_e \tau \]

\[ \text{for } 0 \leq y_i \leq 4 \text{ and } 0 \leq h_e \leq 5 \]

The friction at the container-billet interface contributes significantly to the complexity. During the extrusion process, the force necessary to overcome the friction between the billet and the container results in an increase in the overall load for extrusion. The friction may be defined as the same, as the Tresca friction law, which assumes the proportionality between the friction and the current shear flow stress of the material, and may be written as follows:

\[ \tau = mk \text{ and } \tau = m\sigma / \sqrt{3} \]

where,

‘m’ is the factor of proportionality and is commonly referred to as a friction factor, and varies between m = 0 for perfect lubrication and m = 1 for sticking friction. ‘k’ is the shear flow stress.

At the extreme condition between the billet and the container, the frictional stress at the interface does not exceed the shear strength of the material.
Substituting $\tau = m\sigma/\sqrt{3}$ in Equation 6.6,

$$F_{i} = \pi DLm \frac{\sigma}{\sqrt{3}} + \pi \left( \frac{1+\beta}{\beta} \right) \left( \frac{\pi (D + 12 - 2y_{i} \tan \alpha)}{2} y_{i} \right) + \pi dh_{m} m \frac{\sigma}{\sqrt{3}}$$  \hspace{1cm} (6.7)

for $0 \leq y_{i} \leq 4$ and $0 \leq h_{m} \leq 5$

Existing term  \hspace{2cm} Modified term  \hspace{2cm} Added term

Equation 6.7 is the Depierre modified approach; in this equation, the follower load is considered in addition to the die load and container load. This approach is used to find the intermediate load for each and every movement of the billet in the container, with respect to different area reductions and different interface conditions. The theoretical extrusion load is determined by substituting the flow stress value (drawn from the tensile test), experimental friction factor and other factors in Equation 6.7. The load-displacement curve is drawn for a small billet movement in the container part, and it is discussed in section 8.3.

Similarly, the equations for the determination of the $y_{i}$ values of 12:4, 12:2 and 8:4 die are shown in Table 6.3.

**Table 6.3 Cubic equation for different dies**

<table>
<thead>
<tr>
<th>S.No</th>
<th>Dimension of Die</th>
<th>Determination of $y_{i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12:4</td>
<td>$y_{i}^3 - 18y_{i}^2 + 108y_{1} - 108h = 0$ for $0 \leq h \leq 2.0$</td>
</tr>
<tr>
<td>2</td>
<td>12:2</td>
<td>$y_{i}^3 - 15.10y_{i}^2 + 76.05y_{1} - 76.05h = 0$ for $0 \leq h \leq 1.8$</td>
</tr>
<tr>
<td>3</td>
<td>8:4</td>
<td>$y_{i}^3 - 24y_{i}^2 + 192y_{1} - 192h = 0$ for $0 \leq h \leq 2.4$</td>
</tr>
</tbody>
</table>
6.2 THEORETICAL MODELS FOR THE FRICTION FACTOR

For determining the experimental friction factor, the Avitzur, Altan and Bakhshi-Jooybari approaches are considered.

6.2.1 Avitzur Approach

The extrusion pressure can be determined in the theoretical analysis by the use of the upper bound model proposed by Avitzur (1965), which is rotationally symmetric and assumes a spherical velocity field at the die inlet. The model assumes that the flow stress of the material being extruded is constant, and it follows the Von Mises law. The extrusion pressure ‘P’ is determined as

\[ P = P_i + P_s + P_{sf} + P_f \]

where

- \( P \) - Extrusion pressure (N/m\(^2\)),
- \( P_i \) - Pressure required for internal deformation (N/m\(^2\)),
- \( P_s \) - Pressure required for shear deformation in the reduction zone (N/m\(^2\)),
- \( P_{sf} \) - Pressure required for overcoming friction along the die surface in the reduction zone (N/m\(^2\)),
- \( P_f \) - Pressure required for overcoming friction along the billet-container interface (N/m\(^2\)).

\[ P_i = \sigma f'(\alpha) \ln(R) \]

\[ P_s = 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} - \cot \alpha \right) \]
\[ P_{sf} = \frac{\sigma}{\sqrt{3}} m_3 \cot \alpha \ln(R) \]

\[ P_f = 2 \frac{\sigma}{\sqrt{3}} m_4 \left[ \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \]

\[ P = \left[ \sigma f(\alpha) \ln(R) \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) + \left[ \frac{\sigma}{\sqrt{3}} m_3 \cot \alpha \ln(R) \right] \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} m_4 \left[ \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \right] \]

\text{(6.8)}

where

- \( \sigma \) - Material flow stress (N/mm\(^2\)),
- \( f(\alpha) \) - Geometric factor,
- \( R \) - Extrusion ratio,
- \( \alpha \) - Semi cone die angle,
- \( L \) - Billet length (mm),
- \( r_0 \) - Billet radius (mm),
- \( m_3 \) - Friction factor along the die surface,
- \( m_4 \) - Friction factor along the container surface.

The extrusion force is calculated by multiplying the cross sectional area of the billet with the extrusion pressure

\[ F = \Pi r_0^2 \left[ \left[ \sigma f(\alpha) \ln(R) \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) + \left[ \frac{\sigma}{\sqrt{3}} m_3 \cot \alpha \ln(R) \right] \right] \right] \]
\[ \frac{F}{\Pi r_0^2} = \left[ \sigma f(\alpha) \ln(R) \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] + \left[ \frac{m}{\sqrt{3}} \cot \alpha \ln(R) \right] \]

\[ + \left[ 2 \frac{\sigma}{\sqrt{3}} \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \] (6.10)

\[ \frac{F}{\Pi r_0^2} = \left[ \sigma f(\alpha) \ln(R) \right] - \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] = \left[ \frac{m}{\sqrt{3}} \right] \]

\[ + \left[ 2 m \frac{\sigma}{\sqrt{3}} \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \]

\[ \frac{F}{\Pi r_0^2} = \left[ \sigma f(\alpha) \ln(R) \right] - \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] = \left[ \frac{m}{\sqrt{3}} \right] \]

\[ \times \left[ \cot \alpha \ln(R) + \left[ 2 \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \right] \]

\[ \frac{F}{\Pi r_0^2} = \left[ \sigma f(\alpha) \ln(R) \right] - \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] = \left[ \frac{m}{\sqrt{3}} \right] \]

\[ \frac{\sqrt{3}}{\Pi r_0^2} \left[ \sigma f(\alpha) \ln(R) \right] - \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] = \left[ \frac{m}{\sqrt{3}} \right] \]

\[ m = \frac{\sqrt{3} \left[ \sigma f(\alpha) \ln(R) \right] - \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cdot \cot \alpha \right) \right]}{\sigma \left[ \cot \alpha \ln(R) + \left[ 2 \frac{L}{r_0} \left( 1 - \frac{1}{\sqrt{R}} \right) \cot \alpha \right] \right]} \] (6.11)
6.2.2 Altan Approach

The Extrusion load can be determined in the theoretical analysis, by the use of the slab model for axisymmetric extrusion, proposed by Altan et al (1983), which assumes that the flow stress of the extruded material is constant, and it follows the Von Mises law. The total extrusion load $F_t$ is calculated as

$$F_t = F_i + F_s + F_{sf} + F_f$$

where

$F_t$ - Total extrusion load (N),

$F_i$ - Load required for internal homogeneous deformation (N),

$F_s$ - Load required for shear deformation (N),

$F_{sf}$ - Load required for overcoming friction along the die surface (N),

$F_f$ - Load required for overcoming friction along container surface (N).

$$F_i = \Pi r_0^2 \sigma \ln(R)$$

$$F_s = 2 \Pi r_0^2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cot \alpha \right)$$
\[
F_{s_f} = \Pi r_0^2 \frac{\sigma}{\sqrt{3}} m_3 \frac{\ln(R)}{\sin\alpha a}
\]

\[
F_r = 2 \Pi r_0 L m_4 \frac{\sigma}{\sqrt{3}}
\]

\[
F = \Pi r_0^2 \left[ \sigma \ln(R) \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cot \alpha \right) \right] + \left[ \frac{\sigma}{\sqrt{3}} m_3 \frac{\ln(R)}{\sin \alpha a} \right] + \left[ 2 L m_4 \frac{\sigma}{r_0 \sqrt{3}} \right] \quad (6.12)
\]

where

\[\sigma\] - Material flow stress (N/mm\(^2\)),

\[R\] - Extrusion ratio,

\[\alpha\] - Semi cone die angle,

\[L\] - Billet length (mm),

\[r_0\] - Billet radius (mm),

\[m_3\] - Friction factor along the die surface,

\[m_4\] - Friction factor along the container surface.

Assume that \(m_3 = m_4 = m\)

\[
F = \Pi r_0^2 \left[ \sigma \ln(R) \right] + \left[ 2 \frac{\sigma}{\sqrt{3}} \left( \frac{\alpha}{\sin^2 \alpha} \cot \alpha \right) \right] + \left[ \frac{\sigma}{\sqrt{3}} m \frac{\ln(R)}{\sin \alpha a} \right] + \left[ 2 L m \frac{\sigma}{r_0 \sqrt{3}} \right] \quad (6.13)
\]
\[ F = \Pi r_0^2 \left[ \sigma \ln(R) + 2 \frac{\sigma}{\sqrt{3}} \left( \frac{a}{\sin^2 \alpha} \cdot \cot \alpha \right) \right] + \left[ \frac{\sigma}{\sqrt{3}} m \right] \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] \]

\[ \frac{F}{\Pi r_0^2} = \left[ \sigma \ln(R) \right] + 2 \frac{\sigma}{\sqrt{3}} \left( \frac{a}{\sin^2 \alpha} \cdot \cot \alpha \right) \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] \]

\[ \left[ \frac{\sigma}{\sqrt{3}} m \right] \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] = \frac{F}{\Pi r_0^2} - \left[ \sigma \ln(R) \right] - 2 \frac{\sigma}{\sqrt{3}} \left( \frac{a}{\sin^2 \alpha} \cdot \cot \alpha \right) \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] \]

\[ \frac{\sigma}{\sqrt{3}} m = \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] \]

\[ m = \sqrt{3} \left[ \frac{F}{\Pi r_0^2} - \left[ \sigma \ln(R) \right] - 2 \frac{\sigma}{\sqrt{3}} \left( \frac{a}{\sin^2 \alpha} \cdot \cot \alpha \right) \left[ \frac{\ln(R)}{\sin \alpha \cos \alpha} + \frac{2}{r_0} L \right] \right] \]

6.2.3 Bakhshi-Jooybari Approach

Depierre (1970) found that, the total extrusion load \( F_t \) is the sum of the die load \( F_d \), the friction load between the container and billet \( F_c \) and the frictional load between the extruded billet and container \( F_e \) (Extruded portion). Generally \( F_e = 0 \).

\[ F_t = F_c + F_d \]

The frictional load between the billet and the container can be written as

\[ F_e = \pi D \tau L \]
The die load $F_d$ is given by

$$F_d = \sigma \left( \frac{1+\beta}{\beta} \right) (R^\theta - 1) \left( \frac{\pi (D+d)}{2} \right) l$$

Therefore, the total extrusion load

$$F_t = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) (R^\theta - 1) \left( \frac{\pi (D+d)}{2} \right) l$$

(6.15)

where

- $\tau$ - Billet shear stress (N/mm$^2$),
- $L$ - Billet length in the container (mm),
- $D$ - Initial billet diameter (mm),
- $\sigma$ - Material flow stress (N/mm$^2$),
- $R$ - Extrusion ratio,
- $d$ - Extruded billet diameter (mm),
- $l$ - Length of die (mm).

The term $\tau = m k$ can be used to define the frictional shear stress in the container (Dieter 1988)

where

- $m$ - Friction factor,
- $k$ - Shear flow stress (N/mm$^2$).
According to the Von Mises (1913) proposal, \( k = \frac{\sigma}{\sqrt{3}} \)

By substituting ‘\( k \)’ in equation 6.15,

\[
F_t = \frac{\pi D L m \sigma}{\sqrt{3}} + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta} - 1 \right) \left( \frac{\pi (D+d)}{2} \right) l
\]

(6.16)

‘\( m \)’ can be determined by the following assumption by Bakhshi-Jooybari (2002) and Wagener (1994). Two billets of different length were taken and extruded for the same distance. In Equation 6.16, the load required to fill the die part is constant for lengths \( L_1 \) and \( L_2 \). So it can be rewritten in terms of \( L_1 \) and \( L_2 \).

The total extrusion load required for \( L_1 \) length

\[
F_{t1} = \frac{\pi D L_1 m \sigma}{\sqrt{3}} + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta} - 1 \right) \left( \frac{\pi (D+d)}{2} \right) l
\]

(6.17)

Similarly, the total extrusion load required for \( L_2 \) length

\[
F_{t2} = \frac{\pi D L_2 m \sigma}{\sqrt{3}} + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta} - 1 \right) \left( \frac{\pi (D+d)}{2} \right) l
\]

(6.18)

Subtract the Equation 6.17 with 6.18

\[
F_{t1} - F_{t2} = \frac{\pi D L_1 m \sigma}{\sqrt{3}} - \frac{\pi D L_2 m \sigma}{\sqrt{3}} = \frac{\pi D m \sigma}{\sqrt{3}} (L_1 - L_2)
\]

\[
\Delta F = \frac{\pi D m \sigma \Delta L}{\sqrt{3}}
\]

Hence the friction factor is,
\[ m = \frac{\Delta F \sqrt{3}}{\pi D \Delta L \sigma} \]  

(6.19)

The above model is extended by considering the friction force in the exit container too. The Depierre modified approach is written as

\[ F_t = \pi D L \tau + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta-1} \right) \left( \frac{\pi (D + D - 2 y \tan \alpha)}{2} y \right) + \pi d h_e \tau \]  

(6.20)

‘m’ can be determined by the following assumption of Bakhshi-Jooybari (2002). Two billets of different length were taken and extruded for the same distance. In Equation 6.20, the load required to fill the die part is constant for lengths \( L_1 \) and \( L_2 \). So it can be rewritten in terms of \( L_1 \) and \( L_2 \).

The total extrusion load required for \( L_1 \) length

\[ F_{t1} = \pi D L_1 \tau + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta-1} \right) \left( \frac{\pi (D + D - 2 y \tan \alpha)}{2} y \right) + \pi d h_e \tau \]  

(6.21)

The total extrusion load required for \( L_2 \) length

\[ F_{t2} = \pi D L_2 \tau + \sigma \left( \frac{1+\beta}{\beta} \right) \left( R^{\beta-1} \right) \left( \frac{\pi (D + D - 2 y \tan \alpha)}{2} y \right) + \pi d h_e \tau \]  

(6.22)

Subtract Equation 6.21 with 6.22

\[ F_{t1} - F_{t2} = \pi D L_1 \tau - \pi D L_2 \tau = \pi D (L_1 - L_2) \tau = \frac{\pi D m \sigma}{\sqrt{3}} (L_1 - L_2) \]

\[ \Delta F = \frac{\pi D m \sigma \Delta L}{\sqrt{3}} \]

Friction factor

\[ m = \frac{\Delta F \sqrt{3}}{\pi D \Delta L \sigma} \]  

(6.23)
Even by considering the follower load in the Depierre modified approach, the same expression is obtained for the determination of the friction factor ‘m’ by the Bakhshi-Jooybari approach. For the determination of the experimental friction factor ‘m’ the Altan and Avitzur approaches have one billet extruded, but in the Bakhshi-Jooybari approach two billets are extruded.

6.3 SUMMARY

The instantaneous height and load were determined, using the theoretical model. The Cubic polynomials relating the instantaneous height and the movement of the billet in the container were framed for 12:8, 12:4, 12:2 and 8:4 extrusion dies. The Depierre modified approach is used to find the intermediate load for each and every movement of the billet in the container, with respect to different area reductions and different interface conditions. Three approaches, namely, Altan, Avitzur and Bakhshi-Jooybari were used to determine the experimental friction factor ‘m’.