CHAPTER 5

FAULT ANALYSIS OF INTEGRATED THREE PHASE AND SIX PHASE TRANSMISSION SYSTEM

5.1 INTRODUCTION:-

So far, no work has been reported on fault analysis of three - phase / six - phase integrated system. In this work, an integrated three phase - six phase system is considered with a six phase transmission line connected to three phase systems x and y via three phase to six phase transformers as shown in fig 5.1. The single line diagram of the system is as shown in fig 5.2.

Fig 5.1 Integrated three phase - six phase power system. (The two three phase transformers are identical and have star/double star winding connection with grounded neutral).

Fig 5.2 Single line diagram of mixed power system.
The Thevenin's equivalent diagram of the system is shown in fig 5.3, up to the six phase transmission line.

Fig 5.3 Sequence networks of the integrated 3-ϕ/6-ϕ system for shunt fault analysis.

For the analysis of the three phase parts of the system, such as the calculation of fault currents etc, it is useful to replace the six phase part by an equivalent three phase system. Such that the total system can be analysed as a three phase power system using three phase symmetrical components. Vice-versa, for the analysis of six phase part of the power system, it is convenient to replace the three phase elements by equivalent six phase components, such that the system can be analysed as a system which is completely six phase system using six phase symmetrical components or Dual Three Phase Transformation in terms of the familiar three phase symmetrical components, treating the six phase system as two mutually coupled three phase systems. Here equivalent representation for analysing the fault currents of an integrated three phase to six phase transmission system for faults on either the six phase part or the three phase parts have been developed. Only certain basic shunt faults on either three phase or six phase sides, which are likely to occur have been analysed. The modeling of three phase/ six phase transformers and six phase generators etc., as developed by J.L. Williams [25] have been made use.
5.2 MODELLING OF THREE PHASE / SIX PHASE TRANSFORMER:-

5.2.1 Three Phase Sequence Components In Terms of Six Phase Components:-

Consider a six phase transmission line connected to a three phase transmission line via a three phase/six phase transformer as shown in fig 5.4. The terminal conditions are

\[
V_p^3 = \left(\frac{1}{2}\right) N^T V_p^6 \quad ; \quad I_p^3 = \left(\frac{1}{2}\right) N^T I_p^6
\]

\( (5.1) \)

Where \( V_p^3 = [V_a \ V_b \ V_c]^T \) ; \( V_p^6 = [V_A \ V_B \ V_C \ V_D \ V_E \ V_F]^T \)

\( I_p^3 = [I_a \ I_b \ I_c]^T \) ; \( I_p^6 = [I_A \ I_B \ I_C \ I_D \ I_E \ I_F]^T \)

Also \( N^T = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \)

we know \( V_p^3 = [T_3] V_s^3 \) \( (5.2) \)

\( V_s^3 = [T_3]^{-1} V_p^3 \) \( (5.2a) \)
\[ V_S^3 = (1/2)[T_3]^{-1} N^T V_P^6 \]  \hspace{1cm} (5.3)

\[ I_S^3 = (1/2) [T_3]^{-1} N^T I_P^6 \]  \hspace{1cm} (5.3a)

Where  
\[ V_S^3 = [V_{a0} \ V_{a1} \ V_{a2}]^T \]

\[ I_S^3 = [I_{a0} \ I_{a1} \ I_{a2}]^T \]

\[ [T_3] = \text{Three phase symmetrical components transformation matrix.} \]

5.2.2 Six phase sequence components in terms of three phase components:-

5.2.2.1 Using six phase symmetrical components method:-

From the fig5.4, the terminal conditions are

\[ V_P^6 = N \ V_F^3 \; ; \; I_P^6 = N \ I_P^3 \]  \hspace{1cm} (5.4)

\[ V_P^6 = [T_6] \ V_S^6 \]  \hspace{1cm} (5.5)

\[ V_S^6 = [T_6]^{-1} V_P^6 \]  \hspace{1cm} (5.5a)

\[ V_S^6 = [T_6]^{-1} N \ V_P^3 \]  \hspace{1cm} (5.6)

\[ I_S^6 = [T_6]^{-1} N \ I_P^3 \]  \hspace{1cm} (5.6a)

Where \( N = \)

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & -1 \\
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0 \\
\end{pmatrix}
\]

\[ [T_6] = \text{Six phase symmetrical component matrix} \]

\[ V_S^6 = [V_{A0} \ V_{A1} \ V_{A2} \ V_{A3} \ V_{A4} \ V_{A5}]^T \]

\[ I_S^6 = [I_{A0} \ I_{A1} \ I_{A2} \ I_{A3} \ I_{A4} \ I_{A5}]^T \]
5.2.2.2 Using Dual Three Phase Transformation (DTPT) Method:

From fig 5.4, the terminal conditions are

\[ V_p^6 = N V_p^3; \quad I_p^6 = N I_p^3 \]

\[ V_p^6 = [T_D] [V_{SD}]^6 \]  \hspace{5cm} (5.7)

\[ [V_{SD}]^6 = [T_D]^{-1} V_p^6 \]

\[ [V_{SD}]^6 = [T_D]^{-1} N V_p^3 \]  \hspace{5cm} (5.8)

\[ [I_{SD}]^6 = [T_D]^{-1} N I_p^3 \]  \hspace{5cm} (5.8a)

where \[ [T_D] = \text{Dual Three Phase Transformation matrix} \]

\[ [V_{SD}]^6 = [V_{a0} \ V_{a1} \ V_{a2} \ V_{a1} \ V_{a1} \ V_{a12}]^T \]

\[ [I_{SD}]^6 = [I_{a0} \ I_{a1} \ I_{a2} \ I_{a1} \ I_{a1} \ I_{a12}]^T \]

5.3 Modelling of six phase Transmission Line interconnected to Two three phase Systems:

5.3.1 Three phase equivalent of six phase line in terms of three phase symmetrical components:

The series voltage drop for the transposed line from fig 5.1 is

\[ V_x^3 - V_y^3 = (\frac{1}{2}) N^T Z_p^6 N I_p^3 \]  \hspace{5cm} (5.9)

\[ V_x^3 - V_y^3 = \Delta V_p^3 = Z_{p_{eq}}^3 I_p^3 \]  \hspace{5cm} (5.9a)

Where \[ Z_{p_{eq}}^3 = (\frac{1}{2}) N^T Z_p^6 N \]

\[ [T_3][\Delta V_s^3] = [Z_{p_{eq}}^3][T_3][I_s^3] \]

\[ \Delta V_s^3 = [T_3]^{-1} [Z_{p_{eq}}^3][T_3][I_s^3] \]  \hspace{5cm} (5.10)

\[ \Delta V_s^3 = [Z_{s_{eq}}^3][I_s^3] \]  \hspace{5cm} (5.11)

where \[ [Z_{s_{eq}}^3] = [T_3]^{-1} [Z_{p_{eq}}^3][T_3] \]
In the expanded form

\[
\begin{bmatrix}
V_{a0} \\
V_{a1} \\
V_{a2}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & Z_{1eq} & 0 \\
0 & 0 & Z_{1eq}
\end{bmatrix}
\begin{bmatrix}
Z_{0eq} & 0 & 0 \\
0 & Z_{1eq} & 0 \\
0 & 0 & Z_{1eq}
\end{bmatrix}
\begin{bmatrix}
I_{a0} \\
I_{a1} \\
I_{a2}
\end{bmatrix}
\]

Where \( Z_{eq} = Z_{1eq} = Z_{2eq} = (Z_s - Z_m) \) for a transposed line

5.3.2 Six phase equivalent of three phase symmetrical components:

5.3.2.1 In terms of six phase symmetrical components:

The series voltage drop for the transposed six phase line is

\[
V_S^6 - V_R^6 = \frac{1}{2} \left[ N Z p^3 N^T \right] I_p^6
\]

\[
\Delta V_p^6 = \left[ Z_p^6 \right] I_p^6
\]

where

\[
\left[ Z_p^6 \right] = \left[ \frac{1}{2} \right] \left[ N Z p^3 N^T \right]
\]

\[
\left[ T^6 \right] \Delta V_s^6 = \left[ Z_s^6 \right] I_s^6
\]

\[
\Delta V_p^6 = \left[ Z_s^6 \right] I_s^6
\]

In the expanded form

\[
\begin{bmatrix}
V_{a0} \\
V_{a1} \\
V_{a2} \\
V_{a3} \\
V_{a4} \\
V_{a5}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z_{1eq} & 0 & 0 & 0 & 0 \\
0 & 0 & Z_{2eq} & 0 & 0 & 0 \\
0 & 0 & 0 & Z_{3eq} & 0 & 0 \\
0 & 0 & 0 & 0 & Z_{4eq} & 0 \\
0 & 0 & 0 & 0 & 0 & Z_{5eq}
\end{bmatrix}
\begin{bmatrix}
I_{a0} \\
I_{a1} \\
I_{a2} \\
I_{a3} \\
I_{a4} \\
I_{a5}
\end{bmatrix}
\]

Where \( Z_{0eq} = Z_{2eq} = Z_{4eq} = 0 \)

\( Z_{1eq} = Z_{5eq} = (Z_s - Z_m) \)

\( Z_{3eq} = (Z_s + 2Z_m) \)
5.3.2.2 In terms of DTPT Components:

From equation 5.12a

$$\Delta V_p^6 = [Z_p^6] I_p^6$$

Where

$$[Z_p^6] = \left[ \begin{array}{c} 1/2 \end{array} \right] [NZ_p^3N^T]$$

$$[TD] \Delta V_s^6 = [Z_p^6] \left[ \begin{array}{c} T_D \end{array} \right] I_s^6$$  \hspace{1cm} \text{(5.15)}

$$\Delta V_s^6 = [T_D]^{-1} [Z_p^6] [T_D] [I_s^6]$$  \hspace{1cm} \text{(5.16)}$$

In the expanded form

$$\begin{bmatrix} V_{z0} \\ V_{a1} \\ V_{a2} \\ V_{a1}^1 \\ V_{a2}^1 \\ V_{a2}^2 \end{bmatrix} = \begin{bmatrix} 0 & E_{a1} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{1eq} & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{0eq} & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{1eq} & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{2eq} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \\ I_{a1}^1 \\ I_{a2}^1 \\ I_{a2}^2 \end{bmatrix}$$

Where

$$Z_{0eq} = (Z_s + 2Z_m)/2 = Z_{00eq}; \ Z_{1eq} = Z_{2eq} = (Z_s - Z_m)/2$$

5.4 Fault Analysis:

Only certain basic faults, which are more likely to occur are considered in this work. The leakage reactances of the three phase / six phase transformer are neglected.

5.4.1 Fault on Six phase Line and its effect on Three phase System:

5.4.1.1 Single Line to ground fault:

This fault is the most probable type of fault to occur on a power system. Let the fault on phase A, the valid relations from figure 5.4 are

$$V_A = 0 \hspace{1cm} I_B = I_C = I_D = I_E = I_F = 0$$  \hspace{1cm} \text{(5.17)}$$
Using the inverse transformation equation (5.17) results
\[ V_{a0} + V_{a1} + V_{a2} = V_a/2 \quad ; \quad I_{a0} = I_{a1} = I_{a2} \]  \hspace{1cm} (5.18)

Using equation (5.18) in equation (5.11) gives
\[ I_{a1} = \frac{[E_{a1} - V_0/2]}{(Z_{0eq} + Z_{1eq} + Z_{2eq})} \]  \hspace{1cm} (5.19)

5.4.1.2 Double Line fault:-
Let the double line fault be on the phases A and D. The terminal conditions from fig 5.4 are
\[ V_A = V_D \quad ; \quad I_A + I_D = 0 \quad ; \quad I_B = I_C = I_E = I_F = 0 \]  \hspace{1cm} (5.20)

Using equation (5.20) the sequence relations of voltages and current becomes.
\[ V_{a0} + V_{a1} + V_{a2} = 0 \quad ; \quad I_{a0} = I_{a1} = I_{a2} \]  \hspace{1cm} (5.21)

Using equation (5.21) in equation (5.11) gives
\[ I_{a1} = \frac{E_{a1}}{(Z_{1eq} + Z_{2eq} + Z_{0eq})} \]  \hspace{1cm} (5.22)

5.4.1.3 Double Line to ground fault:-
Let the fault be on phases A and D. The conditions at the location of the fault are
\[ V_A = V_D = 0 \quad ; \quad I_B = I_C = I_E = I_F = 0 \]  \hspace{1cm} (5.23)

Using inverse transformation on the equation (5.23) results
\[ V_{a0} + V_{a1} + V_{a2} = 0 \quad ; \quad I_{a0} = I_{a1} = I_{a2} \]  \hspace{1cm} (5.24)

The above equations clearly suggest that all the sequence networks should be connected in series for simulating fault, using these equations in equation (5.11) results
\[ I_{a1} = \frac{E_{a1}}{(Z_{0eq} + Z_{1eq} + Z_{2eq})} \]  \hspace{1cm} (5.25)
Hence it can be observed that the double line fault and double line to ground fault on phases A and D of six phase transmission line is equivalent to single line to ground fault on phase “a” of the three phase system.

5.4.1.4 Four Line Fault:-

Let the fault be on phases B, C, E and F of the six phase transmission line. The valid relations are

\[ V_B = V_C = V_E = V_F ; \quad I_B + I_C + I_E + I_F = 0 ; \quad I_A = I_D = 0 \]  

Using the inverse transformation gives

\[ V_{a0} = V_{a1} = V_{a2} \quad ; \quad I_{a0} + I_{a1} + I_{a2} = 0 \]  

Using these equations in (5.11) gives

\[ I_{a1} = \frac{E_{a1}}{Z_{1eq.} + (Z_{2eq.}/l) + Z_{0eq.}} \]  

5.4.1.5 Four Line to Ground fault:-

Consider that the phases B, C, E and F are short circuited to ground. The valid relations are

\[ V_B = V_C = V_E = V_F = 0 ; \quad I_A = I_D = 0 \]  

Using the inverse transformation

\[ V_{a0} = V_{a1} = V_{a2} \quad ; \quad I_{a0} + I_{a1} + I_{a2} = 0 \]  

Using these equations in (5.11) results

\[ I_{a1} = \frac{E_{a1}}{Z_{1eq.} + (Z_{2eq.}/l) + Z_{0eq.}} \]
The four line fault and four line to ground fault on phases B, C, E, and F of six phase transmission line is equivalent to double line to ground fault on phases b, c of the three phase system.

5.4.1.6 Six Line fault:-

Let the fault on all phases of the six phase line. The valid relations are

\[ V_A = V_B = V_C = V_D = V_E = V_F \]  
\[ \text{(5.32)} \]

With the aid of inverse transformations

\[ V_{ao} = V_{a1} = V_{a2} = 0 \]  
\[ \text{(5.33)} \]

Therefore the equation for positive sequence current becomes

\[ I_{a1} = \frac{E_{a1}}{Z_{eq.}} \]  
\[ \text{(5.34)} \]

5.4.1.7 Six Line to ground fault:-

The valid relations at the fault location are

\[ V_A = V_B = V_C = V_D = V_E = V_F \]  
\[ \text{(5.35)} \]

Using the inverse transformation on equation (5.35)

\[ V_{ao} = V_{a1} = V_{a2} = 0 \]

Hence

\[ I_{a1} = \frac{E_{a1}}{Z_{eq.}} \]  
\[ \text{(5.36)} \]

It can be observed that the six phase fault and six phase to ground fault is equivalent to three phase to ground fault on a b c of the three phase system.
5.4.2 Fault on Three phase Line and its effect on Six phase System:

5.4.2.1 Using six phase symmetrical components Method:

5.4.2.1 (a) Single Line to ground fault:

Let the single line to ground fault on phase ‘a’ of the three phase system. The valid relations are

\[ V_a = 0 \quad ; \quad I_b = I_c = 0 \]

The sequence component relation of voltages and currents becomes

\[ V_{A1} + V_{A3} + V_{A5} = 0 \quad ; \quad I_{A1} = I_{A3} = I_{A5} \]

Using equation (5.38) in (5.14) yields

\[ I_{A1} = \frac{E_{A1}}{(Z_{1eq} + Z_{2eq} + Z_{5eq})} \]

It can be observed that single line to ground fault on phase ‘a’ of the three phase system is equivalent to double line to ground fault on phases A and D of the six phase line as given in 5.22.

5.4.2.1 (b) Double line to ground fault:

Let the fault be on phases b and c to ground of three phase system. The terminal conditions are

\[ V_b = V_c = 0 \quad ; \quad I_b = 0 \]
Using the inverse transformations

\[ V_{A1} = V_{A3} = V_{AS} \quad ; \quad I_{A1} + I_{A3} + I_{AS} = 0 \]  \hspace{1cm} (5.41)

Substituting equation (5.41) in (5.14) results

\[ I_{A1} = \frac{E_{A1}}{Z_{eq} + (Z_{eq} || Z_{seq})} \]  \hspace{1cm} (5.42)

Hence it observed that the double line to ground fault on phases b and c of three phase system is equivalent to four line to ground fault on phases B, C, E and F of six phase line as given in 5.28.

5.4.2.1(c) Triple Line to ground fault:-

Let fault be on phases a b c to ground. The terminal conditions are

\[ V_a = V_b = V_c \]  \hspace{1cm} (5.43)

Using Inverse transformations

\[ V_{A1} = V_{A3} = V_{AS} = 0 \]  \hspace{1cm} (5.44)

Using the equation (5.14) results

\[ I_{A1} = \frac{E_{A1}}{Z_{eq}} \]  \hspace{1cm} (5.45)

Hence this fault is equivalent to six line to ground fault on six phase line as given 5.36.

5.4.2.2 Using Dual Three Phase Transformation (DTPT) Method:-

5.4.2.2(a) Single Line to ground fault:-

Let the fault be on phase “a” of the three phase system, the valid relations are

\[ V_a = 0 \quad ; \quad I_b = I_c = 0 \]  \hspace{1cm} (5.46)
The sequence components of voltages and currents from equation (5.46) are

\[ I_{a0} = I_{a1} = I_{a2} \quad ; \quad I_{a0}^1 = I_{a1}^1 = I_{a2}^1 \]  
\[ V_{a0} + V_{a1} + V_{a2} = 0 \quad ; \quad V_{a0}^1 + V_{a1}^1 + V_{a2}^1 = 0 \]  

Using equation (5.47) in equation (5.16) gives

\[ I_{a1} = \frac{E_{a1}}{[Z_{1eq} + Z_{2eq} + (Z_{0eq} + Z_{0eq})]} \]  

5.4.2.2(b) Double Line to ground fault:

Let the fault be on phases b c to ground. The terminal conditions are

\[ I_a = 0 \quad ; \quad V_b = V_c = 0 \]  

The sequence components are

\[ I_{a0} + I_{a1} + I_{a2} = 0 \quad ; \quad I_{a0}^1 + I_{a1}^1 + I_{a2}^1 = 0 \]  
\[ V_{a0} = V_{a1} = V_{a2} \quad ; \quad V_{a0}^1 = V_{a1}^1 = V_{a2}^1 \]  

Using the above equation in (5.16) yields,

\[ I_{a1} = \frac{E_{a1}}{Z_{1eq} + [Z_{2eq} | e | (Z_{0eq} - Z_{0eq})]} \]  

5.4.2.2(c) Triple line to ground fault:

Let the fault be on phases a b c to ground. The valid conditions are

\[ V_a = V_b = V_c = 0 \]  

Imposing inverse transformations

\[ V_{a1} = V_{a2} = V_{a0} = 0 \]  

From equation (5.16)

\[ I_{a1} = \frac{E_{a1}}{Z_{1eq}} \]
5.5 SUMMARY:

Assuming that each three phase group of conductors are located on either side of the tower, as will be the case when three phase double circuit line is converted into a six phase line, based on the DTPT analysis of some of typical faults (a) single line to ground, double line, double line to ground on one three phase group (b) 4L, 4LG (2L on each group of three phase group) (c) 6L and 6LG the following conclusions are made.

(i.) AD and ADG faults led to the same results as SLG on phase “a” of the three phase group.

(ii) BCEF (bcb'c') and BCEFG faults led to the same results as “bcg” faults on three phase group.

(iii) 6L and 6LG faults led to the same result as “abc” of three phase group.

The above results are confirmed by the fact that,

(a) a g fault on three phase side led to the same result as double line to ground fault on A D phases on six phase side.

(b) b c g fault on three phase side to ground fault led to the same results as BCEFG fault on six phase side.

(c) a b c fault on three phase side led to the same result as 6LG fault on six phase side.

All the above have been confirmed by six phase symmetrical components analysis also.