CHAPTER-I

INTRODUCTION

1.1.1 INTRODUCTION:

Remarkable development in the system synthesis and design has been noted in the recent past. The synthesis is only a generalized statement of the term and starting from the simple electrical networks to any sophisticated innovation it holds a broad implication. The basic approach in such synthesis problems start with the mathematical modelling of the individual components which, however, may require considerable hortie technique as lots of non-linear elements are likely to be involved.

Sometimes there are physical systems evolving from the natural course of design. Such systems are in general nonlinear. The problem associated with such cases is to derive a sufficiently accurate solution.

But because of inherent nonlinear nature of either the system or the system components, mathematical descriptions can rarely be made adequate in generalized cases and therefore helps of the computers are necessary for evaluating the system characteristics with the required amount of accuracy.

However, reasonably accurate results may still be derived starting from the mathematical description, without requisitioning the services of a sophisticated computer as long as the systems are of low complexity.

It is reasonable to assume that certain basic elements go to build up specific components or system blocks; these blocks, in turn, when properly arranged and cascaded produce a system of desired characteristics. Synthesis of the system blocks and the systems can be attempted with necessary generality and appropriate rigour only when the nomenclatures of the basic elements are given or known. Finally the performance characteristics of the system are evaluated from the solution of certain equations which may be formed as being
the representative mathematical models of the system.

Thus a physical system is initially to be realized and this has in association with it statements regarding the properties of the system blocks and realizability conditions. Then the analysis of the system in the proper perspective is to be made. For an electrical oscillatory system, the synthesis of frequency determining networks is necessary. The analysis of the system is made from the system equations using the usual techniques for a linear system. For a nonlinear system the periodic solution is obtained using the so-called nonlinear mathematics whose application in form and rigour depend entirely on the type and complexity of the system.

1.2.1 NETWORK SYNTHESIS:

Three things are of primary concern in network theory: (a) an excitation (b) a response and (c) a network. Generally two of these are known and the third is to be found out. When the excitation and the response are known the process of solution or finding the appropriate network is what is known as network synthesis. The initial step in the synthesis methods is more commonly termed as approximation problem.

Depending on the alignment of the excitation and the response the synthesis problems are more generally classified in (i) driving point immittance functions synthesis and (ii) transfer function synthesis.

Since the turn of this century till the mid-fifties the bulk of the synthesis works were confined to the linear, lumped, finite, passive and bilateral (LLFPB) networks. These networks are occasionally casually referred to as passive networks because passivity is the main feature of the synthesis. Lots of restrictions were necessary in this synthesis procedure which were described with rigorous mathematical formulation and were termed as realizabilit:
conditions. The basic concise conditions due to O. Brune\textsuperscript{1.1} came as late as 1931 which stated that the driving point immittance functions must have positive real character. This became subsequently the necessary and sufficient conditions for the synthesis. But still there were many formidable problems in the synthesis of LLFPB networks. The contributions made by Cauer\textsuperscript{1.2}, Brune, Bode\textsuperscript{1.3}, Darlington\textsuperscript{1.4}, Guillemin\textsuperscript{1.5}, Dasher\textsuperscript{1.6}, Belevitch\textsuperscript{1.7}, Wienberg\textsuperscript{1.8} and a lot others helped to solve the problems obtaining profound results, and the network theory advanced considerably. In fact the field is so highly developed now that the state of the art can be said to be quite mature. The few unsolved problems remain so because they are not likely to be solved with the reasonable advancement already made.

From the descriptions of the LLFPB, passive network synthesis would mean synthesis including positive resistances, capacitances, inductances and ideal transformers. But the stringent restrictions of such a network made the synthesis little less attractive. Around mid-fifties the solid-state and other active devices were sufficiently developed and network design and synthesis became easier with far fewer restrictions when these devices were incorporated. Passivity naturally found the exit, bilateralness and reciprocity closely followed. With the restrictions gone to a certain extent the process was now termed active network synthesis in contrast to the passive case. In actuality, the LLFPB elements are again used in conjunction with the active and non-reciprocal devices. The general synthesis procedure is, therefore, not very much different except the embedding of one or more active parts which however completely alter the prescribed conditions of synthesis.

The younger group of circuit theorists made concentrated and concerted efforts to formulate the realizability conditions for different types of active synthesis procedures with that mathematical rigour as was done in the case of passive synthesis two decades ago. Consequently new techniques evolved, and a
tremendous amount of information was made available in such a short period that this branch deserved to be considered as a separate branch of study.

1.2.2 ACTIVE NETWORK ELEMENTS:

The active network synthesis has many techniques either in common with or which are mere modifications of those used in passive synthesis. Often times active elements are lumped, linear and finite and sometimes bilateral as well. In the passive network synthesis each circuit element is given a mathematical representation discounting any imperfections inherently present in its physical behaviour. However the parasitic loss effects in the energy storage elements can be considered and the mathematical models suitably readjusted. This method known as the predistortion technique still uses a simplified mathematical model of the elements by considering prescribed insertion loss characteristics. For an element like a negative resistance a reverse predistortion technique is proposed.

Unlike the passive network elements active elements are rather less easily representable by suitable simple mathematical models. Firstly the models so attempted and derived are reasonably accurate within limited ranges only; secondly, the parasitic effects are almost always present. A derived model may work well in alternating current within a narrow frequency band but in direct current and other frequency ranges it may fail entirely. Finally, the stability of an active device is a major problem and unless properly used the model is quite sure to fail the designer. But the final word has not yet been spoken about the development of the active devices and the constant efforts directed in this line might one day lead to realization of these elements as idealized as their mathematical models. Therefore, inspite of all the present day limitations and imperfections mathematical descriptions of the elements are necessarily justified, as, otherwise, the network theorists find no suitable starting point in the synthesis procedure.
Of the few commonly used active network elements negative resistance has been the first to be in application in the network synthesis. Since this element is a power supplying one, its real characteristic must be nonlinear and, therefore, is true only over a limited range of voltages and currents. Depending on the types of nonlinearity the negative resistance characteristic may be said to be voltage controlled or current controlled. The negative resistance is realized in practice from a number of possible sources. The older sources are vacuum tubes, arc discharge tubes, space-charge-grid tubes etc. Then the semiconductor devices came up some of which have negative resistance characteristics. A semiconductor diode made of very highly doped material with a very abrupt junction has an effective negative resistance for a small forward bias. When used as a two terminal negative resistance device by biasing at the centre of the negative slope region, the capacitance of the device is generally not negligible and the equivalent circuit model is therefore a negative resistance in parallel with a positive capacitance. Other semiconductor devices showing negative resistance effects are, double-base diodes made up of one p-n junction and two ohmic contacts; space-charge diodes which have a large depletion layer between the p and n regions. There are in fact numerous semiconductor devices designed to have negative resistance characteristics. Many circuit arrangements are possible with either vacuum tubes or transistors that have the negative resistance effects. Practical oscillator circuits are in general, in part negative resistance circuits. A simple and an early example of a negative resistance circuit using two triodes in series is given in Fruhhauf's circuit.

In 1948 Tellegen came with a new active element called gyrator. Ideal gyrators are impedance invertors. Symbol of a gyrator strongly represents its nonreciprocity and such a device is sometimes referred to as antireciprocal. Gyrators are made with vacuum tubes, transistors, Hall-effect devices etc. and have found wide applications in network synthesis.
By far the most important types of active network elements are controlled sources, negative immittance converters and infinite gain devices. A controlled source is a source whose voltage or current at any part is proportional to either voltage or current in some other part of the network. There are various names for the controlled sources such as ideal amplifiers, transactors, dependent sources etc. There are four basic types of controlled sources, (i) the voltage controlled voltage type (VCVS), (ii) the voltage controlled current type (VCIS), (iii) the current controlled voltage type (ICVS) and (iv) the current controlled current type (ICIS). Each of these sources has a constant associated with it which is usually referred to as the gain of the source. Quite a few electronic devices can be represented by the combination of controlled sources and some circuit elements. Even a gyrator can be realized by the appropriate combination of two controlled sources. Controlled source realization technique is of no less importance and has been discussed in some details in the literature.

The negative immittance converter (NIC) is a two port device that converts an impedance at any port to its negative at the other. The conditions of such a behaviour of the two port were derived in terms of g-parameters. A fundamental classification of practical NIC's is made in terms of their observed behaviour. There are two general classes referred to as (i) the current inversion type and (ii) the voltage inversion type. In recent times NIC has been extensively used in the active circuit synthesis. The realization technique for the NIC's themselves has also received wide attention. Not all circuit schemes that have appeared in the literature are versatile, but quite a few are noteworthy.

Infinite gain devices are not entirely new elements. The controlled sources with their gain parameters taken to infinity are termed as infinite gain devices. The active network synthesis procedure with these devices are similar to the ones using controlled sources.
There are a few other active network elements like the circulators\(^1\), nullators and norators\(^1\) etc. Although the circulator is realizable by using a single gyrator, the nullators and the norators are primarily of a conceptual nature and have, therefore, not received appropriate attention so far.

1.2.3 THE ACTIVE NETWORK SYNTHESIS - PREFACE:

It has already been pointed out that the active synthesis, for all common purposes, merely means synthesis in terms of LLF networks. It is, therefore, important to study the basic properties of such networks. A few basic elements can be arranged to yield an unlimited number of elements which can be used to cover the synthesis of equally numerous LLF networks. It would therefore seem impossible to make a generalized approach to the study of the active synthesis procedure.

The commonly employed elements in the synthesis are positive and negative resistances, capacitances, inductances, gyrators and ideal transformers and the general trend of synthesis with these elements is to realize any specific network function overlooking many practical problems like economy and stability. However attention is to be given when the theoretical realisation needs practical conformity.

The basic properties that are to be satisfied are expressed in terms of the Brune form of the immittance matrix. It was shown\(^1\) that properties that are satisfied by the driving point immittances must also be satisfied by the Brune form of immittance matrix for any class of network.

Such a generalized statement in the line of synthesis barely satisfies the practical need of the specialized classes of networks and attempts have been made for the formulation of simpler and explicit criteria for the more practical types of networks which are worked with by professional groups.
In the course of the development of the network theories, it has been observed that inductance and the ideal transformers are rather unsatisfactory elements as their practical behaviours in the circuit are far away from their assumed approximate mathematical models. Inductance is never completely lossless nor without parasitic capacitance effects. The coreless in high frequency operation mount in large steps. Ideal transformer is all the more less practical. There is never an ideal coupling between the coils and the requisite of the winding inductances to be infinite is an impossible proposition. Furthermore, in the low frequency ranges inductors are likely to acquire unwieldy sizes non-coupled with truncated cost functionals. It is therefore undesirable to include inductances and ideal transformers as elements in the circuit synthesis. This restriction obviously limits the complexity of network functions as long as passive realization is attempted. But by using suitable active elements in addition to passive RC structure the defect can be eliminated.

As an introduction to the subject of active RC circuits, it is pertinent to define several classes of such circuits and discuss their properties. Including the passive RC type the following broad classification of the network can be made:

1. The passive RC class of networks
2. The + R, + C class of networks
3. The RC-gyrator class of networks
4. The + R, + C class of networks
5. The RC-NIC class of networks
6. The RC-controlled source and the infinite gain classes of networks.

A very concise discussion on the properties of these networks is made by specifying the pole-zero locations on the complex frequency plane of both the driving point functions and the transfer functions. Possible locations of stable natural
frequencies for class (1) and (2) are shown in Fig.1.1, which can also represent the possible locations of the zeros of the driving point functions for class (1) networks only. Locations of the zeros of the transfer functions of class (1) networks are shown in Fig.1.2. Locations of the zeros of the driving point functions of class (2) networks are shown in Fig.1.3, whereas the possible locations of the stable natural frequencies of classes (3) and (4) networks and the zeros of the driving point functions of class (3) types are shown in Fig.1.4. Fig.1.5 indicates the possible locations of the zeros of transfer functions of classes (2), (3) and (4) and the zeros of the driving point functions of class (4) networks. Classes (5) and (6) networks are as general as the \pm R, \pm C networks and therefore the possible pole zero locations are similar. Besides, NIC, controlled sources and infinite gain devices being nonreciprocal devices, the classes of networks with these elements also exhibit nonreciprocal behaviour.

The general properties enumerated above only set the guideline for the synthesis procedure. For the desired realization purposes other results are to be specified in terms of the necessary and sufficient conditions.

The basic mathematical approach to the analysis and synthesis of all active RC networks is derived from the concepts of sensitivity and polynomial decomposition. These simultaneously help to develop some criteria by means of which the performance of various types of active RC circuits can be evaluated and compared.

Numerous works relating to active RC synthesis have been published in recent times. Starting from the assessment of the suitability of certain devices as network elements to the discussion on the sophisticated techniques like optimization in the active RC synthesis, the advancement in this line has really been remarkable.
1.2.5 THE ACTIVE RC SYNTHESIS

From the general classification of active RC networks it would appear that \( +R, +C \) networks have to a certain extent restricted application, because, only a limited types of functions can be generated with \( +R, +C \) elements. The realizability criteria of this class were obtained by Kinariwala\(^{1.44}\). A convenient \( -R \) element is the tunnel diode. The realizability conditions of tunnel diode-RC networks have been discussed\(^{1.45}\) but because of the imperfect negative resistance model of the tunnel diode, it is not widely accepted in the active RC synthesis. The popular active elements in this type of synthesis are the controlled sources (inclusive of the infinite gain devices) and the NIC's. Synthesis techniques in terms of the controlled sources have been described by quite a few investigators both for driving point\(^{1.46,1.47}\) and transfer\(^{1.49,1.49}\) functions. The numbers of controlled sources and the RC elements required for the synthesis of different functions are varying and are dependent on the degree of complexity of the network functions. However, the sufficiency of a single controlled source in conjunction with a passive RC network for the realization of an arbitrary driving point or transfer function has also been demonstrated\(^{1.50}\). Synthesis of active RC networks with negative impedance converters received full attention as early as 1954 when Linvill\(^{1.51}\) used an NIC and two RC two-ports to realize any given open circuit transfer impedance function. Yanagisawa\(^{1.52}\) subsequently demonstrated the realization technique of any open circuit voltage ratio that is expressed as a rational function of \( p \). Problems of synthesizing RC two ports whose transmission zeros can be specified independently in the two directions were also tackled with a single negative impedance converter. It was shown that the overall two port is capable of realizing any two of the four short circuit admittance functions simultaneously\(^{1.53}\).
Almost all the synthesis procedures with controlled sources and NIC's require the synthesis of passive RC two or one ports and the circuit arrangements with the active elements are then made as would be appropriate for the realization of the specific functions. Synthesis of passive RC networks should therefore receive careful consideration. Conditions and realizability criteria were found for different types of RC synthesis \cite{1.54,1.55} and application of these realizations in active RC synthesis is quite general.

Besides these semigeneralized synthesis procedure realization of specific functions has gained importance for innumerable practical applications. Design of active filters such as low, high and band-pass structures has been considered with controlled source \cite{1.56}. Realization of RC all-pass functions also is important as these are used in increased proportion as delay networks and in current literature of active RC synthesis a considerable bulk is devoted to its realization. In a subsequent chapter the synthesis technique of RC all pass networks will be considered.

1.3.1 **LINEAR SYSTEMS**

A linear oscillatory system without friction is generally known as harmonic oscillator in the classical literature. It is known to be governed by an equation of the form

$$\ddot{x} + \omega_0^2 x = 0 \quad \ldots(1.1)$$

The characteristic properties of such a system are well known. These quantities characterizing the oscillations are (1) amplitude, (2) frequency and (3) phase. The amplitude and phase of a harmonic oscillatory motion are determined by the initial conditions and frequency is determined by the system parameters only.
In the presence of the friction, the system may be either damped oscillatory or damped aperiodic depending on the relation between the system parameters. Analyses of all the nature of a linear system have been made in classical mechanics and have been successfully applied to mechanical, electrical and other systems of interest.

Earliest electrical frequency generating devices like alternators and spark gap generators have now been replaced with more simple, stable and versatile electronic oscillators. Development of active devices like vacuum tubes, transistors and the like coupled with the continued efforts in the line of network theory has practically revolutionised the art of frequency generating techniques.

Oscillation in a resonant circuit having no resistance may serve as an example of electrical oscillation occurring in a linear conservative system. The period of oscillation is determined from the energy relations in such a system. Naturally the system should have an inductance and a capacitance in the circuit. A wide range of passive circuits are capable of generating sine wave oscillations in a system containing a real gain amplifying device. The condition of oscillation has also been enunciated from the principle of feedback and is written as

\[ A \cdot B = 1 \]  \hspace{1cm} \text{{(1.2)}}

where, 
\[ A = \text{gain of the amplifier, and} \]
\[ B = \text{amount of back-coupling} \]

With this criterion satisfied and the Hamilton's equations made valid (which however, are rarely demonstrated in the analysis) the systems under study are mostly seen to be conservative in nature and the system equations are generally derived to be, of the form of (1.1). Analysis of the linear systems of oscillations can be made with the help of conventional linear techniques. The added advantage is that many nonlinear techniques can be applied with appropriate limiting conditions to derive the solutions.
1.3.2 THE SINE WAVE GENERATORS

Active devices coupled with passive networks are capable of generating sine wave oscillations with the appropriate conditions specified. The passive circuit parameters are generally the determining factors of the system frequency.

Low frequency sine wave generation is inconvenient with an L-C network because of their large values. RC networks are more advantageous in that respect. An oscillator with a 3-section CR phase-shifter network and a vacuum tube was described by Ginzton and Hollingsworth. The transistorized version of this circuit is due to Nichols. Continued efforts directed in this line have produced different RC oscillators with special types of networks like Wien-bridge, Bridged-T, Twin-T etc.

The main consideration in sine wave generation is to obtain variable frequency with good frequency and amplitude stability all over the range. The requirements are rarely met by an individual proposed circuit. Frequency variation is normally made by the variation of a set of RC elements and generally at low frequencies the stability is not as expected.

Active RC filters may be convenient in so far as the choice of the frequency selective networks are considered. The basic limitations are conceivable in terms of the drifts in the active parts. But it is not unusual to find that the active elements' drift with time and temperature is no worse than that of a passive element. Variation of the active element function (such as the gain of a transactor) may be used with advantage in a variable frequency oscillator. The basic move is to derive all the benefits of the active filters, although, such a filter in the system leads to increased complexity in the circuit analysis as a whole.

Irrespective of the types of the filter circuits used, the derivation of the equations, pertinent to the analysis of harmonic oscillations, is made,
following any suitable technique dependent on the system alone. In the feedback oscillators, the frequency and starting conditions are often determined from the conventional loop or nodal analysis of the circuit. Matrix technique is quite helpful where the above methods lead to chores of algebraic manipulations or when a generalized approach with the two-port concept is desired.63

1.4.1 NONLINEAR SYSTEMS

Investigations of physical problems start with the idealization of the system properties. A very important and effective method of idealization is linearization. Depending on the point of interest some approximations are required to be introduced for this purpose. These assumptions, in general, lead only to approximate analysis and solution of the problems involving actual physical system, but, from the practical point of view this is much no less welcome. The neglected elements in the process contribute to the distortions which can be evaluated only if an exact analysis is found possible.

Majority of the problems relating to real physical systems are possibly investigated mathematically involving solution of differential equations ordinary linear and nonlinear and partial. Even the idealized physical processes are rarely solvable by simple ordinary linear differential equations. Consequently the equations are themselves quite involved and are oftentimes solved with intricate techniques like solution of nonlinear algebraic or nonalgebraics or are approximated by piecewise linearized model and the derived linear equations are solved with appropriate boundary conditions. Investigation of the autonomous systems reveal that a finite non-zero solution of the linearized equations representing these systems at the equilibrium state can hardly be obtained at any arbitrary time. Also, linearized equations of nonautonomous systems, in general, do not yield the desired results.
General solution of the nonlinear systems should always be concerned with the stability problems. For that reason linearization is almost essential with certain assumptions, as otherwise, the intricacy does not permit to judge the stability problems with the required amount of rigor.

For the solution of nonlinear problems, the mathematics involved is, in the present day state of the art, referred to as "nonlinear mathematics". Procedure in the analytical treatment of the nonlinear problems is hindered due to insufficient mathematical development in this line. The engineers however have the complementing tools in computers for the analysis of the dynamic behaviour of the physical systems. In fact, even the highly complicated nonlinear problems which occur in practice, have been successfully solved when the analytical approach is coupled with the aid of computers.

It appears that the nonlinear oscillatory systems are describable by autonomous differential equations. Certain basic concepts like phase plane technique etc. in nonlinear mathematics help to obtain a time domain behaviour of the physical systems with appropriate stress on what is known as system stability.

1.4.2 THE PERIODIC SOLUTION:

Attempting to solve the autonomous and nonautonomous oscillatory systems the primary interest grows around the steady state x\text{\textit{m}} oscillations - existence of a periodic solution, ensuring the stability. Poincare\textsuperscript{1.64,1.65} was the first to present a suitable analytical approach for obtaining the periodic solution. The basic principle is the parameter (\mu) - dependence of the certain terms of non-autonomous differential equation. The major difficulty in the Poincare method is in finding the characteristic exponents required for the solution. Minorsky's stroboscopic method\textsuperscript{1.66,1.67} solves this difficulty by replacing the original non-autonomous differential equation by another autonomous
one such that the existence and the stability of its singular points is a
criterion of the existence and stability of the periodic solution of the original
problem. In Poincare's method variation of the parameter, $\mu$, is limited within
a short range of small values and the method of solution is suitable only for
quasilinear systems, the periodic solution of which is close to the harmonic
solution. Physical interpretation of the method is due to Lienard$^{1.68}$ who drew
out the correlation between the existence of a periodic solution and a physical
state of the system basing on conservation of energy principle. The classic
van der Pol$^{1.69}$ equation describes many of the operating features of the
oscillator. The equation is quite simple but the small parameter approximation
method is not applicable for a large value of the parameter $\mu$ associated with
the first derivative term. A study of van der Pol equation for both small and
large values of $\mu$ has led to the appropriate understanding of the operation
of self-oscillations. Van der Pol's original method of solution was graphical
and was presented in the form of an integral curve in the phase plane. The
latter efforts for an analytical solution$^{1.70,1.71}$ of the equation acquired
little significance although it was demonstrated that for larger and larger
values of $\mu$ quasilinear systems approach relaxation modes. Subsequent works$^{1.72}$
for a quantitative study of nonlinear oscillations evolved mostly from this
source but were not very successful. The method of asymptotic expansions$^{1.73}$
for example, require some knowledge of the integral curve for proper solution.
Methods of obtaining a periodic solution of the oscillatory processes assuming
fictitious discontinuity existing in the system found ground because such an
assumption could explain the dynamic characteristics of many a physical systems
with success. The first modern approach in this direction made by Mandel'shtam$^{1.7}$
consists of transforming the differential equation of the system to the equation
of an integral curve over the phase plane. When a periodic solution exists
the closed integral curve is found to consist of four segments, two of them
representing slow motions and the other two very rapid movements. Each type is however alternated by the other. The slow motions are actually guided by the differential equation while over the segments representing the rapid movements the differential equation ceases to govern the system and a discontinuous 'jump' is said to occur governed by a physical invariant. This principle of physical invariance retains a physical continuity in the solution of the system where the analytical discontinuity is presupposed. When the discontinuous theory does not explain the system behaviour with a 'strong' nonlinearity still prevalent, the best method of obtaining a periodic solution is the point transformation method. The foundation of this method was laid down quite early in but a rigorous mathematical formulation has been developed only recently. In this method the periodic solution is sought to obtain on the basis of specially defined point transformation which is transformation into itself. Here also, a closed integral curve, in the phase plane or phase space, finally demonstrates the existence of a periodic solution. The method can be applied, in principle, to the entire general systems of nonlinear differential equations.

1.4.3 STUDY OF THE NONLINEAR OSCILLATORY SYSTEMS IN THE PHASE PLANE:

From what has already been said it would appear that the solution of autonomous differential equations representing nonlinear oscillatory systems is better performed in the phase plane or phase space using the discontinuous theory or the point transformation method. The approach of the phase plane technique may be demonstrated starting with a system differential equation with the lowest degree of freedom such that it is still capable of describing an oscillatory system. The degree of freedom in that case is one. A second order differential equation of a system with lumped parameters can be considered. It should be realized that for higher order systems concept of phase space is more appropriate but extensive work is yet to be oriented in this generalization.
The basic starting equation in the linearized autonomous form may be written as

$$x + 2hx + \omega_0^2 x = 0 \quad \ldots (1.3)$$

where $x$ is the fundamental variable which in the mechanical analogue represents displacement and, therefore, $\ddot{x}$ and $\dot{x}$ are velocity and acceleration respectively. $2h$ denotes the ratio of friction coefficient, $B$, to mass, $M$, and $\omega_0^2$ is the ratio of restraint constant, $K$, to mass. In the electrical case the nomenclatures are only changed whereas the form of the equation remains unaltered. The equation of integral curve is derived from equation (1.3) as

$$\frac{\dot{x}}{x} = \frac{-2hx - \omega_0^2 x}{x} \quad \ldots (1.4)$$

Thus the solution, the trajectory and the vector field can be investigated in the $(x, \dot{x})$-plane, known as the phase plane of the corresponding system of differential equation.

Although the equation (1.4) is quite amenable to investigation of the physical system, dynamics of the system are not properly explainable all over the trajectory. These are the points where the system equation (1.3) ceases to govern the motion of the system representative point. At these points the discontinuous theory is applied and the continuity in the trajectory is maintained till equation (1.4) again guides the motion in the phase plane. In all physical systems there cannot be an abrupt change in the energy as it is physically impossible. Andronov's approach to such a system is quite interesting. In his method the equation to be investigated is converted to the one with a degree of freedom one-half by what is known as the principle of degeneracy. A degenerate differential equation has the coefficient of the highest order derivative small compared with the coefficients of the other terms. Thus a differential equation of degree of freedom one easily transforms to a first order equation as long as the coefficients of $\dot{x}$ can be assumed zero. A 'jump' condition is added to the first order equation which explains the alternate slow motions due to
the effectiveness of the first order model of the system. The 'jump' postulate is made to be compatible with the system conditions and is based normally on the energy consideration. The 'jumps' occur where discontinuity with reference to the governing equation appears. These points are critical points. In some systems, instead of isolated critical points there are certain critical lines. The discontinuous theory of Mandel'shtam and Chaikin exposes that at such critical lines a discontinuous stretch begins ending at a point where the analyticity is recovered.

The point transformation method also is one of the methods of finding the limit cycles and determining their stability. It consists in deriving the law of a certain point transformation and evaluating the corresponding sequence function. Poincaré first introduced the concept of sequence function. If in the phase plane a line 'ab' (Fig.1.6) through non-singular points is drawn such that the phase paths of the system under investigation intersect it without being tangent to it, such a line is known as the 'arc without contact'. P is the phase path passing through q at time $t = t_0$. At time $t > t_0$, let $\bar{t}$ be the first value of $t$ when P intersects 'ab' and $\bar{q}$ be the point of intersection. The point q is then said to possess a consecutive point $\bar{q}$ on 'ab'. $s$ and $\bar{s}$ are the distances of q and $\bar{q}$ from a. The sequence function, the law of a certain point transformation, is derived from the theorem of continuity of the dependence of the initial conditions and is written as

$$\bar{s} = f(s) \ldots (1.5)$$

If a certain value $s = s_0$ corresponds to a closed path then $s = f(s_0) = s_0$ and q and its consecutive point $\bar{q}$ are coincident. Such points are known as fixed points. If the sequence function of a certain arc 'ab' is known the fixed point $s_f$ and the limit cycles are found by solving

$$f(s_f) = s_f \ldots (1.6)$$
FIG. 1-6. POINCARE'S SEQUENCE FUNCTION
The stability of the limit cycle is adjudged from Koenig's theorem which states that the fixed points \( s \) of a point transformation \( s = f(s) \) is stable if
\[
\left| \frac{ds}{ds} \right|_{s=s_f} < 1 \quad \ldots (1.7)
\]
and unstable if
\[
\left| \frac{ds}{ds} \right|_{s=s_f} > 1 \quad \ldots (1.8)
\]

It should be stressed that the system differential equations may be nonlinear and piecewise linear approximations are used.

1.4.4 EXTENSION TO HIGHER ORDER SYSTEMS

Analysis of quasilinear and the discontinuous oscillations for the periodic solution is extendable for higher orders, when \( n \)-dimensional phase space should be considered instead of a simple phase plane. The complexity of the mathematical model also increases. The parameters that are consequential at different stages determine the complexity in the analysis and, are, therefore, restricted to the minimum in number such that a convenient qualitative and quantitative approach following the line of Mandel'shtam and Chaikin can be made for the required solution.

The point transformation method is extended for the higher order system by dividing the phase space in hyperplanes of contact. The solution is represented by the fixed point of the point transformation into itself of a certain hyperplane, chosen as the initial hyperplane of contact which belongs

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\* A set of points, the coordinates of which satisfy one of the equations:

\[
\frac{1}{n} \sum_{k=1}^{n} \alpha_k x(k) = \beta(i)
\]

is called the hyperplane of contact.
to a certain closed convex polyhedron*. The polyhedrons are part of an n-dimensional Euclidean space which has the significance of a phase space 1.80. It should, however, be emphasized that when the system order is high, the complexity is also quite large and it deserves special consideration when the method is to be applied.

1.4.5 PRACTICAL ASPECTS OF RELAXATION OSCILLATIONS:

Relaxation oscillations differ considerably from sinusoidal or quasisinusoidal oscillations. Relaxation oscillations are rich in harmonics which can attain very high orders.

In practical relaxation oscillators, effectively there is a single reactive element capable of storing energy. Thus energy can be stored only in one part of the period. In the other part this energy is dissipated in a resistance. Naturally the exciting device should supply an amount of energy equal to that stored or dissipated in the period. In an autonomous system, the exciting device is formed in the system which in the part required for supplying the energy becomes a negative resistance device or a generator 1.81, 1.82, 1.83.

* A set S which belongs to a vector space \( R \) is said to be convex if each two of its points can be connected by a line lying in S.

A closed convex polyhedron P in \( R \) of dimension \( r \) consists of the convex set P of points that satisfy a system of inequalities

\[
\sum_{k=1}^{r} a_k (1) x(k) \leq \varepsilon(1)
\]

where \( a_k (1) \) and \( \varepsilon(1) \) are certain constants.
The presence of the negative resistance in the circuit of the system may be made apparent in two general ways. First an actual negative resistance is used for the circuit synthesis. Secondly a feedback is allowed in the system in such a way as to produce an effective negative resistance in the system during the part energy is to be supplied. Negative resistance oscillators are numerous in operation consisting of transitron, dynatron, glow discharge, tunnel diode etc. Basic multivibrator is a simple relaxation oscillator of the feedback type. The other commonly known feedback type is the blocking oscillator. It was due to the study of van der Pol that a method of obtaining a continuous transition from the sinusoidal to relaxation oscillation is known. This follows from the nonlinear equation when the coefficient $\mu$ gradually becomes larger and larger. Physically this is interpreted as in an LRC circuit one of the reactive elements is gradually reduced to zero, and the system being beyond the limit state, the oscillation that occurs may be considered as kind of jumping from one state of equilibrium to another.

There is no practical limit to the design of the relaxation oscillator circuits. The older existing and often employed circuits are also improved upon some with reference to the rise and fall times by eliminating the integrating effects of the timing capacitors. The period of oscillation in such circuits is generally controlled by the alteration of the passive elements which simultaneously affect the performance of the multivibrator. Synthesis of multivibrator circuits where time periods can be altered by varying a single circuit parameter other than the formal frequency obtaining network elements without affecting the operating characteristics of the generator can be an welcome addition to the list of the range of the multivibrators.

1.5.1 SCOPE OF THE WORK

Although the theoretical studies of oscillatory systems - both
qualitative and quantitative—start from the formulation of the mathematical models in the form of the differential equations, systems of general practical interest are synthesized in terms of the available requisites and techniques.

It is already known that a quasilinear and relaxation oscillations occur in a system with a definite nonlinearity. For harmonic oscillation a nonlinearity is not essential, but from the point of view of amplitude and frequency stabilization, inclusion of an appropriate nonlinear element in the system is not precluded. The conventional oscillatory circuit consists, besides a suitable nonlinear element, essentially of a passive filter and an amplifier, adequate regenerative feedback providing the condition of sustained oscillations.

Variation of frequencies in such cases is obtainable by the variation of the time constant of the passive filter circuit elements and the wave shape is altered by the adjustment of a reactive element in the filter.

The present study shows that active RC filters conveniently replace passive filters specially for low frequency generation. Naturally, the synthesis of these filters is important and is done so as to make them suitable for the desired system design. When such filters are used change of the oscillation period of the system may be made by adjusting only the system gain or some such parameter—indeed independent of the passive RC elements. This adds an advantage to the system that the frequency range can be enlarged when RC elements are simultaneously variable.

The work starts with a few methods of active RC all-pass filter synthesis presented in Chapter II. Two methods of realization of the second order filter—one with controlled sources and another with an NIC—are described indicating advantageous phases of the realization. Two other methods both using controlled sources and two port RC networks, are generalized for the n-th order all pass function synthesis. One of these procedures uses the root
locus technique which probably is quite novel in the RC all-pass synthesis. Literature abounds in other active filter synthesis techniques.\cite{1.89,1.90,1.28}

Active low and high pass filters are used in a system with a linear amplifier to generate sine wave oscillations. Chapter III presents this method mentioning its importance and advantages.

Both sinusoidal and nonlinear oscillations occur in an active bandpass filter. Reversion of the phases of the active parts in the filter and the system amplifier determines the frequency spans obtainable in such oscillatory systems. A detailed study of this is made in Chapter IV.

Chapter V demonstrates that stable sine wave, quasilinear and relaxation oscillations are obtainable in RC all-pass networks. Besides a first or a second order all-pass filter the system consists of a linear amplifier and a clipper all cascaded and complete in a feedback loop. Filter orders higher than two are not considered, for, the inclusion of these does not materially improve the performance of the system, on the contrary, brings in extra complexity.

First order all-pass filter cascaded with either an integrator or a differentiator are seen to produce both continuous and discontinuous oscillations, when a system is designed to include a saturating element. Chapter VI is intended to make studies of these systems on the phase plane and demonstrate the performance of such systems.

Specified transfer function synthesis is presented in Chapter II and the application of such filters in oscillatory systems is described in Chapter V and VI. In Chapter VII an inductive driving point impedance synthesis is illustrated with a negative immittance converter and a couple of RC elements. It has further been shown that by appropriate adjustments of the RC elements the Q factor of the inductor can be increased to infinity so that a sine wave oscillator can be constructed with this simulated element when a capacitor is connected across it.

Analysing with the help of the method of chain matrices, certain NIC imperfections can also be compensated for.
Two special type RC oscillatory systems are described in Chapter VIII. The first one is a sinusoidal oscillator consisting of two idealized transactors, an RC differentiator and a special RC network. Very low frequency sine wave generation is possible by this method without much of the constraints required in other conventional designs. The second one consists of the modified form of the Wien network in a transistorized circuit. This circuit is conveniently used both as an amplifier and an oscillator whose frequency of oscillation is nearly determined by the resonance frequency of the filter. Transmission matrices and circuit determinants are used for the analysis of these systems.

Finally in Chapter IX concluding remarks with necessary critical discussions are made.
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