7.1 INTRODUCTION

In Chapter II, RC synthesis with active elements like controlled sources and negative immitance converters have been presented for specified transfer characteristics and the use of such filters in oscillatory systems has been demonstrated subsequently in Chapter V and VI. A circuit consisting of a negative immittance converter (NIC) and a couple of RC elements suitable for the realization of a specified driving point impedance is described in this chapter. The impedance is inductive and hence a capacitor connected across this element can produce sine wave oscillation provided the Q-factor of the inductor can be enhanced by simple adjustments in the circuit parameters.

Quite a few interesting inductor realization methods with RC elements are already known. In most of these cases the Q-factor of the simulated inductor is possible to be enhanced without making it infinitely large. The scope of application of the realized element is consequently restricted. In the present chapter a method of inductor realization is described where Q-peaking with frequency is obtained as usual and with the variation of the passive parameters the Q of the inductor can be made infinity at specified frequencies.

The idealized inductor thus obtained is effectively used to form a sine wave oscillator whose frequency of oscillation is determined by its circuit conditions. The frequency of the sinusoidal oscillator can, however, be varied by varying the circuit elements maintaining the condition of oscillation.

Analysis of the oscillator circuit is presented both from the tuned circuit concept and with the help of the method of chain matrices. The latter method has the advantage of taking into consideration of the gain of the NIC.
FIG. 7-1 THE SCHEMATIC CIRCUIT OF THE INDUCTOR SIMULATION

FIG. 7-2 FREQUENCY VERSUS $L_{\text{IN}}, R_{\text{IN}}$ AND $Q$ PLOTS
(EXPERIMENTAL POINTS: X)
Results of the derived analysis are shown in various graphs along with the experimental findings.

7.2 INDUCTOR SIMULATION

7.2.1 Inductor and its frequency dependence

The circuit considered for analysis with an ideal NIC is shown by a block with its open circuit stable (ocs) and the short circuit stable (scs) sides marked (fig. 7.1). For the associated passive elements the input impedance

\[ Z_{in} = \frac{R' - R + j\omega C R^2 R'}{1 + \omega^2 C^2 R^2} \quad \ldots(7.1) \]

is effectively inductive with an inductance

\[ L_{in} = \frac{CR^2}{1 + \omega^2 C^2 R^2} \quad \ldots(7.2) \]

and its associated resistance

\[ R_{in} = \frac{R' - R + j\omega C R^2 R'}{1 + \omega^2 C^2 R^2} \quad \ldots(7.3) \]

Both the inductance and the resistance of the inductor are frequency dependent, and the Q-factor is given by

\[ Q = \frac{\omega C R^2}{R' - R + \omega^2 C^2 R^2 R'} \quad \ldots(7.4) \]

For \( R' = 10 \, \text{k} \, \text{ohm}, R = 9.4 \, \text{k} \, \text{ohm} \) and \( C = 0.1 \, \text{uF} \), the plots of \( L_{in}, R_{in} \) and \( Q \) for various values of \( \frac{\omega}{2\pi} \) are shown in fig. 7.2. The experimental values are shown by crossmarks in the same figure.

The maximum value of \( Q \) (eqn. (7.4)) occurs at a frequency \( \frac{\omega}{2\pi} \), where

\[ \omega_0 = \frac{1}{CR} \sqrt{\frac{R' - R}{R'}} \quad \ldots(7.5) \]

and

\[ Q_{max} = \frac{R}{R' + \frac{1}{R R^{\prime}}} \quad \ldots(7.6) \]
Both $\omega_0$ and $Q_{\text{max}}$ can be selected by appropriate choice of the passive parameters while infinite value of $Q_{\text{max}}$ is realized only at zero frequency. Since this is of no special practical consequence a more reasonable approach to realize an ideal inductor ($Q = \infty$) at specific frequencies would be to adjust passive elements like $R$, $R'$ and $C$, either independently or simultaneously. From the classical definition, however, the $\omega$-sensitivity of $Q$ is given as

$$S^Q_\omega = \frac{R' - R - \omega^2 C^2 R^2 R'}{(R' - R + \omega^2 C^2 R^2 R')} \quad \ldots(7.7)$$

This relation clearly specifies the performance characteristics of the inductor with respect to frequency.

### 7.2.2 Infinite $Q$-factor and other aspects of the inductor

It has already been noted that an ideal inductor at various frequencies is realized by varying the passive circuit elements. From eqn.(7.4) it is seen that $Q$ is $\infty$ when

$$R' - R + \omega^2 C^2 R^2 R' = 0 \quad \ldots(7.8)$$

For constant values of $R'$ and $C$, $\omega$ and $R$ are simply related either as

$$R = \frac{1 \pm \sqrt{(1 - 4 \omega^2 C^2 R^2)}}{2 \omega^2 C^2 R'} \quad \ldots(7.9)$$

or as

$$\omega = \frac{1}{GR} \sqrt{\frac{R - R'}{R'}} \quad \ldots(7.10)$$

Equation (7.10) shows an unique relation between $\omega$ and $R$ for positive values of $\omega$. Equation (7.9), however, shows that there may be two values of $R$ for each frequency at which $Q$ is infinity. This is obvious from eqn.(7.8) which is quadratic in $R$. These two values of $R$ become identical when
FIG. 7-3  $R-Q$ CURVES FOR
$\omega_1 = 400 \ \text{r/s}$
$\omega_2 = 500 \ \text{r/s}$
$\omega_3 = 600 \ \text{r/s}$
\[ \omega = \frac{1}{2CR} \]  

and for \( \omega > \frac{1}{2CR} \), eqn. (7.9) becomes incompatible as \( R \) assumes a complex value.

\[ Q - R \text{ curves for three different values of } \omega \]

- \( \omega_1 < \frac{1}{2CR} \)
- \( \omega_2 = \frac{1}{2CR} \)
- \( \omega_3 > \frac{1}{2CR} \)

are shown in fig. 7.3. From the curves it is interesting to note that for 
\( \omega_1 < \frac{1}{2CR} \), the portion of the curve between the two values of \( R \) for which \( Q \) is infinity lies in the negative \( Q \) region. This is due to the negative values of associated resistance, \( R_{in} \), in this range, as can be seen from the \( R_{in} - R \) curves drawn in fig. 7.4 for the four different frequencies. Two curves are drawn for 
\( \omega < \frac{1}{2CR} \) one for \( \omega = \frac{1}{2CR} \) and one for \( \omega > \frac{1}{2CR} \). For 
\( \omega = \frac{1}{2CR} \), \( R_{in} \) decreases with increase in \( R \), becoming zero for a critical value of \( R \) and then again increases without being negative. Correspondingly the \( Q \)-factor, at this frequency, increases with \( R \), becoming infinity for the critical value after which it gradually falls down all the time remaining positive. This is shown in fig. 7.3 for \( \omega = \omega_2 \). This is true as the variation in \( L_{in} \) with \( R \) for the normal range of frequencies is never discontinuous. The correlation between the sets of curves in fig. 7.3 and fig. 7.4 are thus easily found.

It needs stressing however that the choice of the passive elements especially of \( R \) and \( R' \) is to be made judiciously such that for the required driving point function realization the stability of the system is not impaired. The specific criteria for these conditions are not well laid down, but the qualitative aspects are discussed in the literature. As the terminating resistance \( R \), in the ssc port is gradually increased the system tends to be...
**FIG. 7-4**  **R - R\text{IN} CURVES FOR**
\[ \omega = 300 \text{ r/s}, 400 \text{ r/s}, 500 \text{ r/s}, \text{ AND } 600 \text{ r/s} \]

**FIG. 7-5**  **R - R\text{IN} AND R - L\text{IN} CURVES FOR**
\[ f = 41.5 \text{ Hz}, 100 \text{ Hz}, \text{ AND } 1000 \text{ Hz} \]

(EXPERIMENTAL POINTS: \(x\))
unstable. For the system to form an oscillator, this provides no serious consequence, although for general stable operation this instability is to be checked strictly. The $R_{in} - R$ and the $L_{in} - R$ curves for three different frequencies are shown in fig. 7.5 with the experimental points marked. It is clear from the graphs that with the increase in frequency, slopes of these curves decrease. The realized inductor is therefore essentially suitable for low frequency applications.

7.3 SINEUSOIDAL OSCILLATIONS

7.3.1 Tuned circuit method

If the Q of an inductor is large at a particular frequency without being infinity, generally such an inductor is suitable for developing a frequency selective amplifier. If the Q-factor is infinitely large the circuit is well adopted for an oscillator.

It has been shown in the foregoing analysis that an ideal inductor can be realized by adjusting the passive elements. Therefore if across the terminals $a$ of fig. 7.1, a capacitor of a suitable value is connected, sinusoidal oscillation occurs in the system. However, since the realized inductor is ideal only at a frequency dependent on the passive circuit elements, the choice of the capacitor is dependent and can be found from the relations

$$R' - R + \omega^2 C^2 R^2 R' = 0 \quad \cdots(7.12)$$

$$L_{in} = \frac{C R^2}{1 + \omega^2 C R^2} \quad \cdots(7.13)$$

and

$$\omega = \frac{1}{\sqrt{L_{in} C'}} \quad \cdots(7.14)$$
where \( C^* \) denotes the capacitance required for the sine-wave oscillation.

Combining eqns. (7.12), (7.13) and (7.14) on obtains

\[
C^* = \frac{C}{1 - \frac{R^*}{R}} \quad \ldots (7.15)
\]

Equation (7.15) indicates that for a sine-wave oscillation to occur in the system, the passive circuit elements are uniquely related. It is observed from figs. 7.3 and 7.4 that oscillation is possible when \( R_{in} \) (eqn. (7.3)) is zero. However, there are two values of \( R \) when \( Q \) is infinity for the same frequency and in between them \( Q \) is negative implying the possibility of occurrence of nonlinear oscillations. However, the higher value of \( R \) is not acceptable because of practical limitations. When both \( R_{in} = 0 \) and \( \frac{dR_{in}}{dR} = 0 \), \( Q \) is infinite, only at a single value of \( R \) and is never negative. From the condition \( \frac{dR_{in}}{dR} = 0 \), \( \omega \) is obtained as

\[
\omega = \frac{1}{CR} \quad \ldots (7.16)
\]

Using this value of \( \omega \) in the condition \( R_{in} = 0 \), one derives

\[
R = 2R^* \quad \ldots (7.17)
\]

Expressing \( R \) as

\[
R = KR^* \quad \ldots (7.18)
\]

where nominally \( K = 2 \), and using relation (7.16), \( Q \) can be written as

\[
Q = \frac{2K^2}{(K-2)^2} \quad \ldots (7.19)
\]

A plot of \( Q \) versus \( K \) around the nominal value of \( K = 2 \) is shown in fig. 7.6.

Fig. 7.7 shows the \( \frac{dQ}{dR} - K \) curve illustrating \( R \) selectivity of the \( Q \) factor.

Equations (7.15) and (7.17) also imply that \( C \) and \( C^* \) are related as

\[
C^* = 2C \quad \ldots (7.20)
\]
Hence the oscillation frequency is written as

$$\omega = \frac{1}{CR} = \frac{1}{C^* R^*} = \frac{1}{\sqrt{(CC'RR')}} \quad \ldots(7.21)$$

### 7.3.2 The chain matrix approach

- Conditions of oscillation derived for $R_{in} = 0$ and $\frac{dR_{in}}{dR} = 0$ is only a particular instance of the general case obtained for $R_{in} = 0$ alone, as can be seen from eqns. (7.9) and (7.10) as also from eqns. (7.15), (7.18) and (7.20).

The general condition is more simply derived from the chain matrix analysis of the circuit. Fig. 7.8 shows the oscillator circuit with the ideal NIC represented by its chain matrix.

Writing

$$\begin{align*} 
Y' &= pC' \\
Z' &= R' \\
Y &= (pCR + 1)/R 
\end{align*} \quad \ldots(7.22)$$

the transmission matrix of the overall system is given by

$$\begin{bmatrix} a_T \end{bmatrix} = \begin{bmatrix} 1 & Z' \\ Y' & 1+Y'Z' \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \quad \ldots(7.23)$$

Therefore,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 - YZ' & -Z' \\ Y' - Y(1+Y'Z') & -(1 + Y'Z') \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \ldots(7.24)$$

Hence the transmission equations are written as

$$V_1 = (1 - YZ')V_2 + Z'I_2 \quad \ldots(7.25a)$$
Figure 7.9
(a) and (b) C' versus frequency curves (experimental points: X)
and

\[ I_1 = \left[ Y' - Y(1 + Y'Z') \right] V_2 + (1 + Y'Z')I_2 \]  \tag{7.25b}

If now the system is oscillatory and autonomous, \( I_1 = I_2 = 0 \), then from eqn. (7.25b), as \( V_2 \neq 0 \)

\[ Y' = Y(1 + Y'Z') \]  \tag{7.26}

Combining eqns. (7.22) and (7.26) and simplifying, the following relations for the frequency and the condition of oscillation are derived

\[ \omega = \frac{1}{\sqrt{(C'CC')}} \]  \tag{7.27}

and

\[ \frac{C'}{C} = \frac{1}{1 - \frac{R'}{R}} \]  \tag{7.28}

For an NIC with a gain parameter \( \beta \), the converter chain matrix is given by

\[
\left[ \begin{array}{c}
\beta_{\text{NIC}} \\
\end{array} \right] = \left[ \begin{array}{cccc}
1 & 0 \\
0 & -1 / \beta \\
\end{array} \right]
\]

so that the condition (7.28) is modified to

\[ \frac{C'}{C} = \frac{1}{\beta - \frac{R'}{R}} \]  \tag{7.29}

Whereas the frequency of oscillation remains same. Thus the chain matrix method has the advantage that it accounts for the nonunity gain of the NIC.

### 7.3.3 Experimental Results

The inductor simulation results are shown in figs. 7.2 and 7.5. A dc-coupled version of an INIC as proposed by Franklin was used with necessary compensation. Figs. 7.9(a) and (b) show the frequency - C' graphs when conditions (7.18) and (7.20) are satisfied. The experimental points are
Fig. 7-10 Verification of relation

\[ \frac{c'}{c} = \frac{1}{1 - \frac{R'}{R}} \]

(experimental points: X)

Fig. 7-11 \( \frac{c'}{c} \)-frequency curve
(experimental points: X)
also shown on the curves. Verification of the relation (7.28) is shown in fig.7.10 and the oscillation frequencies for different \( \frac{C_1}{C} \) values are shown in fig.7.11 with the experimental points duly marked. All the experimental values are seen to be in good agreement with the theoretical analysis.

7.4 REMARKS

The inductor realization method described is simple and straightforward and can easily be adopted to microcircuit techniques especially when the simulated element is not required for an oscillator. The oscillator circuit formed with the inductor does not require any amplifier and the conditions of oscillations are quite simple to be implemented. The frequency of oscillation is variable over a wide range by the variation of a set of ganged elements. It must be noted that for an inappropriate choice of the element values, distorted sine wave oscillations result. However a careful consideration can easily annul such situations.
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* Material in Chapter VII is based mainly upon the results given in these publications.