Chapter-4

Almost $S^*_g$-Continuous Functions In Topological Spaces

4.1 Introduction

Almost continuous functions in topological spaces was defined and instigated by authors such as Singal M.K and Singal Asha Rani[65], Hussain[29], Long and Carnahan[38], Noiri.T[39]. They characterized the results of almost continuity. The purpose of this chapter is to introduce almost $S^*_g$-continuous functions, almost contra $S^*_g$-continuous functions and completely $S^*_g$-irresolute functions. We connect almost continuous function with almost $S^*_g$-continuous functions and establish their relationship. Also we relate $S^*_g$-continuous functions and almost $S^*_g$-continuous functions. The basic properties of almost contra $S^*_g$-continuous functions and completely $S^*_g$-irresolute functions are also discussed. Further the compositions of various functions are verified.

4.2 Preliminaries

Definition 4.2.1: A subset $A$ of a space $(X, \tau)$ is called a regular open set [66] if $A = \text{Int}(\text{Cl}(A))$ and a regular closed set if $A = \text{Cl}(\text{Int}(A))$.

Definition 4.2.2: A function $f: X \rightarrow Y$ is called almost continuous [56] if $f^{-1}(V)$ is open in $X$ for each regular open subset $V$ of $Y$.

Definition 4.2.3: A function $f: X \rightarrow Y$ is said to be regular set connected [20] if $f^{-1}(V)$ is clopen in $X$ for each regular open subset $V$ of $Y$. 
4.3 Almost $S_g^*$-Continuous Function

A new function called almost $S_g^*$-continuous functions in topological spaces are introduced and compared with almost continuous functions. Also its implications and compositions are discussed.

**Definition 4.3.1:** A mapping $f: X \to Y$ is said to be **almost $S_g^*$-continuous** if $f^{-1}(V)$ is $S_g^*$-open in $X$ for every regular open set $V$ in $Y$.

**Example 4.3.2:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Here $S_g^*O(X, \tau) = \{\emptyset, X, \{a\}, \{a, b\}\}$ Define $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Then $f$ is almost $S_g^*$-continuous.

**Theorem 4.3.3:** Every almost continuous function is almost $S_g^*$-continuous.

**Proof:** Let $f: X \to Y$ be a almost continuous function. Let $O$ be a regular open set in $Y$. Since $f$ is almost continuous, $f^{-1}(O)$ is open in $X$. Since every open set is $S_g^*$-open, $f^{-1}(O)$ is $S_g^*$-open in $X$. Hence $f: X \to Y$ is almost $S_g^*$-continuous.

**Remark 4.3.4:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.3.5:** Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{a, b, c\}\}$, $S_g^*O(X, \tau) = \{\emptyset, X, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b, c\}\}$, $RO(Y, \sigma) = \{\emptyset, Y, \{a\}, \{b, c\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = d$ and $f(d) = a$. Here $f^{-1}\{b, c\} = \{a, b\}$ which is $S_g^*O$ but not open. Hence $f$ is almost $S_g^*$-continuous but not almost continuous.
**Theorem 4.3.6:** If $f: (X, \tau) \to (Y, \sigma)$ is almost $S^*_g$-continuous and $(X, \tau)$ is a $S^*_g$-$T_{1/2}$ space, then $f$ is almost continuous.

**Proof:** Let $U$ be a regular open set in $Y$. Since $f$ is almost $S^*_g$-continuous, $f^{-1}(U)$ is $S^*_g$-open in $X$. Since $X$ is a $S^*_g$-$T_{1/2}$ space, $f^{-1}(U)$ is open in $X$. Hence $f$ is almost continuous.

**Theorem 4.3.7:** Every $S^*_g$-continuous function is almost $S^*_g$-continuous.

**Proof:** Let $f: X \to Y$ be a $S^*_g$-continuous function. Let $V$ be a regular open set in $Y$. Since every regular open set is open, $V$ is open in $Y$. Since $f$ is $S^*_g$-continuous, $f^{-1}(V)$ is $S^*_g$-open in $X$. Hence $f: X \to Y$ is almost $S^*_g$-continuous.

**Remark 4.3.8:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.3.9:** Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{b\}, \{c\}, \{a, b, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Here $S^*_gO(X, \tau) = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $RO(Y, \sigma) = \{\emptyset, Y\}$. Define $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Here $\{a, b\}$ is open in $Y$ but $f^{-1}\{a, b\} = \{a, b\}$ is not $S^*_g$-open in $X$. Hence $f$ is almost $S^*_g$-continuous but not $S^*_g$-continuous.

**Theorem 4.3.10:** Every $S^*_g$-irresolute function is almost $S^*_g$-continuous.

**Proof:** Let $f: X \to Y$ be a $S^*_g$-irresolute function. Let $V$ be a regular open set in $Y$. Since every regular open set is open, $V$ is open in $Y$. Also $V$ is $S^*_g$-open in $Y$. Since $f$ is $S^*_g$-irresolute, $f^{-1}(V)$ is $S^*_g$-open in $X$. Hence $f: X \to Y$ is almost $S^*_g$-continuous.

**Remark 4.3.11:** The converse of the above theorem need not be true as can be seen from the following example.
Example 4.3.12: Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b, c\}\}$ and
$\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Here $S^*_g O(X, \tau) = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$, $RO(Y, \sigma) = \{\emptyset, Y\}$ and $S^*_g O(Y, \sigma) = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly $f$ is almost $S^*_g$-continuous but not $S^*_g$-irresolute since $\{a, d\}$ is $S^*_g$-open in $Y$ but $f^{-1}\{a, d\} = \{a\}$ is not $S^*_g$-open in $X$.

Theorem 4.3.13: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions and $gof: (X, \tau) \rightarrow (Z, \eta)$ is a composition function. Then the following properties hold.

(i) If $f$ is $S^*_g$-irresolute and $g$ is almost $S^*_g$-continuous then $gof$ is almost $S^*_g$-continuous.

(ii) If $f$ is strongly $S^*_g$-continuous and $g$ is almost $S^*_g$-continuous then $gof$ is almost continuous.

(iii) If $f$ is $S^*_g$-continuous and $g$ is almost continuous then $gof$ is almost $S^*_g$-continuous.

(iv) If $f$ is slightly $S^*_g$-continuous and $g$ is regular set connected then $gof$ is almost $S^*_g$-continuous.

Proof:

(i) Let $F$ be a regular open set in $Z$. Since $g$ is almost $S^*_g$-continuous, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Also since $f$ is $S^*_g$-irresolute, $f^{-1}(g^{-1}(F))$ is $S^*_g$-open in $X$. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$. Hence $gof$ is almost $S^*_g$-continuous.

(ii) Let $F$ be a regular open set in $Z$. Since $g$ is almost $S^*_g$-continuous, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Since $f$ is strongly $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is open in $X$. Hence $gof$ is almost continuous.
(iii) Let $F$ be a regular open set in $Z$. Since $g$ is almost continuous, $g^{-1}(F)$ is open in $Y$. Since $f$ is $S_g^*$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S_g^*$-open in $X$. Hence $gof$ is almost $S_g^*$-continuous.

(iv) Let $F$ be a regular open set in $Z$. Since $g$ is regular set connected, $g^{-1}(F)$ is clopen in $Y$. Since $f$ is slightly $S_g^*$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S_g^*$-open in $X$. Hence $gof$ is almost $S_g^*$-continuous.
4.4 Almost Contra $S^*_g$-Continuous Function

In this section, almost contra $S^*_g$-continuous functions in topological spaces are introduced and its properties are investigated.

**Definition 4.4.1:** A mapping $f: X \to Y$ is said to be *almost contra $S^*_g$-continuous* if $f^{-1}(V)$ is $S^*_g$-closed in $X$ for every regular open set $V$ in $Y$.

**Example 4.4.2:** Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}\}$ and $S^*_gC(X, \tau) = \emptyset, X, \{b\}, \{c\}, \{b, c\}$ and $Y = \{a, b, c\}$ with $\sigma = \emptyset, Y, \{a\}, \{a, b\}$. Define $f: (X, \tau) \to (Y, \sigma)$ be an identity map. Then $f$ is almost $S^*_g$-continuous.

**Theorem 4.4.3:** Every contra $S^*_g$-continuous function is almost contra $S^*_g$-continuous.

**Proof:** Let $f: X \to Y$ be a contra $S^*_g$-continuous function. Let $U$ be a regular open set in $Y$. Since every regular open set is open, $U$ is open in $Y$. Since $f$ is contra $S^*_g$-continuous, $f^{-1}(U)$ is $S^*_g$-closed in $X$. Hence $f: X \to Y$ is almost contra $S^*_g$-continuous.

**Remark 4.4.4:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.4.5:** Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{b\}, \{d\}, \{b, d\}, \{a, b, c\}\}$, and $\sigma = \emptyset, Y, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}$. Here $S^*_gC(X, \tau) = \emptyset, X, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}$ and $RO(Y, \sigma) = \emptyset, Y, \{a\}, \{b, c, d\}$.

Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = d$ and $f(d) = a$. Clearly $f$ is almost contra $S^*_g$-continuous but not contra $S^*_g$-continuous because $\{a, b, c\}$ is open in $Y$ implies $f^{-1}\{a, b, c\} = \{a, b, d\}$ which is not $S^*_g$-closed in $X$. 
**Theorem 4.4.6:** If $f: (X, \tau) \to (Y, \sigma)$ is almost contra $S^*_g$-continuous and $(X, \tau)$ is $S^*_g$-locally indiscrete space, then $f$ is almost continuous.

**Proof:** Let $V$ be a regular open set in $Y$. Since $f$ is almost contra $S^*_g$-continuous, $f^{-1}(V)$ is $S^*_g$-closed in $X$. Since $X$ is a $S^*_g$-locally indiscrete space, $f^{-1}(V)$ is open in $X$. Hence $f$ is almost continuous.

**Theorem 4.4.7:** Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent.

(i) $f$ is almost contra $S^*_g$-continuous

(ii) The inverse image of every regular closed set in $Y$ is $S^*_g$-open in $X$.

**Proof:**

(i)$\implies$(ii): Let $F$ be a regular closed set in $Y$. Then $Y - F$ is regular open in $Y$. By (i), $f^{-1}(Y - F) = X - f^{-1}(F)$ is $S^*_g$-closed in $X$. This implies $f^{-1}(F)$ is $S^*_g$-open in $X$.

(ii)$\implies$(i): Let $G$ be a regular open set in $Y$. Then $Y - G$ is regular closed in $Y$. By (ii), $f^{-1}(Y - G) = X - f^{-1}(G)$ is $S^*_g$-open in $X$. This implies $f^{-1}(G)$ is $S^*_g$-closed in $X$. Hence $f$ is almost contra $S^*_g$-continuous.

**Theorem 4.4.8:** Let $f: (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent.

(i) $f$ is almost contra $S^*_g$-continuous

(ii) $f^{-1}(\text{Int}(\text{Cl}(A)))$ is $S^*_g$-closed in $X$ for every open set $A$ in $Y$.

(iii) $f^{-1}(\text{Cl}(\text{Int}(B)))$ is $S^*_g$-open in $X$ for every open subset $B$ of $Y$.

**Proof:**

(i)$\implies$(ii): Let $A$ be an open set in $Y$. Then $\text{Int}(\text{Cl}(A))$ is regular open in $Y$. By (i), $f^{-1}(\text{Int}(\text{Cl}(A)))$ is $S^*_g$-closed in $X$.

(ii)$\implies$(i): The proof is obvious.
(i) $\implies$(iii): Let $B$ be an open set in $Y$. Then $\text{Cl}(\text{Int}(B))$ is regular closed in $Y$. By (i), $f^{-1}(\text{Cl}(\text{Int}(B)))$ is $S_g^*$-open in $X$.

(iii)$\implies$(i): The proof is obvious.

**Theorem 4.4.9:** Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions. Then the following properties hold.

(i) If $f$ is $S_g^*$-irresolute and $g$ is almost contra $S_g^*$-continuous then $gof$ is almost contra $S_g^*$-continuous.

(ii) If $f$ is $S_g^*$-continuous and $g$ is perfectly continuous then $gof$ is almost contra $S_g^*$-continuous and almost $S_g^*$-continuous.

(iii) If $f$ is contra $S_g^*$-continuous and $g$ is almost continuous then $gof$ is almost contra $S_g^*$-continuous.

(iv) If $f$ is contra $S_g^*$-continuous and $g$ is regular set connected then $gof$ is almost $S_g^*$-continuous and almost contra $S_g^*$-continuous.

**Proof:**

(i) Let $F$ be a regular open set in $Z$. Since $g$ is almost contra $S_g^*$-continuous, $g^{-1}(F)$ is $S_g^*$-closed in $Y$. Also since $f$ is $S_g^*$-irresolute, $f^{-1}(g^{-1}(F))$ is $S_g^*$-closed in $X$. But $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$. Hence $gof$ is almost contra $S_g^*$-continuous.

(ii) Let $F$ be a regular open set in $Z$ implies $F$ is open in $Z$. Since $g$ is perfectly continuous, $g^{-1}(F)$ is clopen in $Y$. Since $f$ is $S_g^*$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S_g^*$-closed and $S_g^*$-open in $X$. Hence $gof$ is almost contra $S_g^*$-continuous and almost $S_g^*$-continuous.
(iii) Let $F$ be a regular open set in $Z$. Since $g$ is almost continuous, $g^{-1}(F)$ is open in $Y$. Since $f$ is contra $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S^*_g$-closed in $X$. Hence $gof$ is almost contra $S^*_g$-continuous.

(iv) Let $F$ be a regular open set in $Z$. Since $g$ is regular set connected, $g^{-1}(F)$ is clopen in $Y$. Since $f$ is contra $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S^*_g$-open and $S^*_g$-closed in $X$. Hence $gof$ is almost $S^*_g$-continuous and almost contra $S^*_g$-continuous.

**Theorem 4.4.10:** If a map $f: (X, \tau) \to (Y, \sigma)$ is almost contra $S^*_g$-continuous, almost continuous and $(X, \tau)$ is a $S^*_g-T_{1/2}$ space then $f$ is regular set connected.

**Proof:** Let $V$ be a regular open set in $Y$. Since $f$ is almost contra $S^*_g$-continuous and almost continuous, $f^{-1}(V)$ is $S^*_g$-closed and open in $X$. Since $(X, \tau)$ is a $S^*_g-T_{1/2}$ space, $f^{-1}(V)$ is closed. Thus $f^{-1}(V)$ is clopen in $(X, \tau)$. Hence $f$ is regular set connected.
4.5 Completely $S^*_g$-Irresolute Function

A new class of function called completely $S^*_g$-irresolute functions in topological spaces are introduced and certain characterizations of these functions are obtained.

**Definition 4.5.1:** A mapping $f: X \rightarrow Y$ is said to be completely $S^*_g$-irresolute if the inverse image of each $S^*_g$-open subset of $Y$ is regular open in $X$.

**Example 4.5.2:** Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$. Here $RO(X, \tau) = \{\emptyset, X\}$, $S^*_g O(Y, \sigma) = \{\emptyset, Y, \{a, b\}\}$. Define a map $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = f(c) = b$. Then $f$ is completely $S^*_g$-irresolute.

**Theorem 4.5.3:** Every completely $S^*_g$-irresolute function is $S^*_g$-irresolute.

**Proof:** Let $f: X \rightarrow Y$ be a completely $S^*_g$-irresolute function. Let $U$ be a $S^*_g$-open set in $Y$. Since $f$ is completely $S^*_g$-irresolute, $f^{-1}(O)$ is regular open in $X$. Since every regular open set is open, $f^{-1}(U)$ is open in $X$. Also since every open set is $S^*_g$-open, $f^{-1}(U)$ is $S^*_g$-open in $X$. Hence $f: X \rightarrow Y$ is $S^*_g$-irresolute.

**Remark 4.5.4:** The converse of the above theorem need not be true as can be seen from the following example.

**Example 4.5.5:** Let $X = Y = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\} = S^*_g O(X, \tau)$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\} = S^*_g O(Y, \sigma)$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c$ and $f(c) = b$. Here $\{a, b\}$ is $S^*_g$-open in $(Y, \sigma)$ but $f^{-1}\{a, b\} = \{a, c\}$ is $S^*_g$-open in $(X, \tau)$ but not regular open in $(X, \tau)$. Hence $f$ is $S^*_g$-irresolute but not completely $S^*_g$-irresolute.
Theorem 4.5.6: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be a function. Then the following are equivalent.

(i) \( f \) is completely \( S^*_g \)-irresolute.

(ii) \( f^{-1}(F) \) is regular closed set in \( X \) for each \( S^*_g \)-closed set \( F \) in \( Y \).

Proof:

(i)\( \Rightarrow \)(ii): Let \( F \) be a \( S^*_g \)-closed set in \( Y \). Then \( Y - F \) is \( S^*_g \)-open in \( Y \). By (i), \( f^{-1}(Y - F) = X - f^{-1}(F) \) is regular open in \( X \). This implies \( f^{-1}(F) \) is regular closed in \( X \).

(ii)\( \Rightarrow \)(i): Let \( G \) be a \( S^*_g \)-open set in \( Y \). Then \( Y - G \) is \( S^*_g \)-closed in \( Y \). By (ii), \( f^{-1}(Y - G) = X - f^{-1}(G) \) is regular closed in \( X \). This implies \( f^{-1}(G) \) is regular open in \( X \). Hence \( f \) is completely \( S^*_g \)-irresolute.

Theorem 4.5.7: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions and \( gof: (X, \tau) \rightarrow (Z, \eta) \) is a composition function. Then the following properties hold.

(i) If \( f \) is completely \( S^*_g \)-irresolute and \( g \) is \( S^*_g \)-irresolute then \( gof \) is completely \( S^*_g \)-irresolute.

(ii) If \( f \) is \( S^*_g \)-continuous and \( g \) is completely \( S^*_g \)-irresolute then \( gof \) is \( S^*_g \)-irresolute.

(iii) If \( f \) is strongly \( S^*_g \)-continuous and \( g \) is completely \( S^*_g \)-irresolute then \( gof \) is strongly \( S^*_g \)-continuous.

(iv) If \( f \) is almost \( S^*_g \)-continuous and \( g \) is completely \( S^*_g \)-irresolute then \( gof \) is \( S^*_g \)-irresolute.

(v) If \( f \) is perfectly \( S^*_g \)-continuous and \( g \) is completely \( S^*_g \)-irresolute then \( gof \) is perfectly \( S^*_g \)-continuous.

(vi) If \( f \) is regular set connected and \( g \) is completely \( S^*_g \)-irresolute then \( gof \) is perfectly \( S^*_g \)-continuous.
Proof:

(i) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is $S^*_g$-irresolute, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Since $f$ is completely $S^*_g$-irresolute, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is regular open in $X$. Hence $gof$ is completely $S^*_g$-irresolute.

(ii) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is completely $S^*_g$-irresolute, $g^{-1}(F)$ is regular open in $Y$. Since every regular open set is open, $g^{-1}(F)$ is open in $Y$. Since $f$ is $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S^*_g$-open in $X$. Hence $gof$ is $S^*_g$-irresolute.

(iii) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is completely $S^*_g$-irresolute, $g^{-1}(F)$ is regular open in $Y$. Since every regular open implies open and open implies $S^*_g$-open, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Since $f$ is strongly $S^*_g$-continuous, $f^{-1}(g^{-1}(F))$ is open in $X$. Hence $gof$ is strongly $S^*_g$-continuous.

(iv) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is completely $S^*_g$-irresolute, $g^{-1}(F)$ is regular open in $Y$. Since $f$ is almost $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is $S^*_g$-open in $X$. Hence $gof$ is $S^*_g$-irresolute.

(v) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is completely $S^*_g$-irresolute, $g^{-1}(F)$ is regular open in $Y$. Since every regular open implies open and open implies $S^*_g$-open, $g^{-1}(F)$ is $S^*_g$-open in $Y$. Since $f$ is perfectly $S^*_g$-continuous, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is open and closed in $X$. Hence $gof$ is perfectly $S^*_g$-continuous.

(vi) Let $F$ be a $S^*_g$-open set in $Z$. Since $g$ is completely $S^*_g$-irresolute, $g^{-1}(F)$ is regular open in $Y$. Since $f$ is regular set connected, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(F)$ is clopen in $X$. Hence $gof$ is perfectly $S^*_g$-continuous.
Lemma 4.5.8[38]: Let $S$ be an open subset of a space $(X, \tau)$. Then the following hold.

(i) If $U$ is regular open in $(X, \tau)$, then so is $U \cap S$ in the subspace $(S, \tau_s)$.

(ii) If $B \subset S$ is regular open in $(S, \tau_s)$, then there exists a regular open set $U$ in $(X, \tau)$, such that $B = U \cap S$.

Theorem 4.5.9: If $f: (X, \tau) \to (Y, \sigma)$ is completely $S^*_g$- irresolute and $A$ is any open subset of $X$, then the restriction $f_{/A}: (A, \tau_A) \to (Y, \sigma)$ is completely $S^*_g$- irresolute.

Proof: Let $V$ be a $S^*_g$- open subset of $Y$. By hypothesis, $f^{-1}(V)$ is regular open in $X$. Since $A$ is open in $X$, it follows from Lemma 4.5.8 that $(f_{/A})^{-1}(V) = f^{-1}(V) \cap A$ is regular open in $A$. Hence $f_{/A}$ is completely $S^*_g$- irresolute.

Remark 4.5.10: From the above results of this chapter we have the following diagrams.

1. completely $S^*_g$- irresolute $\rightarrow$ $S^*_g$- irresolute $\rightarrow$ $S^*_g$- continuous $\rightarrow$ Almost continuous $\rightarrow$ Almost $S^*_g$- continuous

2. Contra $S^*_g$- continuous $\rightarrow$ Almost contra $S^*_g$- continuous